CMPT 733 Hypothesis Testing

Instructor Zhengjie Miao

Course website https://coursys.sfu.ca/2025sp-cmpt-733-gl/pages/

Based on the slides by Jiannan Wang

Why Hypothesis Testing?

We want to make a claim from our data

But, data is just a sample

How to prove our claim in this situation?

Using Hypothesis Testing

Example:

- Claim: A data scientist earns more money than a data engineer
- Data: A sample of 50 data scientists and 50 data engineers
- Result: 100k vs. 70k

that our claim is correct?

More examples

Example:

Send out surveys to MPCS students about the program. Are the survey results significant?

Design a new MPCS course on GenAl. 20 students take the course. Is the new course helpful?

Hypothesis Testing

Equivalent Terms

- Hypothesis == Claim
- Hypothesis Testing == Claim Proving

Key Idea

Prove by contradiction

Analogy

- How to prove: There exists no smallest positive rational number.
- Hint: a rational number is any number that can be expressed as the fraction a/b of two integers

Alternative and Null Hypotheses

Alternative Hypothesis (H_a)

- The claim that you want to prove it's correct (e.g., the new course is helpful)
- Example H_a : $\mu \neq \mu_0$ (two-tailed), $\mu < \mu_0$ (left-tailed), $\mu > \mu_0$ (right-tailed)

Null Hypothesis (H₀)

- The opposite side of H_a (e.g., if our results are due to a random chance)
- Example H_0 : $\mu = \mu_{0}$, $\mu_1 = \mu_2$

Possible Outcomes

- Reject H_0 (a contradiction is found) \rightarrow Accept H_a
- Fail to reject H₀ (no contradiction is found)

Example

Alternative Hypothesis (H_a)

- $^{\circ}$ A data scientist earns more money than a data engineer NULL Hypothesis (H_0)
- A data scientist earns less (or equal) money than a data engineer If H_0 is true, what's the probability of seeing:
 - Data Scientist (100 K) vs. Data Engineer (70 K)

This is called P-value

Salary(Data Scientist) – Salary(Data Engineer) >= 30 -

Make a decision based on p-value

We hope that

• p-value is as low as possible so that we can reject H_0 (i.e., accept H_a)

Level of Significance (e.g., $\alpha = 0.01, 0.05, 0.1$)

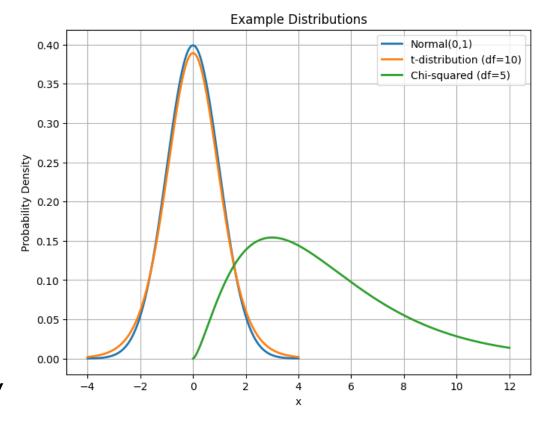
- How low do we want p-value to be?
- P-value $< \alpha$: reject null hypothesis
- P-value > α : not enough evidence to reject null hypothesis
 - does not mean we can accept null hypothesis

Level of Confidence (e.g., $c = 1 - \alpha = 99\%$)

• How confident are we in our decision?

