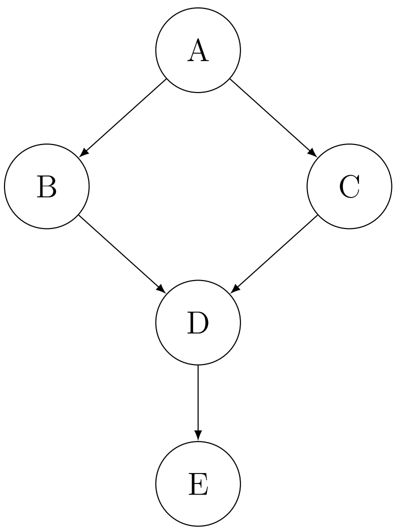
Quiz 10

CMPT 727
Spring 2026

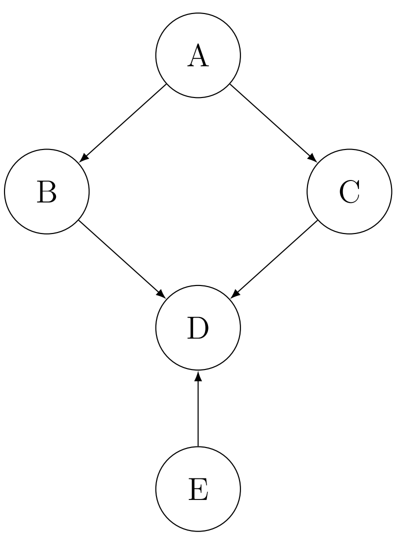
1 Markov equivalence

Consider the following two DAGs over variables $\{A, B, C, D, E\}$:

Graph G_1:



Graph G_2:

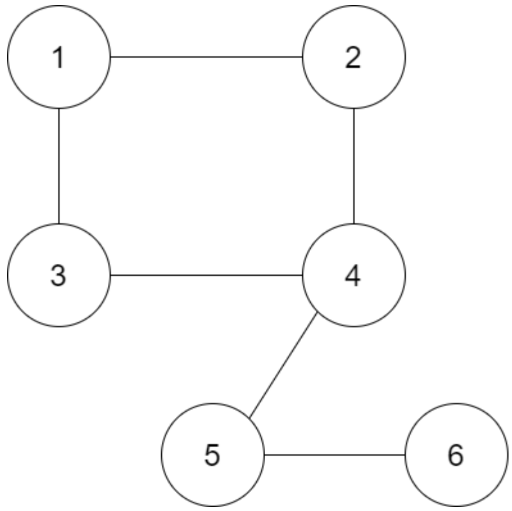


1. List all v-structures in G_1 and in G_2.
2. Are G_1 and G_2 Markov equivalent? I.e. do they express the same conditional independence assertions? Justify your answer.

Quiz 10

Variable elimination in a MRF

Consider the MRF in the following Figure.



1. Suppose we want to compute the partition function using the elimination ordering $\prec = (1, 2, 3, 4, 5, 6)$, i.e.,

$$\sum_{x6}\sum_{x5}\sum_{x4}\sum_{x3}\sum_{x2}\sum_{x1} \psi_{12}(x_1, x_2)\psi_{13}(x_1, x_3)\psi_{24}(x_2, x_4)\psi_{34}(x_3, x_4)\psi_{45}(x_4, x_5)\psi_{56}(x_5, x_6)$$

   If we use the variable elimination algorithm, we will create new intermediate factors. What is the largest intermediate factor?

2. Add an edge to the original MRF between every pair of variables that end up in the same factor. (These are called fill in edges.) Draw the resulting MRF. What is the size of the largest maximal clique in this graph?

3. Now consider elimination ordering $\prec = (4, 1, 2, 3, 5, 6)$, i.e.,

$$\sum_{x6}\sum_{x5}\sum_{x3}\sum_{x2}\sum_{x1}\sum_{x4} \psi_{12}(x_1, x_2)\psi_{13}(x_1, x_3)\psi_{24}(x_2, x_4)\psi_{34}(x_3, x_4)\psi_{45}(x_4, x_5)\psi_{56}(x_5, x_6)$$

   If we use the variable elimination algorithm, we will create new intermediate factors. What is the largest intermediate factor?

4. Add an edge to the original MRF between every pair of variables that end up in the same factor. (These are called fill in edges.) Draw the resulting MRF. What is the size of the largest maximal clique in this graph?