Quiz 3

1 Pairwise independence does not imply mutual independence

We say that two random variables are pairwise independent if

$$P(X_2|X_1) = P(X_2)$$

and hence

$$P(X_2, X_1) = P(X_1)P(X_2|X_1) = P(X_2)P(X_1).$$

We say that n random variables are mutually independent if

$$P(X_i|X_S) = P(X_i) \quad \forall S \subseteq \{1, \ldots n\} \setminus \{i\}.$$

Show that pairwise independence between all pairs of variables does not necessarily imply mutual independence. It suffices to give a counterexample.

2 Traffic on a road

Suppose we want to model traffic on a given road. At each hour i, we measure the number of cars y_i. We think the number of cars is well modeled by a Gaussian distribution. However, we know that some times are rush hour, so we decide to use a mixture of Gaussian distributions with two components (normal and rush hour traffic respectively.) For simplicity, imagine that traffic at each hour is independent of one another.

Draw a graphical model that represents this mixture model. You should use plate notation and include variables $\mathbf{z} = (z_1, ..., z_n)$ and $\mathbf{y} = (y_1, ..., y_n)$ as well as the parameters of the distributions, π, μ_1, μ_2, σ_1 and σ_2 in your graph. (z_i is a traffic type at i-th hour. π is a parameter of a Bernoulli distribution specifying normal vs rush hours. μ_is and σ_is are parameters of Gaussian distributions associated with each traffic type.)