How to calculate p-value

- I. Select Alternative Hypothesis
 - Two-tailed, Right-tailed, Left-tailed
- 2. Determine Test Statistic Distribution
 - e.g., Normal, Student's t, Chi-squared, etc.
- 3. Specify Degrees of Freedom (if applicable)
 - e.g., t-distribution and Chi-squared tests
- 4. Calculate the test statistic from sample data
 - Use the CDF to find the prob. of observing an extreme value under the null hypothesis
 - For two-tailed tests, multiply the one-tail probability by 2



Example

Suppose in the survey, 60% of 100 randomly sampled MPCS alumni said CMPT 733 is helpful for their job

- Null hypothesis H_0 : at most half of MPCS students find CMPT 733 helpful
- Alternative hypothesis H_a : the majority of MPCS students find CMPT 733 helpful
- p-value: the probability that 60% of 100 random students find CMPT
 733 helpful if at most 50% of all students find it helpful

Example

p = probability that 60% of 100 random students respond CMPT 733 Population Mean (null hypothesis)

The **z-statistic** formula, using central limit theorem

Sample mean $z = \frac{\bar{X}_n - \mu}{\sigma/\sqrt{n}} \qquad z \approx \frac{0.6 - 0.5}{0.5/\sqrt{100}} \approx 2.00$ Standard Deviation

• What is the probability of drawing > 2.00 from a standard normal distribution?

```
print(1-scipy.stats.norm.cdf(2.00))

     0.0s

0.02275013194817921
```

- p-value is ~ 0.02 , we reject the null hypothesis
- Strong support that majority of students find
 733 helpful

Smaller Example

p = probability that 60% of 30 random students respond CMPT 733helpful

The **z-statistic** formula, using central limit theorem

$$t = \frac{\bar{X}_n - \mu}{\sigma / \sqrt{n}}$$

$$t = \frac{\bar{X}_n - \mu}{\sigma/\sqrt{n}} \qquad \qquad t \approx \frac{0.6 - 0.5}{0.5/\sqrt{30}} \approx 1.10$$

• What is the probability of drawing > 1.10 from a standard normal distribution?

```
print(1-scipy.stats.norm.cdf(1.10))
    0.0s
0.13566606094638267
```

We CANNOT reject null

hypothesis is ~13.57%

p-value is ~ 0.1357, prob under null

Example with unknown standard deviation

Population Mean

p = probability that 60% of 100 random students respond CMPT 733

helpful

The **t-statistic** formula

Sample mean Sample Standard Deviation

(null hypothesis)

• What is the probability of drawing > 2.04 from a Student's t Distribution with n-I degrees of freedom?

```
print(1-scipy.stats.t.cdf(2.04, 99))
    0.0s
                        Degree of freedom
0.022006777804037148
```

- p-value is ~ 0.02, we reject the null hypothesis
- Strong support that majority of students find 733 helpful

p-Hacking (Cheating on a p-Value)

Common Mistakes

- I. Collect data until the hypothesis testing is passed
- 2. Keep doing analysis on the same data until you find something significant

Solution

- You should know what you're looking for $(H_0 \text{ and } H_a)$ before you start
- Decrease the level of significance (e.g., $\alpha/2$ for two hypothesis tests on the same data)

Pairwise t- tests

Example: You have three treatment groups (A, B, C) and want to determine if the differences in their means are statistically significant Pairwise Comparisons:

- Compare A vs. B
- Compare A vs. C
- Compare B vs. C

Multiple Hypothesis Testing

- Multiple simultaneous statistical tests, each of which has a potential to produce a "discovery"
- A stated confidence level (e.g., I-0.05) generally applies only to each test considered individually
 - When running multiple tests simultaneously, the chance of at least one false positive increases, meaning the overall error rate is higher than 5%.
- Without adjustment, multiple tests can lead to false discoveries
 - Incorrectly identifying a statistically significant effect when there isn't one.

Multiple Hypothesis Testing

- Example: Imagine a clinical trial where researchers evaluate the effectiveness of a new drug. Instead of testing just one outcome, they examine several outcomes simultaneously
 - e.g., Blood Pressure Cholesterol Levels, Heart Rate, Body Weight
- Each separate hypothesis test is performed to determine if the drug has a statistically significant effect at a 95% confidence level (α = 0.05)
- However, because multiple tests are performed, the chance of making at least one false positive (a type I error) across all tests increases.

