

Problem 1

Suppose we observe a set of data points $x_{1..4} = \{-1, 0.5, 1, 100\}$ and we would like to use Gaussian Mixture Model (GMM) for clustering D . Consider the following mixture of two Gaussians: $\mathcal{N}(0, 10)$ and $\mathcal{N}(1, 2)$ and we believe the data points are drawn from $\mathcal{N}(0, 10)$ 40% of the time and $\mathcal{N}(1, 2)$ 60% of the time.

1. Calculate the responsibility of each component for each data point in $x_{1..4}$.
2. For $x_4 = 100$ which component has higher responsibility? Briefly explain why that is the case.

Problem 2

Suppose we want to model traffic on a given road. At each hour i , we measure the number of cars y_i . We think the number of cars is well modeled by a Gaussian distribution. However, we know that some times are rush hour, so we decide to use a mixture of Gaussian distributions with two components (normal and rush hour traffic respectively.) For simplicity, imagine that traffic at each hour is independent of one another.

Draw a graphical model that represents this mixture model. You should use plate notation and include variables $\mathbf{z} = (z_1, \dots, z_n)$ and $\mathbf{y} = (y_1, \dots, y_n)$ as well as the parameters of the distributions $\pi, \mu_1, \mu_2, \sigma_1$ and σ_2 in your graph.

Problem 3

Suppose we have two sensors with known (and different) variances v_1 and v_2 , but unknown (and the same) mean μ . Suppose we observe n_1 observations $y_i^{(1)} \sim \mathcal{N}(\mu, v_1)$ from the first sensor and n_2 observations $y_i^{(2)} \sim \mathcal{N}(\mu, v_2)$ from the second sensor. (For example, suppose μ is the true temperature outside, and sensor 1 is a precise (low variance) digital thermosensing device, and sensor 2 is an imprecise (high variance) mercury thermometer.)

Let D represent all the data from both sensors. What is the posterior $p(\mu|D)$, assuming a non-informative prior for μ (which we can simulate using a Gaussian with a variance of ∞)? Give an explicit expression for the posterior mean and variance.

Problem 4

Please write one thing from this course you found confusing, a topic you would like to hear more about, or something you found particularly interesting.