Today's Plan

Upcoming:

- PQ1 posted
- AQ1 next week

Last time:

- Combinations
- Binomial Theorem

Today's topics:

Chapter 1: Combinatorics

- From last time:
  - 1.3: Combinations with no repetition
- 1.4: Combinations with repetition

Section 1.4: Combinations with Repetition

What if we want to allow repetitions?

In general, the number of selections, ***with repetitions***, of r objects from n distinct objects are:

$$\frac{(n + r - 1)!}{r!(n - 1)!}$$

E.g. Suppose we want to distribute \$1000 to 4 persons (in units of \$100).

Example

A message is made up of 12 different symbols and is to be transmitted through a communication channel. In addition to the 12 symbols, the transmitter will also send a total of 45 blank spaces between the symbols, with at least three spaces between each pair of consecutive symbols. In how many ways can the transmitter send such a message?

Integer Solutions Example

Determine all integer solutions to the equation

$x_1 + x_2 + x_3 + x_4 = 7$ where $x_i >= 0$ for all $1 <= i <= 4$

Alternative Definitions

The following are equivalent:

(a) the number of integer solutions of the equation

$x_1 + x_2 + \ldots + x_n = r$ where $x_i >= 0$ for all $1 <= i <= n$

(b) the number of selections, with repetition, of size *r* from a collection of size *n*

(c) the number of ways *r* identical objects can be distributed among *n* distinct containers

Example

How many nonnegative integer solutions are there to the inequality: $x_1 + x_2 + \ldots + x_6 < 10$