Multiple Hypothesis Testing

Example: There are 20 options we are interested in as independent (predictor) features for your model. For each feature, we use a hypothesis test with level of significance 0.05

What's the probability of having one significant result just due to chance? (Recall that probability of making a type I error is α)

$$I - (I-0.05)^{20} = 0.64$$

P-hacking: Pairwise t-tests

```
num_datasets, sample_size = 100, 100
  samples = np.random.normal(loc=0, size=(num_datasets, sample_size))
   total_pairs, total_siginificant = 0, 0
   for i in range(num_datasets):
       for j in range(i+1, num_datasets):
           p_value = scipy.stats.ttest_ind(samples[i,:], samples[j,:])[1]
           total pairs += 1
           if p value < 0.05:
               total_siginificant += 1
   print(total_siginificant, total_pairs)
 ✓ 3.6s
286 4950
```

Useful scipy.stats functions

Independent Sample Tests

Independent sample tests are typically used to assess whether multiple samples were independently drawn from the same distribution or different distributions with a shared property (e.g. equal means).

Some tests are specifically for comparing two samples.

ttest_ind_from_stats (mean1, std1, nobs1,)	T-test for means of two independent samples from descriptive statistics.
<pre>poisson_means_test</pre> (k1, n1, k2, n2, *[,])	Performs the Poisson means test, AKA the "E-test".
<pre>ttest_ind (a, b, *[, axis, equal_var,])</pre>	Calculate the T-test for the means of <i>two</i> independent samples of scores.

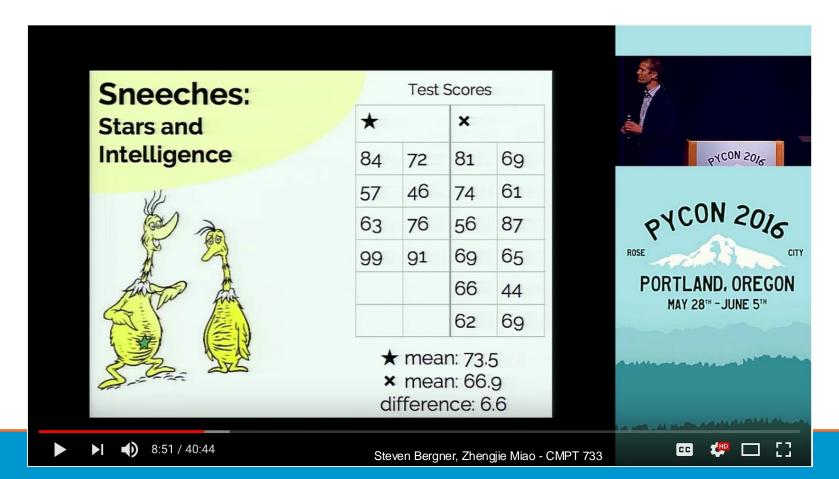
https://docs.scipy.org/doc/scipy/reference/stats.html

Permutation Test

- Resampling Data: Repeatedly "permute" the labels or values in your dataset
 - E.g., if comparing two groups, shuffle the group labels
- Computing a Test Statistic: For each permutation, calculate a test statistic (e.g., the difference in means between groups)
- Estimating the p-value: compare the observed test statistic from the actual data to this permutation distribution
- When to use: does not require the assumptions of normality or equal variances

Permutation Test

https://youtu.be/lq9DzN6mvYA?t=8m9s



Hypothesis Test Using Simulation

- Generate a large number of datasets under the null hypothesis
 - E.g., drawing samples from a known distribution or resampling (as in permutation testing)
- Calculate the test statistic for observed data
- P-value is the empirical probability of seeing a test statistic as extreme as the observed statistic in the simulated datasets
- Recall Monte Carlo methods / Bootstrap
- When to use: no assumptions of normality or equal variances; small sample sizes

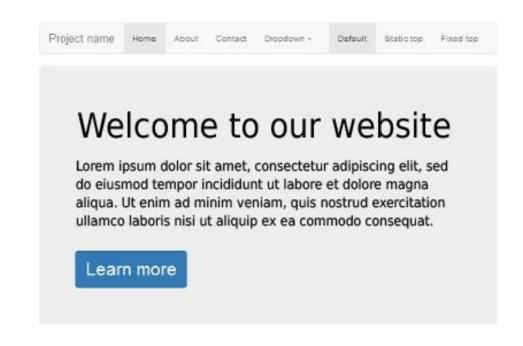
Hypothesis Test Using Simulation

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A/B Testing

What UI is better?





