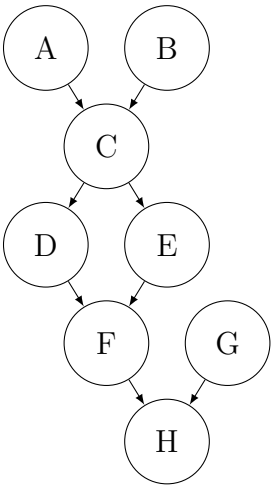
Quiz 13

1 Variable elimination ordering

Given the following BN, find the variable elimination ordering that leads to the minimal maximal clique size.



2 MCMC for encrypted text

You are given a passage of English text that has been encrypted by remapping each symbol to a (usually) different one. For example,

$$\begin{aligned} a &\rightarrow t \\ b &\rightarrow ! \\ < \text{space} > &\rightarrow v \\ \ldots &\rightarrow \ldots \end{aligned}$$

Thus a text like 'a boy...' might be encrypted by 'tv!op...'. Assume that the mapping between symbols is a one-to-one permutation function σ. Decoding the message by brute force is impossible since there are $k!$ possible permutations to try given k symbols. Instead, we will set up a Metropolis-Hasting Markov Chain to find modes in the space of permutations.

We model English text, say $s_1 s_2 \ldots s_n$, where s_i are symbols, as a Markov chain, where each symbol given the immediately previous one is independent of all earlier symbols:

$$p(s_1 s_2 \ldots s_n) = p(s_1) \prod_{i=2}^{n} p(s_i | s_{i-1})$$

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1. The state variable for our MCMC sampler will be the permutation amongst the symbols. Let $\sigma(s)$ be the symbol that stands for symbol s in the encrypted text, e.g. $\sigma(a) = t$ and $\sigma(b) = !$ in the above example. Assume a uniform prior distribution over permutations.

   Are the latent variables $\sigma(s)$ for different symbols s independent?

   Let $e_1 e_2 \dots e_n$ be an encrypted English text. Write down the joint probability of $e_1 e_2 \dots e_n$ and $s_1 s_2 \dots s_n$ given σ.

2. We shall use a Metropolis-Hastings (MH) sampler, with a proposal formed by choosing two symbols s and s' at random and swapping the corresponding encrypted symbols $\sigma(s)$ and $\sigma(s')$.

   What is the probability of a given proposal, and what is the corresponding acceptance probability?