Today's Plan

Upcoming:

- Practice assignment 1

Last time:

- Introduction to the course

Today's topics:

Chapter 1: Combinatorics

- 1.1: Sum & Product Rule
- 1.2: Permutations

Section 1.1: Basic Counting Principles

Counting problems are of the following kind:

- "How many different 8-letter passwords are there?"
- "How many possible ways are there to pick 11 soccer players out of a 20-player team?"

Most importantly, counting is the basis for computing probabilities of discrete events.

- "What is the probability of winning the lottery?"

Group Exercise

Automobiles come with several options. You may choose the exterior color, the interior color, air conditioning, tape or CD player, power windows, security systems, and so on. In the following problems, you will explore how many choices a buyer would have if given certain options.

1. If you choose a sport car, you have the choice of six models. If you choose a van, you have a choice of four models. How many different choices do you have? Explain your answer.

2. As the buyer, you will need to choose from four different exterior colors (white, red, indigo blue, and ocean green) and three different interior colors (white, tan, and gray.) How many different color combinations are possible?

3. You may choose between a sports car (6 models) and a van (4 models) with the interior and exterior choices of problem two. How many different combinations are possible for you to choose from?

The Sum Rule

If a first task can be performed in n_1 *ways, while a second task can be performed in* n_2 *ways, and the two tasks cannot be performed simultaneously, then performing either task can be done in any of* $n_1 + n_2$ *ways*

Example: The department will award a free computer to either a CS student or a CS professor. How many different choices are there, if there are 530 students and 15 professors?

Example: A student can choose one computer project from one of three lists. The three lists contain 23, 15 and 19 possible choices. How many possible projects are there to choose from?

The Product Rule

Suppose that a procedure can be broken down into two successive tasks. If there are n_1 *ways to do the first task and* n_2 *ways to do the second task after the first task has been done, then there are* $n_1 \cdot n_2$ *ways to do the procedure.*

Generalized product rule:

If we have a procedure consisting of sequential tasks $T_1, T_2, \ldots, T_m$ *that can be done in* $n_1, n_2, \ldots, n_m$ *ways, respectively, then there are* $n_1 \cdot n_2 \cdot \ldots \cdot n_m$ *ways to carry out the procedure.*

Product Rule Example

How many different license plates are there that contain exactly three English letters ?

Sum or Product Rule?

The chairs of an auditorium are to be labeled with a letter and a positive integer not exceeding 100. For example, A23 would be one chair label. What is the largest number of chairs that can be labeled differently?

Inclusion-Exclusion Example

How many bit strings of length 8 either start with a 1 or end with 00?

Task 1: Construct a string of length 8 that starts with a 1.

Inclusion-Exclusion Example

Task 2: Construct a string of length 8 that ends with 00.

Inclusion-Exclusion Example

Since there are 128 ways to do Task 1 and 64 ways to do Task 2, does this mean that there are 192 bit strings either starting with 1 or ending with 00 ?

Inclusion-Exclusion Example

How many cases are there, that is, how many strings start with 1 **and** end with 00?

Another Approach – Tree Diagrams

How many bit strings of length four do not have two consecutive 1s?

Task 1	Task 2	Task 3	Task 4
(1st bit)	(2nd bit)	(3rd bit)	(4th bit)

Section 1.2: Permutations - Example

In a class of 10 students, 5 are to be chosen and seated in a row for a picture. How many such linear arrangements are possible?

Consider the individual seating positions:

1^{st} pos	2^{nd} pos	3^{rd} pos	4^{th} pos	5^{th} pos

Permutations - Definitions

A **permutation** of a set of n distinct objects is an *ordered* arrangement of these objects.

For an integer $n \geq 0$, n factorial (denoted $n!$) is defined by

$$0! = 1,$$

$$n! = (n)(n-1)(n-2)\cdots(3)(2)(1), \text{ for } n \geq 1$$

<u>Counting Formula: Permutations with no repetition</u>

The number of ways to arrange r (with $0 \leq r \leq n$) objects from a set of n objects, in order, but with no repetition allowed is:

$$P(n,r) = n\cdot(n-1)\cdot(n-2)\cdots(n-r+1) = n!/(n-r)!$$

Permutations - Definitions

Counting Formula: Permutations with repetition

The number of ways to arrange r objects from a set of n objects, in order, with repetition allowed is: n^r

Example:

(a) What are the permutations of the letters in the word COMPUTER?

(b) If only 5 letters are used from the above, what is the number of permutations?

(c) If repetition of letters are allowed, how many 12-letter sequences are possible using the letters above?

Group Exercise

1. If every phone number was used in the Lower Mainland, how many individual numbers would there be?

   Note: The first number of a 7-digit phone number can not be a 0 or 1. Also there are currently 2 area codes, 604 and 778.

2. Suppose a new area code were made available in the Lower Mainland, area code 603, how many new phone numbers could be issued?

Example

How many different strings can be made reordering the word "BALL"?

Permutations - Definitions

Counting Formula: Permutations w/ indistinguishable objects

The number of different (linear) permutations of n objects, where there are n_1 indistinguishable objects of type 1, n_2 indistinguishable objects of type 2, ... n_k indistinguishable objects of type k, is:

$$\frac{n!}{n_1! \cdot n_2! \cdots n_k!}$$

where $n = n_1 + n_2 + \ldots + n_k$

Example

How many different strings can be made reordering the letters in the word "DATABASES"?

Permutations Example

Six people are seated about a round table, how many different circular arrangements are possible, if arrangements are considered the same when one can be obtained from the other by rotations?

Example - cont'd…