Surprising A/B Tests

A. Get \$10 off the first purchase. Book online now!

B. Get an additional \$10 off. Book online now.



https://www.wordstream.com/blog/ws/2012/09/25/a-b-testing

A/B Testing

- A method to compare two versions (A and B) of a variable to determine which performs better
 - Optimize website design, marketing campaigns, user experience, etc.
- Split users randomly between the two versions and measure the differences in performance

Why A/B Testing?

- Data-Driven Decisions: Replace guesswork with statistical evidence
- Improved Conversion Rates: Identify changes that boost user engagement
- Reduced Risk: Test ideas on a small scale before full implementation
- Continuous Improvement: Iteratively refine products and strategies

A/B Testing Steps

- I. Metrics identification
 - I. Invariant metrics (e.g., # users, user demographics, etc.)
 - 2. Evaluation metrics (what you want to optimize, e.g., CTR, # Orders)

2. Hypothesis definition

	H0 is True	H0 is False
Reject H0	Type I error (False Positive) Prob = α	True Positive Prob = 1- β
Fail to reject H0	True Negative Prob = $1-\alpha$	Type II error (False Negative) Prob = β

• Usually target $\alpha = 0.05$, $\beta = 0.8$

A/B Testing Steps

3. Determine sample size & duration

$$n = \frac{16\sigma^2}{\delta^2}$$

Duration: If the daily active users is greater than 2n, can we finish A/B testing in one day?

- Usually not, because:
 - Bug in your new version
 - Other variants like weekends/holidays/discounts

A/B Testing Steps

- 5. Distribute users
- 6. Invariance test / sanity check
 - Metrics that should not change between variant A & variant B
 - User demographics, traffic sources, device types, ...
 - When it fails:
 - Review randomization
 - External factors
 - Drill down to segments

What can/cannot be A/B Tested?

- New feature of a product
- UI changes
- Backend changes

Question: can we A/B test to decide if ChatGPT should be released?

Question: can we A/B test the new UI of a house selling website

- when the evaluation metric is traffic volume?
- when the evaluation metric is # of transactions?

Conclusion

- Hypothesis Testing
- Null Hypothesis (H₀) and Alternative Hypothesis (H_a)
- P-value and P-hacking
- A/B Testing

CMPT 733 Causal Inference

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Based on the slides by Yi Xie, Nathan Yan, and Jiannan Wang

Outline

Why Should Data Scientists Care?

Basic Concepts

Causal Inference

Questions Data Scientists Can Answer

Is This A or B?	Classification
How much or How Many?	Regression
Is This Weird?	Anomaly Detection
How Is This Organized?	Clustering
What if?	Causal Inference

From Prediction to Causation

Predicting user activity for Xbox

- Y = logins in next month
- X = logins in past month, number of friends,...

What if we increase the number of friends?

Would it increase user activity?

Maybe or May be not (!)

Number of Xbox A causes B Friends Activity B causes A Xbox Number of Activity Friends User interest in gaming C causes A and B Number of Xbox Friends Activity

A/B Testing Helps!

