Quiz 4

1 Exam scores

You are investigating the relationship between students' study hours and their exam scores. You suspect that there is an unobserved latent variable, the students' innate ability, which influences both their study hours and exam scores. You model the latent ability as a Gaussian variable with a mean of 0 and a standard deviation of 1. The study hours and exam scores are also modeled as Gaussian variables, given the latent ability.

The study hours, given the latent ability, follow a Gaussian distribution with a mean of 2 times the latent ability and a standard deviation of 1.5. The exam scores, given the latent ability, follow a Gaussian distribution with a mean of 3 times the latent ability and a standard deviation of 2.

If a student's exam score is observed to be 75, and the student spent 30 hours studying, use Bayes' rule to estimate the posterior distribution of the latent ability.

- Write down the expressions for the likelihood functions, prior probabilities, and posterior probabilities in this context.
- Calculate the posterior distribution of the latent ability given the observed exam score and study hours.

2 Numbers game

Consider the Numbers game with one-sided interval hypotheses $h_{\leq x}$ for numbers 1 up to 10, where $h_{\leq x} = \{1, 2, \dots, x\}$, $x \in \{1 \dots 10\}$. Assume we have a uniform prior over h.

1. Show that the $h_{mle} = h_{\leq \max(S)}$ for a given set of numbers $S = \{x_1, x_2, \dots\}$.
2. Briefly say why the MAP estimate = MLE.