# Assignment 5

## Problem 1

The Poisson distribution is defined as

$$\text{Poi}(x|\lambda) = e^{-\lambda}\frac{\lambda^x}{x!},$$

for $x \in \{0, 1, 2, \dots\}$ where $\lambda > 0$ is the rate parameter. Suppose we have observed $\{x_1, x_2, \dots x_n\}$ drawn from $\text{Poi}(\lambda)$ What is the MLE of $\lambda$?

## Problem 2

Consider samples $x_1, \dots, x_n$ from a Gaussian random variable with known variance $\sigma^2$ and unknown mean $\mu$. We further assume a prior distribution (also Gaussian) over the mean, $\mu \sim \mathcal{N}(m, s^2)$, with fixed mean $m$ and fixed variance $s^2$. Thus the only unknown is $\mu$.

1. Calculate the MAP estimate $\hat{\mu}_{MAP}$ . You can state the result without proof. Alternatively, with a bit more work, you can compute derivatives of the log posterior, set to zero and solve.
2. Show that as the number of samples $n$ increase, the MAP estimate converges to the maximum likelihood estimate.
3. Suppose $n$ is small and fixed. What does the MAP estimator converge to if we increase the prior variance $s^2$?
4. Suppose $n$ is small and fixed. What does the MAP estimator converge to if we decrease the prior variance $s^2$?

## Problem 3

Let $\theta$ be a univariate, continuous parameter. Consider the following optimization, which adds a regularization term to the log likelihood to identify a regularized MLE.

$$\hat{\theta}_{reg} = \arg\min_{\theta} -\log p(D|\theta) + \lambda C(\theta) \qquad \lambda > 0$$

1. Define a probability distribution $p(\theta)$, such that if $p(\theta)$ is used as the prior for $\theta$, the MAP estimate $\hat{\theta}_{MAP} = \hat{\theta}_{reg}$ . You may assume that $C$ has a finite integral.
2. According to $p(\theta)$, what is the relative likelihood of two parameters $p(\theta_1)/p(\theta_2)$?
3. Based on your answer to (2), qualitatively, how does $p(\theta)$ change as we decrease $\lambda$? Increase?

# Assignment 5

## Problem 4

On average, how many hours have you spent on each assignment so far in this course?

## Problem 5

Please write any comments, concerns or suggestions about the course so far here.