Treatment Group

- A random sample of users
- Launch a campaign to increase friends
- Average activity next month

Control Group

- A random sample of users
- Not Launch a campaign to increase friends
- Average activity next month

Hypothesis Testing

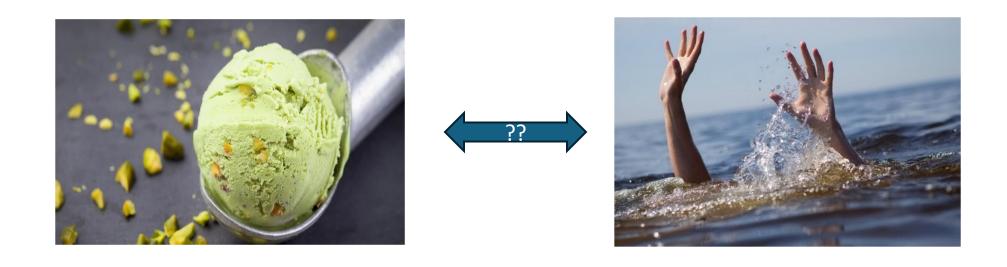
A/B Testing Does Not Work

- It is infeasible to do A/B testing
 - What if you went to UBC rather than SFU, would it help you land a better job?

- It is unethical to do A/B testing
 - What if subscription price is set to \$69 rather than \$99, would it increase revenue?

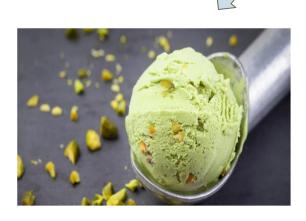
Example 1: Causality ≠ Correlation

When a team was investigating the city death rate data, they found that reported cases of teenage drowning death increase while the sales of ice cream also increase.



Example 1: Causality ≠ Correlation







Example 2: Causality ≠ Correlation

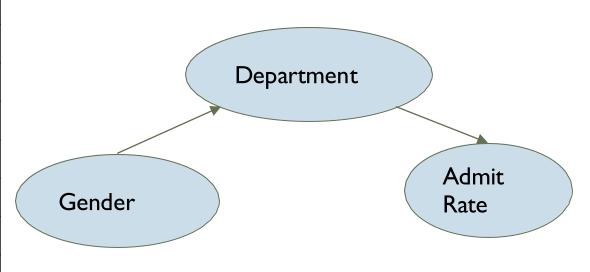
Is Berkeley gender biased?

	Applicants	Admitted
All	12,763	41%
Men	8,442	44%
Women	4,321	35%

Simpson's paradox

Example 2: Causality ≠ Correlation

Donartment	Me	en	Women	
Department	Applicants	Admitted	Applicants	Admitted
Α	825	62%	108	82%
В	560	63%	25	68%
С	325	37%	593	34%
D	417	33%	375	35%
E	191	28%	393	24%
F	373	6%	341	7%
Total	2691	45%	1835	30%



Example 2: Causality ≠ Correlation



- In city areas with nearby trees and natural landscapes, there is less domestic violence.
- Apartment complexes with many trees had 52% fewer crimes.
- On tree-lined streets, people drive more slowly, reducing accident risk.

Outline

Why Should Data Scientists Care?

Basic Concepts

- Outcome / Treatment Variables
- Intervention and Do Operation
- Counterfactual
- Causal Graph

Causal Inference

Outcome / Treatment Variables

What if participating Study Program, would it improve Grade?

Treatment

Outcome

Student	Gender	Class	Study Program	Grade
Jacky	male	I	0	78
Terry	male	I	I	82
Mary	female	I	0	86
Sarah	female	2		83

Intervention and Do Operation

Do operator will signal the experimental intervention (invented by Judea Pearl)

P(grade | do(enrolled in study program))

represents the distribution of grad if the person is enrolled in study program

Intervention and Do Operation

P(A | B = b): probability of A being true given that B is observed as B = b

❖ P(A | do(B) = b): probability of A being true given an intervention that sets B to b

Counterfactual

What would have happened if I had changed "Treatment Variable"

Student	Gender	Class	Study Program	Actual Grade
Jacky	male	1	0	78
Terry	male	I	I	82
Mary	female	I	0	86
Sarah	female	2	I	83

Counterfactual

What would have happened if I had changed "Treatment Variable"

Student	Gender	Class	Study Program	Actual Grade	(You cannot get it in reality)
Jacky	male	I	0	78	82
Terry	male	I	1	82	82
Mary	female	I	0	86	90
Sarah	female	2	I	83	85

