Assignment 10

1 Treewidth of a order-k Markov model

An order-k Markov model is a probablistic model over variables $X_1, \ldots X_n$, where X_i depends on the k variables $X_{i-k}..X_{i-1}$.

1. What is the treewidth of an order-k Markov model?
2. Your colleague wants to do exact inference in a Markov model with $k = 100$. Explain why this is computationally expensive.
3. They suggest that to make inference easier while retaining long-range dependencies, they instead use a Markov-like model where X_i depends only on X_{i-1} and X_{i-k}, thus reducing the number of edges in the network by a factor of 50. Explain why this does not meaningfully improve performance.

2 The min-neighbors heuristic for variable elimination

Finding the best variable elimination ordering in an arbitrary graph is an NP-hard problem. A common heuristic is the *min-neighbors* (or *min-degree*) heuristic: at each step, eliminate the variable that currently has the fewest neighbors in the graph, adding fill edges to make its neighbors into a clique, then remove it.

Recall that the *elimination width* of an ordering is the maximum degree of any vertex at the time it is eliminated. The *treewidth* of a graph is the minimum elimination width over all orderings.

1. Prove that the min-neighbors heuristic achieves the optimal elimination width (i.e., the treewidth) on any tree (i.e. graph with no cycles).
2. Consider the following graph G on 7 vertices. The vertices $\{a, b, c\}$ form a triangle (K_3). Three additional vertices x_1, x_2, x_3 are each connected to all of a, b, c and to a shared vertex v. The vertex v is connected only to x_1, x_2, x_3. The graph is shown below.

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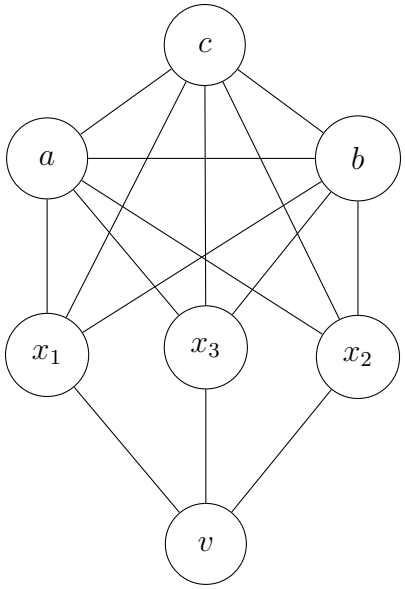

Prove that the min-neighbors heuristic is suboptimal on this graph. That is, show that the largest intermediate factor produced by min-neighbors is strictly larger than the largest intermediate factor produced by some other ordering.