CMPT 307
Summer 2020
Assignment 1
Due Wed June 3 at 23:59
Submit on CS Submission Server/CourSys.

4 problems; 10 points each.

1. Express \( \sum_{i=0}^{n}(3i^3 - 6i + 2) \) as a polynomial \( p(n) \). Then prove that the sum = \( p(n) \) by induction.

2. Aerosort is a sorting algorithm.
   
   \[
   \text{Aerosort}(A, i, j) \quad // A \text{ is array to sort; } i \text{ and } j \text{ are start and end indices.}
   \]
   
   \[
   n = j - i + 1
   \]
   
   If \( n < 10 \) {
   
   sort \( A[i...j] \) by insertion-sort
   
   return
   
   }
   
   \[
   m_1 = i + 3 * n / 4
   
   m_2 = i + n / 4
   \]
   
   Aerosort(A, i, m_1)
   
   Aerosort(A, m_2, j)
   
   Aerosort(A, i, m_1)

   a. What is the asymptotic worst-case running time of Aerosort? Show your work.
   
   b. Prove that Aerosort(A, 1, n) correctly sorts an array A of n elements.

3. Devise a comparison-based algorithm (no bucket or radix sort, for instance) to simultaneously find the minimum and the maximum element in a list of \( n \) numbers using at most \( 3n/2 \) comparisons. Give pseudocode.

4. Give an efficient algorithm to convert a given \( \beta \)-bit (binary) integer to a decimal representation. Argue that if multiplication or division of integers whose length is at most \( \beta \) takes time \( M(\beta) \), then binary-to-decimal conversion can be performed in time \( \Theta(M(\beta) \log \beta) \). (Hint: use a divide-and-conquer approach, obtaining the top and bottom halves of the result with separate recursions.)