Causality

Causality Definition

 The difference between actual outcome and counterfactual outcome

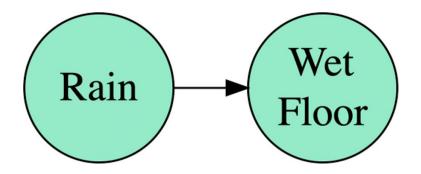
The Fundamental Problem of Causal Inference

We cannot observe the counterfactual outcome

Causal Graph

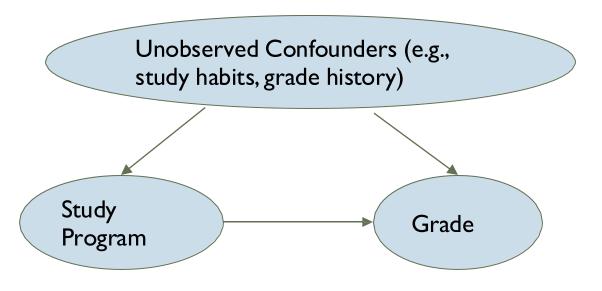
Causal graph is a directed graph

- Nodes: variables
- Directed Edges: Xaffects Y



Causal Graph

Confounding variables: common cause of treatment and outcome



Why Causal Graph?

- Helpful to identify which variables to control for
- Make assumptions explicit

Outline

Why Should Data Scientists Care?

Basic Concepts

Causal Inference

- Statistical Inference vs. Causal Inference
- Average Treatment Effect (ATE) Estimation
- Causal Inference in Practice

Statistical vs. Causal Inferences

Statistical inference

- Data is just a sample
- Your goal is to infer a population
- Think about how to go "backwards" from sample to population

Causal inference

- Derive a treatment group from Data
- Derive a control group (i.e., without treatment) from Data
- Think about how to infer the actual effect of treatment from the derived treatment and control groups

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Jacky	male	I	0	78	82
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Sarah	female	2	I	83	85

Individual Treatment Effect

What is the grade difference between enrolling and not enrolling in the study program?

Student	Gender	Class	Enroll in Study Program	Not Enroll in Study Program	
Jacky	male	I	82	78	4
Terry	male	I	82	82	0
Mary	female	I	90	86	4
Sarah	female	2	83	85	-2

Average Treatment Effect (ATE)

The average of all values for individual treatment effects

ATE =
$$(4 + 0 + 4 + -2) / 4 = 1.5$$

Student	Gender	Class	Enroll in Study Program	Not Enroll in Study Program	
Jacky	male	1	82	78	4
Terry	male	1	82	82	0
Mary	female	1	90	86	4
Sarah	female	2	83	85	-2

ATE: Estimation

ATE estimation methods

- Matching based:
 - Perfect matching
 - > Nearest neighbor matching
 - Propensity score matching
- ML based:
 - Regression method
 - Representation learning

Perfect Matching

student	gender	class	Study program	grade
Terry	male	I	I	82
Sarah	female	2	I	83
Jacky	male	I	0	78
Mary	female	I	0	86

Find the "perfect matching in counterfactual world"

4

Perfect Matching

student	gender	class	Study program	grade
Terry	male	1	I	82
Sarah	female	2	I	83
Jacky	male	I	0	78
Mary	female	1	0	86

Find the "perfect matching in counterfactual world"

Perfect Matching

Student	Gender	Class	Study Program	Grade
Terry	male	I	I	82
Sarah	female	2	I	83
Jacky	male	I	0	78
Mary	female	I	0	86

By perfect matching, we can't find ATE over the population, because for tuple Sarah, we can't find the perfect match in control group

<mark>-3</mark>

Nearest Neighbor Matching

Student	Gender	Class	Study Program	Grade
Terry	male	I	I	82
Sarah	female	2	I	83
Jacky	male	I	0	78
Mary	female	1	0	86

Find the "nearest matching in counterfactual world"

