Today's Plan

Upcoming:

- PQ1 posted
- AQ1 next week

Last time:

- Permutations
- Combinations

Today's topics:

Chapter 1: Combinatorics

- 1.3: Combinations with no repetition

Section 1.3: Combinations with No Repetition Example

The number of arrangements in the letters in TALLAHASSEE is:

How many of these arrangements have no adjacent A's?

Example - cont'd...

Definitions

Summation Notation:

$$a_m + a_{m+1} + a_{m+2} + \cdots + a_{m+n} = \sum_{i=m}^{m+n} a_i$$

The Binomial Theorem:

$(x + y)^n =$

Binomial Theorem Example & Corollaries

What is the coefficient of x^5y^2 in the expansion of $(x + y)^7$?

Corollaries:

Example

An *alphabet* defines the symbols in a language. *Strings* are then formed from these alphabets. For example, if the alphabet consists of the symbols 0, 1, and 2, then 01, 11, 21, 12, and 20 are some of the strings of length 2.

If we have a string $x = x_1x_2x_3\ldots x_n$, then the weight of x is defined as $wt(x) = x_1+x_2+\ldots+x_n$.

Consider the alphabet 0,1, and 2 and n = 3, then wt(101) = 2, wt(210) = 3, and wt(222) = 6.

How many different strings of length 10 are there for the above alphabet?

Example - cont'd...

Among these strings of length 10, how many have even weight?

Example - cont'd...

The Multinomial Theorem

For positive integers n, t, the coefficient of $x_1^{n1}x_2^{n2}x_3^{n3}\cdots x_t^{nt}$ in the expansion of $(x_1 + x_2 + x_3 + \ldots + x_t)^n$ is

$$\frac{n!}{n_1!n_2!n_3!\cdots n_t!}$$

where each n_i is an integer with $0 \le n_i \le n$, for all $1 \le i \le t$, and $n_1 + n_2 + \ldots + n_t = n$.

E.g. What is the coefficient of $x^2y^2z^3$ in the expansion of $(x + y + z)^7$?