Assignment 5

Problem 1

The Poisson distribution is defined as

$$\text{Poi}(x|\lambda) = e^{-\lambda}\frac{\lambda^x}{x!},$$

for $x \in \{0, 1, 2, \dots\}$ where $\lambda > 0$ is the rate parameter. Suppose we have observed $\{x_1, x_2, \dots x_n\}$ drawn from $\text{Poi}(\lambda)$ What is the MLE of λ?

Problem 2

Consider samples $x_1, \dots, x_n$ from a Gaussian random variable with known variance σ^2 and unknown mean μ. We further assume a prior distribution (also Gaussian) over the mean, $\mu \sim \mathcal{N}(m, s^2)$, with fixed mean m and fixed variance s^2. Thus the only unknown is μ.

1. Calculate the MAP estimate $\hat{\mu}_{MAP}$. You can state the result without proof. Alternatively, with a bit more work, you can compute derivatives of the log posterior, set to zero and solve.
2. Show that as the number of samples n increase, the MAP estimate converges to the maximum likelihood estimate.
3. Suppose n is small and fixed. What does the MAP estimator converge to if we increase the prior variance s^2?
4. Suppose n is small and fixed. What does the MAP estimator converge to if we decrease the prior variance s^2?

Problem 3

Let θ be a univariate, continuous parameter. Consider the following optimization, which adds a regularization term to the log likelihood to identify a regularized MLE.

$$\hat{\theta}_{reg} = \arg\min_{\theta} -\log p(D|\theta) + \lambda C(\theta) \qquad \lambda > 0$$

1. As an alternative, we can define the MAP estimate $\hat{\theta}_{MAP}$ for a particular choice of prior $p(\theta)$. Define a probability distribution $p(\theta)$ as a function of $C(\theta)$ such that $\hat{\theta}_{MAP} = \hat{\theta}_{reg}$. You may assume that C has a finite integral.
2. According to $p(\theta)$, what is the relative probability of two parameters $p(\theta_1)/p(\theta_2)$?
3. Based on your answer to (2), qualitatively, how does $p(\theta)$ change as we decrease λ? Increase?

Assignment 5

Problem 4

On average, how many hours have you spent on each assignment so far in this course?

Problem 5

Please write one thing from this course you found confusing, a topic you would like to hear more about, or something you found particularly interesting.