

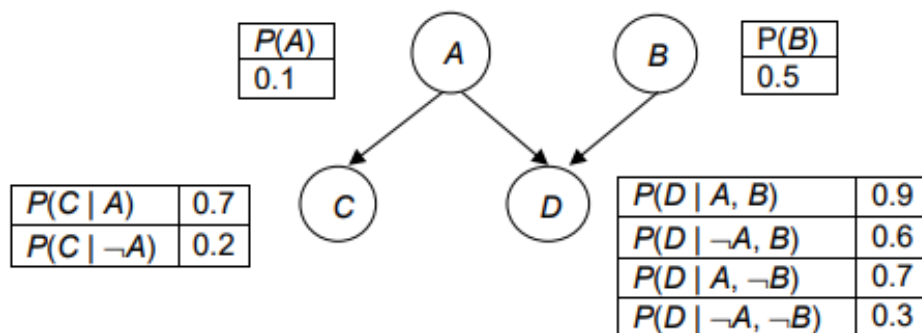
## Quiz 1

### 1 Basketball

Shanille O'Keal shoots free throws on a basketball court with 0.7 accuracy. What is the probability she hits exactly 70 of her first 100 shots? (A mathematical expression that could be plugged into a calculator is sufficient; you do not need to give the number itself.)

### 2 Vancouver weather

Consider the following Bayesian Network containing four Boolean random variables: global warming ( $A$ ), clear sky over Vancouver ( $B$ ), ice melting in arctic ( $C$ ), and high temperature in Vancouver ( $D$ ). Each variable can be either True or False. The probability that variable  $X$  is true is written  $P(X)$ ; the probability that  $X$  is false is written  $P(\neg X) = 1 - P(X)$ .



1. Compute  $P(A|C)$
2. Compute  $P(\neg A, B, \neg C, D)$

### 3 Logarithms

Prove the following:

$$\frac{1}{n} \sum_{i=1}^n \log(a_i) = \log \left( \prod_{i=1}^n \sqrt[n]{a_i} \right) \quad (1)$$

### 4 Matrix calculus

Let  $y \in \mathbb{R}^n$  be a  $n$ -dimensional vector, let  $w \in \mathbb{R}^m$  be a  $m$ -dimensional vector, and let  $\mathbf{X} \in \mathbb{R}^{n \times m}$  be an  $n$ -by- $m$  matrix. Define

$$f(w) = (y - \mathbf{X}w)^\top (y - \mathbf{X}w) \quad (2)$$

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1. Find the multivariate derivative of  $f(w)$  with respect to  $w$ .
2. Find the vector  $w$  that minimizes  $f(w)$ . Hint: Try finding the value of  $w$  such that the derivative equals  $\mathbf{0}$ , the vector of all zeroes.

For the purposes of this question, you may assume that the inverse of any square matrix exists.

(You might recognize your solution as the solution to a least-squares regression problem.)