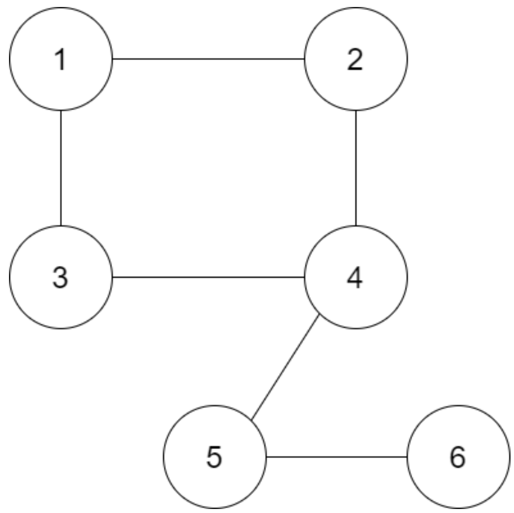
Assignment 11

Problem 1

Consider the MRF in the following Figure.



1. Suppose we want to compute the partition function using the elimination ordering $\prec = (1, 2, 3, 4, 5, 6)$, i.e.,

$$\sum_{x6}\sum_{x5}\sum_{x4}\sum_{x3}\sum_{x2}\sum_{x1} = \psi_{12}(x_1, x_2)\psi_{13}(x_1, x_3)\psi_{24}(x_2, x_4)\psi_{34}(x_3, x_4)\psi_{45}(x_4, x_5)\psi_{56}(x_5, x_6)$$

   If we use the variable elimination algorithm, we will create new intermediate factors. What is the largest intermediate factor?

2. Add an edge to the original MRF between every pair of variables that end up in the same factor. (These are called fill in edges.) Draw the resulting MRF. What is the size of the largest maximal clique in this graph?

3. Now consider elimination ordering $\prec = (4, 1, 2, 3, 5, 6)$, i.e.,

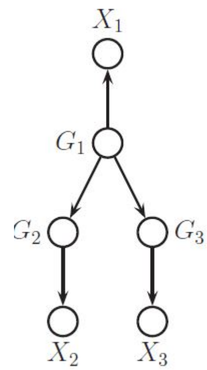
$$\sum_{x6}\sum_{x5}\sum_{x3}\sum_{x2}\sum_{x1}\sum_{x4} = \psi_{12}(x_1, x_2)\psi_{13}(x_1, x_3)\psi_{24}(x_2, x_4)\psi_{34}(x_3, x_4)\psi_{45}(x_4, x_5)\psi_{56}(x_5, x_6)$$

   If we use the variable elimination algorithm, we will create new intermediate factors. What is the largest intermediate factor?

4. Add an edge to the original MRF between every pair of variables that end up in the same factor. (These are called fill in edges.) Draw the resulting MRF. What is the size of the largest maximal clique in this graph?

Assignment 11

Problem 2



Consider the DGM in the which represents the following fictitious biological model. Each G_i represents the genotype of a person: $G_i = 1$ if they have a healthy gene and $G_i = 2$ if they have an unhealthy gene. G_2 and G_3 may inherit the unhealthy gene from their parent G_1. $X_i \in R$ is a continuous measure of blood pressure, which is low if you are healthy and high if you are unhealthy. We define the CPDs as follows

$$p(G_1) = [0.5, 0.5]$$

$$p(G_2|G_1) = \begin{pmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{pmatrix}$$

$$p(G_3|G_1) = \begin{pmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{pmatrix}$$

$$p(X_i|G_i = 1) = N(X_i|\mu = 50, \sigma^2 = 10)$$

$$p(X_i|G_i = 2) = N(X_i|\mu = 60, \sigma^2 = 10)$$

The meaning of the matrix for $p(G_2|G_1)$ is that $p(G_2 = 1|G_1 = 1) = 0.9$, $p(G_2 = 1|G_1 = 2) = 0.1$, etc.

1. Suppose you observe $X_2 = 50$, and X_1 is unobserved. What is the posterior belief on G_1, i.e., $p(G_1|X_2 = 50)$?

2. Now suppose you observe $X_2 = 50$ amd $X_3 = 50$. What is $p(G_1|X_2, X_3)$? Explain your answer intuitively.

3. Now suppose $X_2 = 60$, $X_3 = 60$. What is $p(G_1|X_2, X_3)$? Explain your answer intuitively.:

4. Now suppose $X_2 = 50$, $X_3 = 60$. What is $p(G_1|X_2, X_3)$? Explain your answer intuitively.

Assignment 11

Problem 3

Please write one thing from this course so far that you found confusing, a topic you would like to hear more about, or something you found particularly interesting.