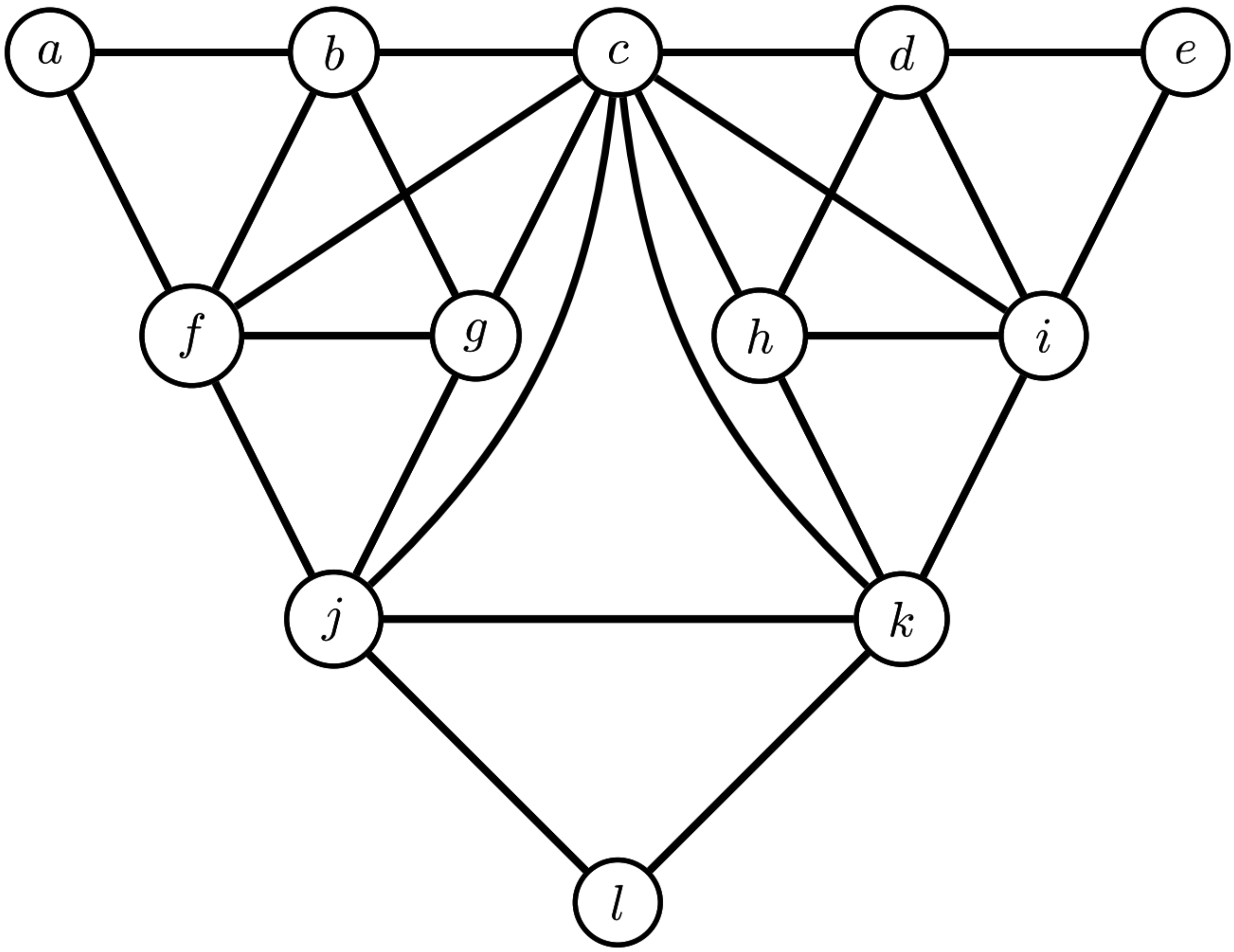
Assignment 9

Problem 1

Given the following MRF, find its corresponding junction tree decomposition with the minimum max clique size.



Problem 2

An order-k Markov model is a probablistic model over variables $X_1, \dots X_n$, where X_i depends on the k variables $X_{i-k}..X_{i-1}$.

1. What is the treewidth of an order-k Markov model?
2. Your colleague wants to do exact inference in a Markov model with $k = 100$. Explain why this is computationally expensive.

3. They suggest that to make inference easier while retaining long-range dependencies, they instead use a Markov-like model where X_i depends only on X_{i-k}, thus reducing the number of edges in the network by a factor of 100. Explain why this does not meaningfully improve performance.

Problem 3

A random variable with density $g(y) = \sqrt{2/\pi}e^{-y^2/2}\mathbb{I}\{y \geq 0\}$ is to be simulated by rejection sampling. The candidate values are realizations from a $Exp(\lambda)$-distributed random variable with density $f(x) = \lambda e^{-\lambda x}\mathbb{I}\{x \geq 0\}$.

1. Determine the smallest value c (with subject to $\lambda > 0$) such that $cf(y) \geq g(y)$.
2. For which value of λ is the theoretical percentage of rejected samples minimal?
   Hint: Recall that the probability of acceptance is $\frac{Z_g}{c}$, where $Z_g = \int_x g(x)dx$.