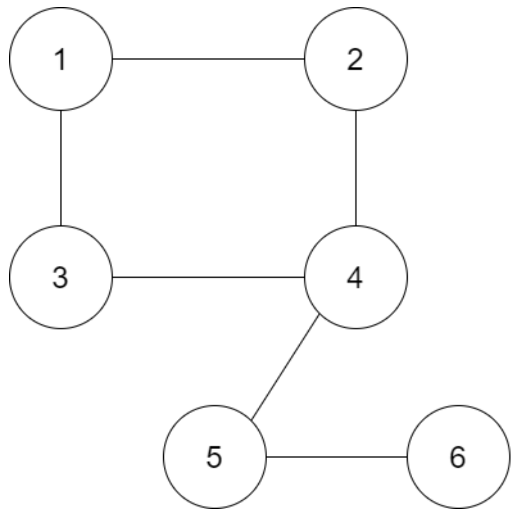
Assignment 8

Problem 1

Consider the MRF in the following Figure.



1. Suppose we want to compute the partition function using the elimination ordering $\prec = (1, 2, 3, 4, 5, 6)$, i.e.,

$$\sum_{x6}\sum_{x5}\sum_{x4}\sum_{x3}\sum_{x2}\sum_{x1}\psi_{12}(x_1, x_2)\psi_{13}(x_1, x_3)\psi_{24}(x_2, x_4)\psi_{34}(x_3, x_4)\psi_{45}(x_4, x_5)\psi_{56}(x_5, x_6)$$

   If we use the variable elimination algorithm, we will create new intermediate factors. What is the largest intermediate factor?

2. Add an edge to the original MRF between every pair of variables that end up in the same factor. (These are called fill in edges.) Draw the resulting MRF. What is the size of the largest maximal clique in this graph?

3. Now consider elimination ordering $\prec = (4, 1, 2, 3, 5, 6)$, i.e.,

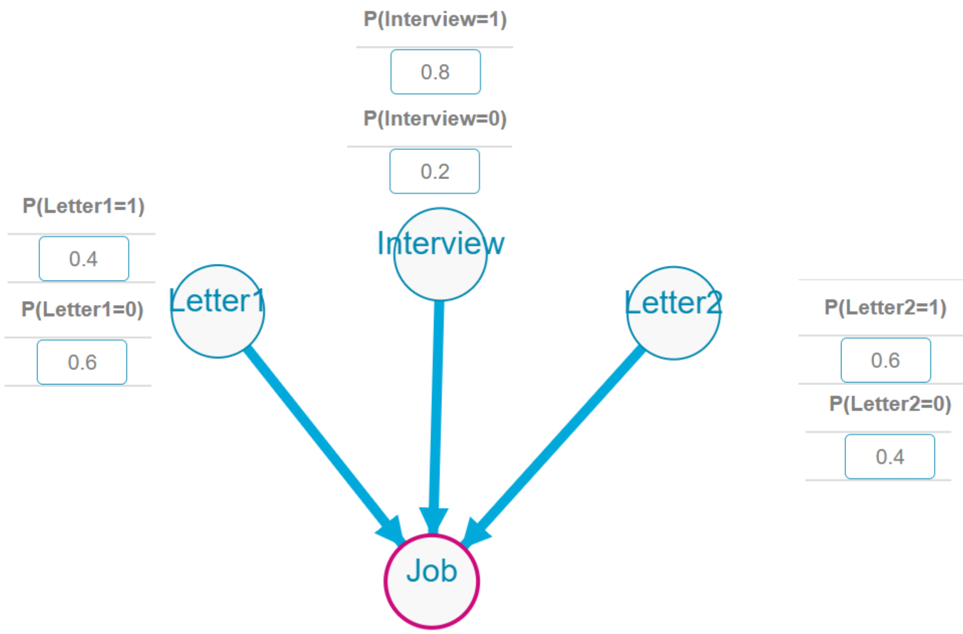
$$\sum_{x6}\sum_{x5}\sum_{x3}\sum_{x2}\sum_{x1}\sum_{x4}\psi_{12}(x_1, x_2)\psi_{13}(x_1, x_3)\psi_{24}(x_2, x_4)\psi_{34}(x_3, x_4)\psi_{45}(x_4, x_5)\psi_{56}(x_5, x_6)$$

   If we use the variable elimination algorithm, we will create new intermediate factors. What is the largest intermediate factor?

4. Add an edge to the original MRF between every pair of variables that end up in the same factor. (These are called fill in edges.) Draw the resulting MRF. What is the size of the largest maximal clique in this graph?

Problem 2

Consider the following simple BN that represents the chance of getting a job. Every variable is binary and the corresponding CPTs are given in the figure.



Interview	Letter2	Letter1	P(Job=1|Interview,Letter2,Letter1)	P(Job=0|Interview,Letter2,Letter1)
1	1	1	0.9	0.1
1	1	0	0.6	0.4
1	0	1	0.6	0.4
1	0	0	0.5	0.5
0	1	1	0.5	0.5
0	1	0	0.2	0.8
0	0	1	0.3	0.7
0	0	0	0.1	0.9

1. Convert the BN to a MRF.
2. Suppose, we received evidence such that we know $Letter1 = 1$; modify the MRF accordingly.
3. Run variable elimination by hand to compute $P(Job|Letter1 = 1)$.