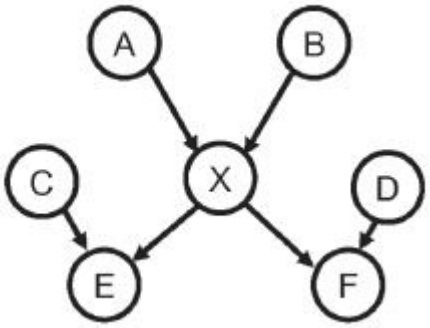
Assignment 8

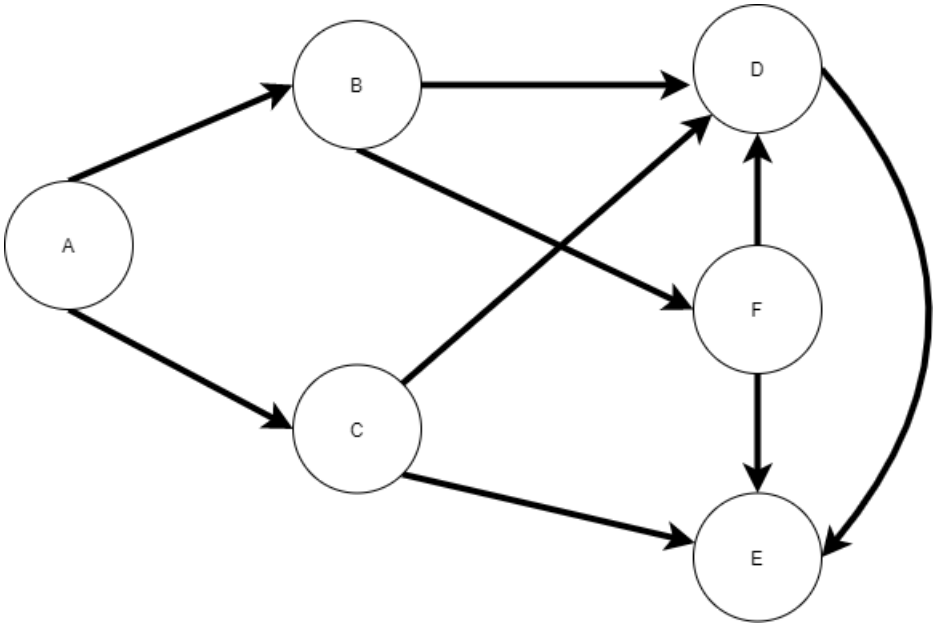
Problem 1

Consider the DAG G in the following figure. Assume it is a minimal I-map for $p(A, B, C, D, E, F, X)$. Now consider marginalizing out X. That is, X is unobserved and we construct $p(A, B, C, D, E, F) = \sum_X p(A, B, C, D, E, F, X)$. Construct a new DAG G' which is a minimal I-map for $p(A, B, C, D, E, F)$. Specify (and justify) which extra edges need to be added.

Hint: Ensure that the new graph does not assert any conditional independence not asserted in the original graph. Does the original graph assert $E \perp\!\!\!\perp F|A, B$ or $E \perp\!\!\!\perp D|A, B, F$?



Problem 2



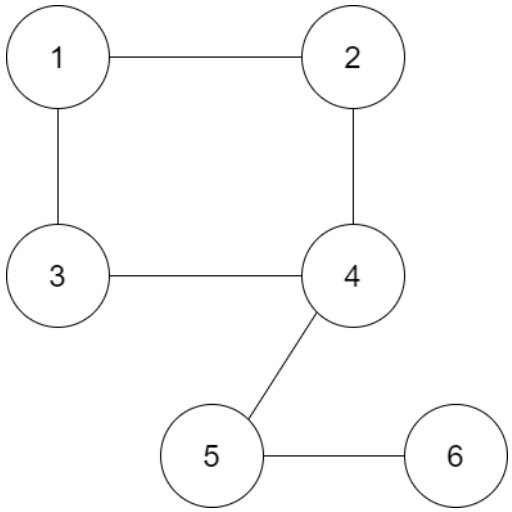
1. Convert the BN above to MRF.

Assignment 8

2. Fill in the blanks to make a true statement: _____ and _____ are d-separated given _____ in the BN, but not d-separated in the MRF.

Problem 3

Consider the MRF in the following Figure.



1. Suppose we want to compute the partition function using the elimination ordering $\prec = (1, 2, 3, 4, 5, 6)$, i.e.,

   $$\sum_{x6}\sum_{x5}\sum_{x4}\sum_{x3}\sum_{x2}\sum_{x1}\psi_{12}(x_1, x_2)\psi_{13}(x_1, x_3)\psi_{24}(x_2, x_4)\psi_{34}(x_3, x_4)\psi_{45}(x_4, x_5)\psi_{56}(x_5, x_6)$$

   If we use the variable elimination algorithm, we will create new intermediate factors. What is the largest intermediate factor?

2. Add an edge to the original MRF between every pair of variables that end up in the same factor. (These are called fill in edges.) Draw the resulting MRF. What is the size of the largest maximal clique in this graph?

3. Now consider elimination ordering $\prec = (4, 1, 2, 3, 5, 6)$, i.e.,

   $$\sum_{x6}\sum_{x5}\sum_{x3}\sum_{x2}\sum_{x1}\sum_{x4}\psi_{12}(x_1, x_2)\psi_{13}(x_1, x_3)\psi_{24}(x_2, x_4)\psi_{34}(x_3, x_4)\psi_{45}(x_4, x_5)\psi_{56}(x_5, x_6)$$

   If we use the variable elimination algorithm, we will create new intermediate factors. What is the largest intermediate factor?

4. Add an edge to the original MRF between every pair of variables that end up in the same factor. (These are called fill in edges.) Draw the resulting MRF. What is the size of the largest maximal clique in this graph?