Quiz 5

1 Numbers game

Consider the Numbers game with one-sided interval hypotheses $h_{\leq x}$ for numbers 1 up to 10, where $h_{\leq x} = \{1, 2, \dots, x\}$, $x \in \{1 \dots 10\}$. Assume we have a uniform prior over h. Suppose $S = \{5, 9\}$.

1. What is the plug-in approximation of the posterior predictive distribution for new data point x?
2. What is the full posterior predictive distribution?

2 Reject option

In many classification problems, one has the option either of assigning x to class j or, if you are too uncertain, of choosing a "reject" option. If the cost for rejects is less than the cost of falsely classifying the object, it may be the optimal action. Let α_i mean you choose action i, for $i = 1 : C + 1$, where C is the number of classes and $C + 1$ is the reject action. Let $Y = j$ be the true (but unknown) state of nature. Define the loss function as follows

$$\lambda(\alpha_i | Y = j) = \begin{cases} 0 & \text{if } i = j \text{ and } i, j \in \{1, \dots, C\} \\ \lambda_r & \text{if } i = C + 1 \\ \lambda_s, & \text{otherwise} \end{cases}$$

In other words, you incur 0 loss if you correctly classify, you incur λ_r loss (cost) if you choose the reject option, and you incur λ_s loss (cost) if you make a substitution error (misclassification). Both λ_r and λ_s are positive numbers.

1. Show that when we decide to choose a class (and not reject), we always pick the most probable one.
2. Show that the minimum expected loss is obtained if we decide to pick the most probable class $j_{max} = \arg\max_j p(Y = j|x)$ and if $p(Y = j_{max}|x) \geq 1 - \frac{\lambda_r}{\lambda_s}$; otherwise we decide to reject.
3. Describe qualitatively what happens as λ_r/λ_s is increased from 0 to 1 (i.e., the relative cost of rejection increases).