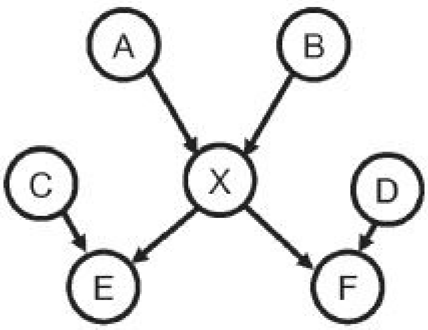
Assignment 3

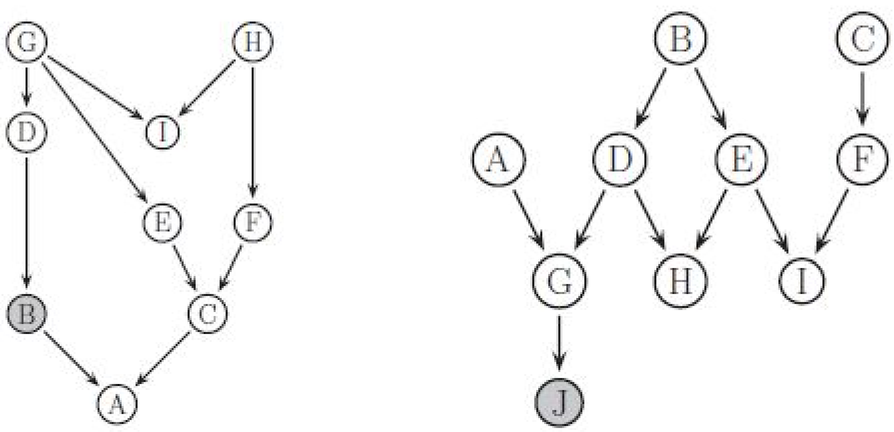
Problem 1 (Textbook exercise 10.1): Marginalizing a node in a DGM

Consider the DAG G in the following figure. Assume it is a minimal $I{-}map$ for $p(A,B,C,D,E,F,X)$. Now consider marginalizing out X. Construct a new DAG G' which is a minimal $I - map$ for $p(A,B,C,D,E,F)$. Specify (and justify) which extra edges need to be added.



Problem 2 (Textbook exercise 10.2): d-separation.

Here we compute some global independence statements from some directed graphical models. You can use the "Bayes ball" algorithm, the d-separation criterion, or the method of converting to an undirected graph (all should give the same results).



(b) (c)

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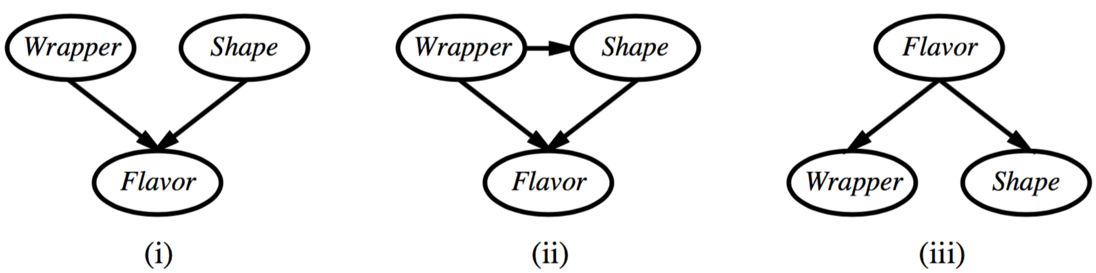
1. Consider the DAG in Figure (b). List all variables that are independent of A given evidence on B.
2. Consider the DAG in Figure (c). List all variables that are independent of A given evidence on J.

Problem 3: Surprise Candy company

The Surprise Candy company makes candy in two flavors: 70% are strawberry flavor and 30% are anchovy flavor. Each new piece of candy starts out with a round shape; as it moves down the production line, a machine randomly selects a certain percentage to be trimmed into a square; then, each piece is wrapped in a wrapper whose color is chosen randomly to be red or brown. 80% of strawberry candies are round and 80% have a red wrapper, while 90% of the anchovy candies are square and 90% have a brown wrapper. All candies are sold individually in sealed, identical black boxes.

Now you, the customer, have just bought a Surprise candy at the store but have not yet opened the box.

Consider these three Bayes nets.



1. Which network(s) can correctly represent $P(\text{Flavor}, \text{Wrapper}, \text{Shape})$?
   (i) (ii) (iii)

2. Which network is the best representation for this problem?
   (i) (ii) (iii)

3. *True/False*: Network (i) asserts that $P(\text{Wrapper}|\text{Shape}) = P(\text{Wrapper})$. __________

4. What is the probability that your candy has a red wrapper?
   (i) 0.8 (ii) 0.56 (iii) 0.59

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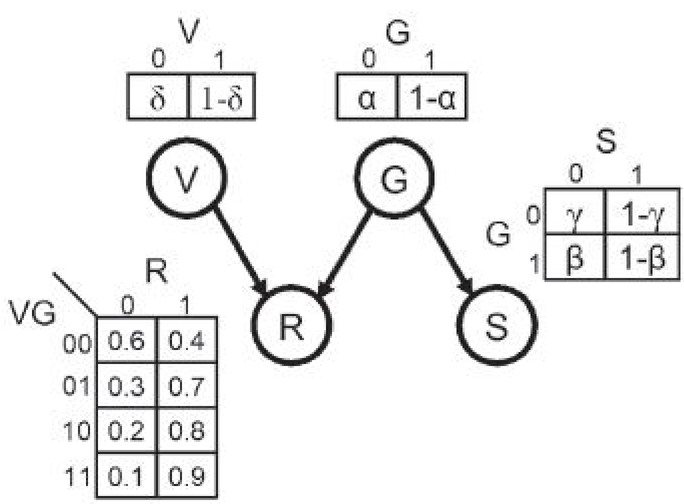
5. In the box is a round candy with a red wrapper. The probability that its flavor is strawberry is
   (i) ≤ 0.7 (ii) Between 0.7 and 0.99 (iii) > 0.99

6. (An unwrapped strawberry candy is worth s on the open market and an unwrapped anchovy candy is worth a. Write an expression for the value of an unopened candy box.

