Regular Languages

CMPT 125
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SFU Computing Science
25/3/2020

Lecture 30

Today:

- Regular Languages
- Regular Expressions
- FSM Implementations
- Finite State Transducers

Formal Languages (Review)

A *formal language* is used to distinguish precisely what sequences are allowed

- expressed mathematically, often recursively

Three important definitions:

- *alphabet* (Σ) - a set of characters / symbols
- *word* (w) - a finite sequence of characters / symbols
- *language* (L) - a [possibly infinite] set of words

Parse a word w to *decide* if it is in the language L

- Accept if w is in L, Reject if not in L

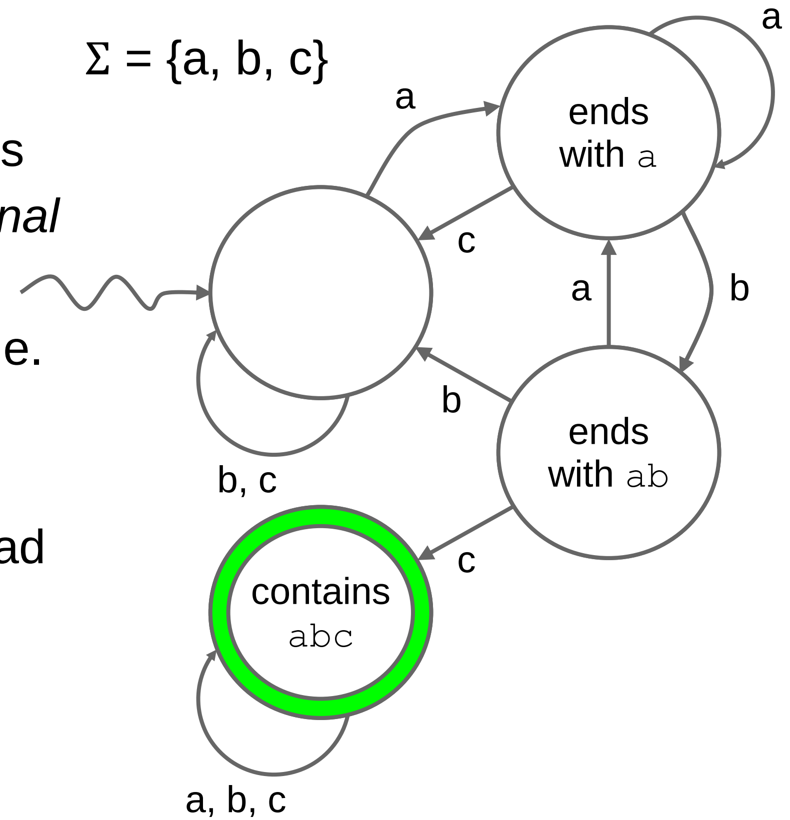
Modelling Computation (Review)

To decide a language, use a *finite state machine* (FSM).

Rules of the Game:

- Finite number of states: one of them is the *Start* state; one or more are the *Final* states.
- The FSM reads one character at a time.
- Transitions are based solely on the current state and the next character.
- A missing transition defaults to the dead state, which is not a Final state.
- If the FSM ends in a final state, then: *Accept*
- Else: *Reject*

