## Problem 1

Reproduce the formatting of the following equation. You might want to use the following commands:
\align; \intertext; \left; \right; \sum; \prod \dots; \frac; \log;

$$
\begin{align*}
\frac{\sum_{i=1}^{n} \log \left(a_{i}\right)}{n} & =\frac{\log \left(a_{1}\right)+\log \left(a_{2}\right)+\cdots+\log \left(a_{n}\right)}{n}  \tag{1}\\
& =\frac{\log \left(a_{1} a_{2} \ldots a_{n}\right)}{n}  \tag{2}\\
& =\log \left[\left(\prod_{i=1}^{n} a_{i}\right)^{\frac{1}{n}}\right] \tag{3}
\end{align*}
$$

because $\log (M N)=\log (M)+\log (N)$ and $\log \left(M^{k}\right)=k \log (M)$

$$
\begin{equation*}
=\log \left(\prod_{i=1}^{n} \sqrt[n]{a_{i}}\right) \tag{4}
\end{equation*}
$$

## Problem 2

Suppose a crime has been committed. Blood is found at the scene for which there is no innocent explanation. It is of a type which is present in $1 \%$ of the population.

1. The prosecutor claims: "There is a $1 \%$ chance that the defendant would have the crime blood type if he were innocent. Thus there is a $99 \%$ chance that he guilty". This is known as the prosecutor's fallacy. What is wrong with this argument?
2. The defender claims: "The crime occurred in a city of 800,000 people. The blood type would be found in approximately 8000 people. The evidence has provided a probability of just 1 in 8000 that the defendant is guilty, and thus has absolutely no bearing on the investigation." This is known as the defender's fallacy. What is wrong with this argument claiming the evidence has absolutely no bearing on the investigation?

## Problem 3

My neighbor has two children. Assume that the gender of a child is a coin flip. Let the genders of the children be $G_{1}$ and $G_{2}$. For both questions, write the probability symbolically (e.g. " $P(A \mid B)$ ") and give the value.

1. Suppose I happen to see one of his children run by, and it is a boy. What is the probability that the other child is a girl?
2. Suppose instead that I ask him whether he has any boys, and he says yes. What is the probability that one child is a girl?

## Problem 4

Consider the Numbers game with one-sided interval hypotheses $h_{\leq x}$ for numbers 1 up to 10 , where $h_{\leq x}=\{1,2, \ldots, x\}, x \in\{1 \ldots 10\}$. Assume we have a uniform prior over $h$.

1. Show that the $h_{m l e}=h_{\leq \max (S)}$ for a given set of numbers $S=\left\{x_{1}, x_{2}, \ldots\right\}$.
2. Briefly say why the MAP estimate $=$ MLE.
3. For Parts 3 and 4, let's suppose $S=\{5,9\}$. What is the plug-in approximation of the posterior predictive distribution for new data point $x$ ?
4. What is the full posterior predictive distribution?