Problem 4 (Textbook exercise 10.5): Rainy day

In this question you must model a problem with 4 binary variables: G ="gray", V ="Vancouver", R ="rain" and S ="sad". Consider the directed graphical model describing the relationship between these variables shown in Figure.



1. Write down an expression for $P(S = 1|V = 1)$ in terms of $\alpha, \beta, \gamma, \delta$.

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2. Write down an expression for $P(S = 1|V = 0)$. Is this the same or different to $P(S = 1|V = 1)$? Explain why.

3. Find maximum likelihood estimates of α, β, γ using the following data set, where each row is a training case. (You may state your answers without proof.)

V	G	R	S
1	1	1	1
1	1	0	1
1	0	0	0

Problem 5 (Textbook exercise 11.3): Bernoulli EM

1. Show that the M step for ML estimation of a mixture of Bernoullis is given by

$$\mu_{kj} = \frac{\sum_i r_{ik} x_{ij}}{\sum_i r_{ik}}$$

2. Show that the M step for MAP estimation of a mixture of Bernoullis with a $Beta(\alpha, \beta)$ prior is given by

$$\mu_{kj} = \frac{(\sum_i r_{ik} x_{ij}) + \alpha - 1}{(\sum_i r_{ik}) + \alpha + \beta - 2}$$

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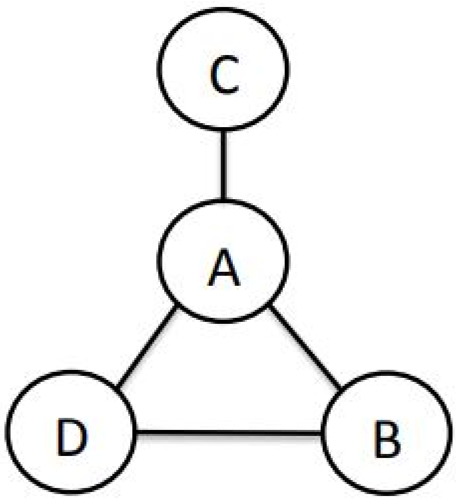
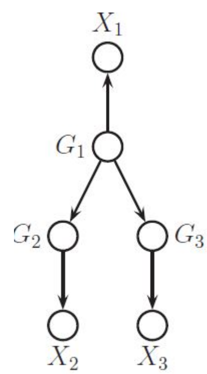
Problem 6 I-Maps and P-Maps

Consider a distribution P over four binary random variables (A, B, C, D), which gives probability 1/8 to assignments $(0, 0, 0, 0)$, and $(1, 1, 0, 0)$, and gives probability 1/4 to assignments $(1, 1, 1, 0)$, $(0, 1, 0, 1)$, and $(1, 0, 1, 1)$. A skeleton for the Bayesian network G is also provided; the skeleton contains all the nodes and edges, but the directions of the edges are missing:



1. Decide whether the following two independencies are in $I(P)$: $(C \perp\!\!\!\perp D)$,$(C \perp\!\!\!\perp B)$
2. Give a direction to each individual edge in G, such that the resulting Bayesian network is a perfect map of P. Is your solution unique? Briefly state why.
3. Give the CPDs for each node in your Bayesian network specified in (2).

Problem 7 (Textbook exercise 20.3): Message passing on a tree



Consider the DGM in the which represents the following fictitious biological model. Each G_i represents the genotype of a person: $G_i = 1$ if they have a healthy gene and $G_i = 2$ if they have an unhealthy gene. G_2 and G_3 may inherit the unhealthy gene from their parent G_1. $X_i \in R$ is a continuous measure of blood pressure, which is low if you are healthy and high if you are unhealthy. We define the CPDs as follows

$$p(G_1) = [0.5, 0.5]$$

$$p(G_2|G_1) = \begin{pmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{pmatrix}$$

$$p(G_3|G_1) = \begin{pmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{pmatrix}$$

$$p(X_i|G_i = 1) = N(X_i|\mu = 50, \sigma^2 = 10)$$

$$p(X_i|G_i = 2) = N(X_i|\mu = 60, \sigma^2 = 10)$$

The meaning of the matrix for $p(G_2|G_1)$ is that $p(G_2 = 1|G_1 = 1) = 0.9$, $p(G_2 = 1|G_1 = 2) = 0.1$, etc.

1. Suppose you observe $X_2 = 50$, and X_1 is unobserved. What is the posterior belief on G_1, i.e., $p(G_1|X_2 = 50)$?
2. Now suppose you observe $X_2 = 50$ amd $X_3 = 50$. What is $p(G_1|X_2, X_3)$? Explain your answer intuitively.
3. Now suppose $X_2 = 60$, $X_3 = 60$. What is $p(G_1|X_2, X_3)$? Explain your answer intuitively.
4. Now suppose $X_2 = 50$, $X_3 = 60$. What is $p(G_1|X_1, X_2)$? Explain your answer intuitively.