Nearest Neighbor Matching

Student	Gender	Class	Study Program	Grade	
Terry	male	I	I	82	4
Sarah	female	2	I	83	_
Jacky	male	I	0	78	
Mary	female	1	0	86	

ATE =
$$\frac{1}{2}$$
 [-3 + 4] = 0.5

Propensity Score Matching

Steps

 \Leftrightarrow Using logistic regression to infer e(x)=Pr[T=1|X=x]

Match users with treatment 0 with users with treatment I based on propensity score, using matching methods

Propensity Score Matching

Student	Gender	Class	Study Program	Grade	PSE
Terry	male	1	I	82	0.7
Sarah	female	2	I	83	0.6
Jacky	male	I	0	78	0.7
Mary	female		0	86	0.55

Based on PSM derived by logistic regression, the match we'll find for Terry is Jacky, and the match we'll find for Sarah is Mary

ATE = 0.5, computation is similar with nearest neighbor matching

Regression Method

Intuition: the distribution of Y given X is different when treatment is different

Train two separate regression models under Treatment = 0 or Treatment = I,infer p(Y | T = 0, X)and p(Y | T = I,X)

Regression Method

Student	Gender	Class	Study Program	Grade
Terry	male	1	1	82
Sarah	female	2	I	83
Jacky	male	I	0	78
Mary	female		0	86

Treat one regression model Ion [Terry, Sarah] where Study Program = I

Treat another regression model 2 on [Jacky, Mary] where Study Program = 0

Using model Ito compute counterfactual outcome of [Jacky, Mary], same for model 2

Other ML-based Methods

- Representation learning
- Intuition: transform dataset into a space where treatment assignment is more evenly distributed
- For other more techniques, please check latest publications

Outline

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- Average Treatment Effect (ATE) Estimation
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Causal Inference Case Study

What if you went to UBC rather than SFU, would it help you land a better job?

- I. What data to collect?
- 2. How to use data to answer this question?

Allocating Policy for Homelessness

Researchers allocate different interventions (like emergency shelter, rapid rehousing) for homelessness based on causal inference

Report published in AAAI 2019

Amanda Kube, Sanmay Das, Patrick J. Fowler: <u>Allocating Interventions Based on Predicted Outcomes</u>: <u>A Case Study on Homelessness Services</u>. AAAI 2019: 622-629

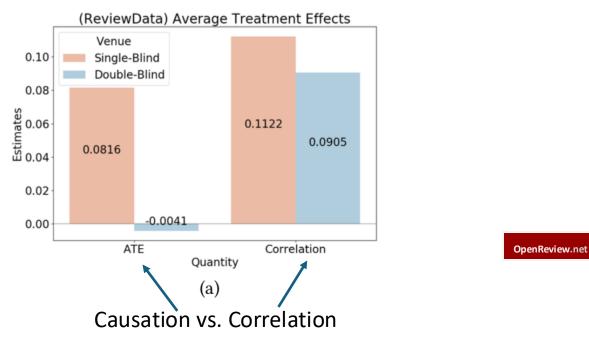
Social Media

For Twitter, the impact of race, gender and closeness on persuasion is studied using causal inference

Result published in NeurIPS 2019

Academic Paper Review

Are reviewers influenced by authors' prestige?



High correlation in both single and double blind High causation only in single blind

[Salimi-Parikh-Kayali-Getoor-Roy-Suciu, SIGMOD'20]

DOWhy Python Library

- DoWhy is a Python library for causal inference that supports explicit modeling and testing of causal assumptions, developed by Microsoft.
- DoWhy is based on a unified language for causal inference, combining causal graphical models and potential outcomes frameworks.

Summary

Why Should Data Scientists Care?

- "What if" Question?
- Why not A/B testing?
- Causality ≠ Correlation

Basic Concepts

- Outcome / Treatment Variables
- Intervention and Do Operation
- Counterfactual
- Causal Graph

Causal Inference

- Statistical Inference vs. Causal Inference
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