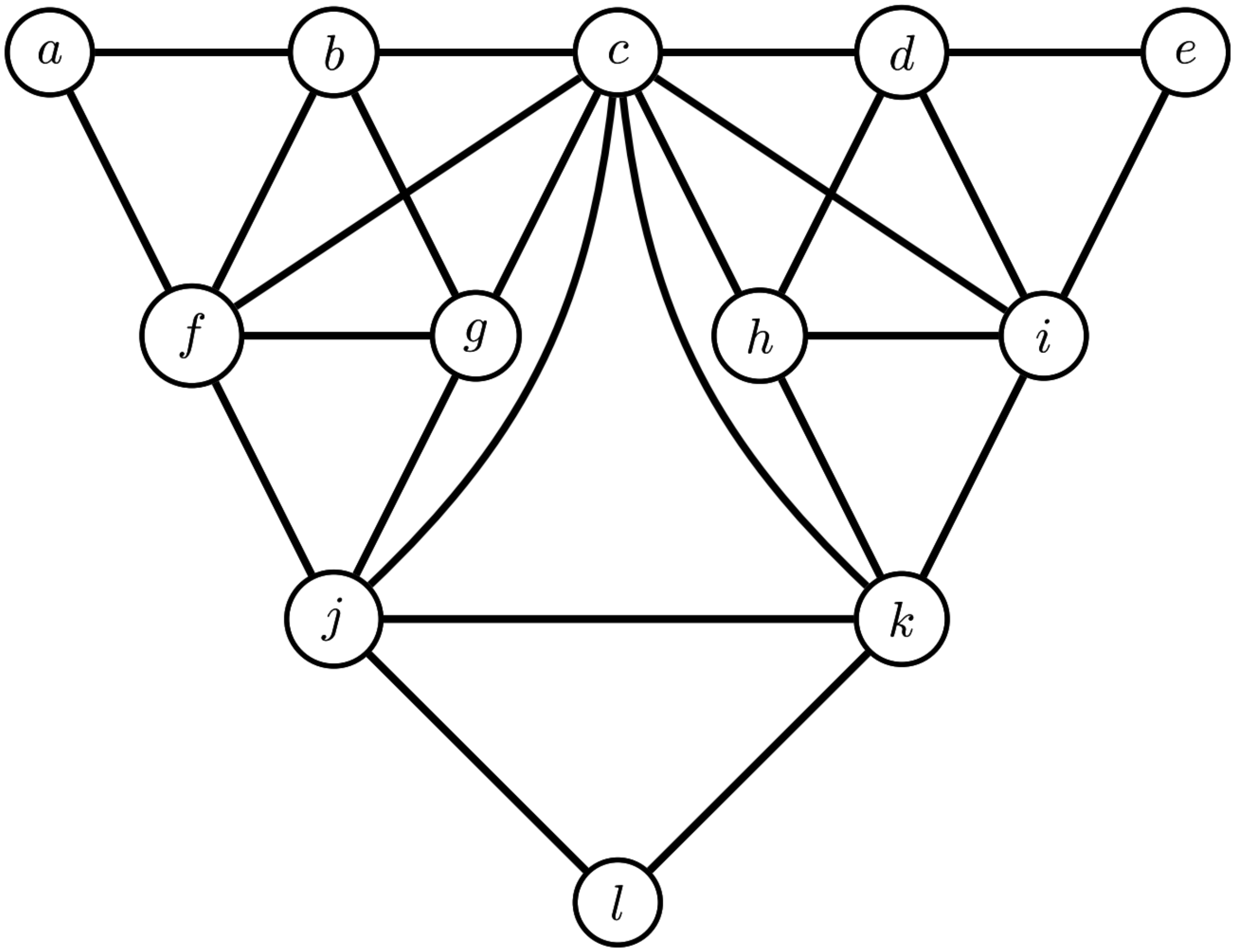
Assignment 9

Problem 1

Given the following MRF, find its corresponding junction tree decomposition with the minimum max clique size.



Problem 2

An order-k Markov model is a probablistic model over variables $X_1, \ldots X_n$, where X_i depends on the k variables $X_{i-k}..X_{i-1}$.

1. What is the treewidth of an order-k Markov model?
2. Your colleague wants to do exact inference in a Markov model with $k = 100$. Explain why this is computationally expensive.

3. They suggest that to make inference easier while retaining long-range dependencies, they instead use a Markov-like model where X_i depends only on X_{i-k}, thus reducing the number of edges in the network by a factor of 100. Explain why this does not meaningfully improve performance.

Problem 3

Consider the goal of sampling from a Gaussian distribution $\mathcal{N}(0, \sigma)$. This is usually done in practice using an approach called the Box-Muller method, but here we will use a naive approach based on rejection sampling: We will sample from $q(x) = \text{Uniform}(-a, a)$ then use rejection sampling to approximate $\mathcal{N}(0, \sigma)$.

1. Complete the rejection sampling algorithm. In particular, derive the acceptance rule.
2. Let $P'(x|0, \sigma, a)$ be the probability of sampling x using our method. Explain why $P'(x|0, \sigma, a) \neq P(x|0, \sigma)$. Give an upper bound on $P(x|0, \sigma) - P'(x|0, \sigma, a)$ and $P'(x|0, \sigma, a) - P(x|0, \sigma, a)$ respectively.

   You may use the Gaussian distribution CDF function, $P(X < x|\mu, \sigma) = \frac{1}{2}\left(1 + \text{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right)$.
3. What is the acceptance probability (as a function of σ and a)?
4. What is the benefit and drawback respectively of increasing a?

Problem 4

Please write one thing from this course you found confusing, a topic you would like to hear more about, or something you found particularly interesting.