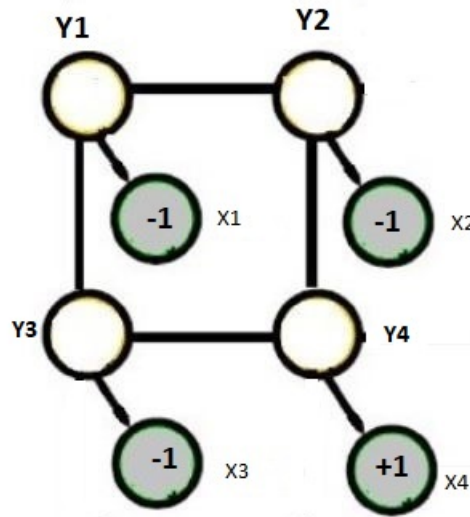


1 MCMC for Ising model

In this question, we will be considering an Ising Model with 4 hidden variables, as shown in the following figure.



Recall that in an Ising model, the variables can take on only the values $\{-1, +1\}$. Suppose we observe that $X_1 = X_2 = X_3 = -1$ and $X_4 = +1$.

1. The factor over every adjacent pairs of Y_i 's is: $\psi_{ij}(y_i, y_j) = \exp(y_i y_j)$, i.e., $\psi_{ij} = \begin{pmatrix} e & e^{-1} \\ e^{-1} & e \end{pmatrix}$ and the evidence factor is $\psi_i(y_i) = \exp(x_i y_i)$. Calculate the posterior belief on Y_4 after one iteration of loopy belief propagation. You do not need to normalize your final result.
2. Suppose we want to use the Metropolis Hastings Algorithm to sample from this Ising Model. We use a transition function that samples all hidden variables uniformly at random. Our initial sample is: $y_1 = y_2 = +1$ and $y_3 = y_4 = -1$.

Which of the following proposed transitions are guaranteed to be accepted?

- (a) $y_1 = y_2 = y_4 = +1$ and $y_3 = -1$
- (b) $y_1 = y_2 = y_3 = y_4 = -1$
- (c) $y_1 = y_2 = y_3 = +1$ and $y_4 = -1$
- (d) $y_1 = +1$ and $y_2 = y_3 = y_4 = -1$