Σ = {a, b, c}



L = {all words that have substring abc}

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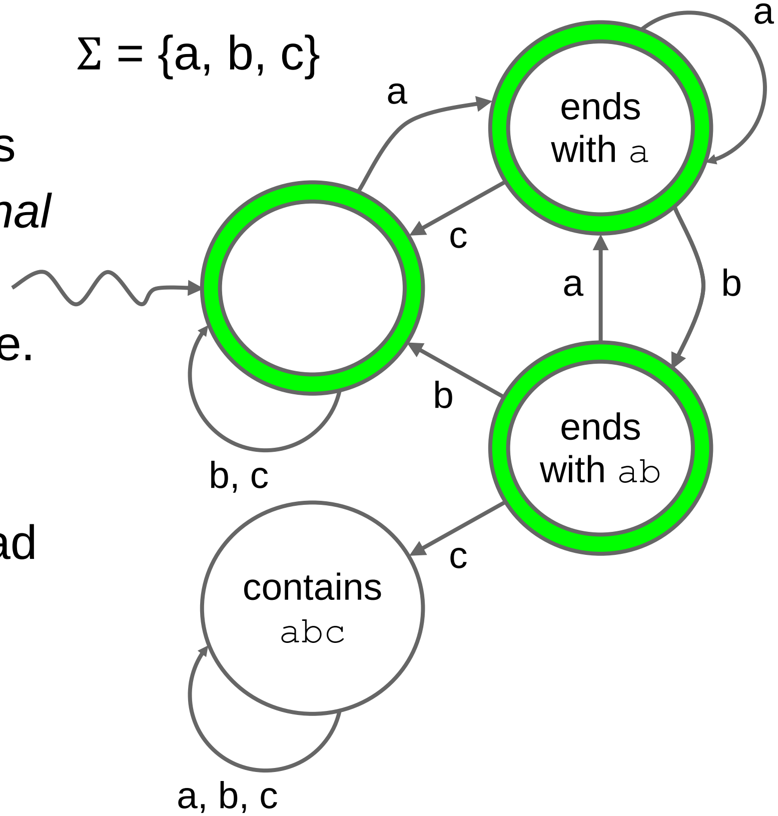


L = {all words that **don't** have substring `abc`}

Regular Languages

A *regular language* can be decided by a FSM.

- If you complement a regular language, i.e., swap Accept ↔ Reject, the result is a regular language.
- Regular languages are *closed* under complement

Regular languages are also closed under:

- union
- catenation
- Kleene star

Write them using *regular expressions*.

Regular Expressions

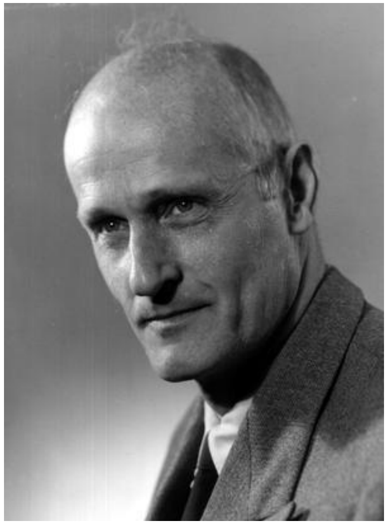
Kleenex
(Regular Tissue)

If L_1 and L_2 are two regular languages, then

- 3rd • $L_1 \mid L_2$ is their union, i.e., use a word from L_1 or a word from L_2
- 2nd • L_1L_2 is their catenation, i.e., use a word from L_1 followed by one from L_2
- 1st • L_1* is its Kleene closure, i.e., use 0 or more catenations of words from L_1

Examples:

- 0 or more b's: b*
- begins with a b: b(a|b)*
- begins and ends with a b: b(a|b)*b
- begins or ends with a b: b(a|b)* | (a|b)*b
- begins and ends with different: λ | a(a|b)*b | b(a|b)*a
- exactly 3 long: (a|b)(a|b)(a|b) OR $(a|b)^3$
- has substring abc: (a|b|c)*abc(a|b|c)*
- even number of a's: b*(ab*ab*)*



Stephen Kleene

(Regular Language Guru)

FSM Implementation

Reasonable Implementations:

- Table Method
- Case Method

Follow transitions in a simple loop.

Algorithm:

state ← Start

while there is still input {

c ← next input symbol

if transition(state, c) exists then

state ← transition(state, c)

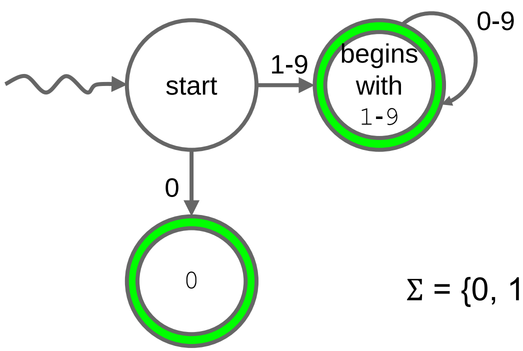
else

Reject (OR . . . state ← Dead)

}

if state is a Final state then Accept

else Reject



$\Sigma = \{0, 1, 2, \ldots, 9\}$

	0	1-9
Start	Begin w/ 0	Begin w/ 1-9
Begin w/ 0	Dead	Dead
Begin w/ 1-9	Begin w/ 1-9	Begin w/ 1-9

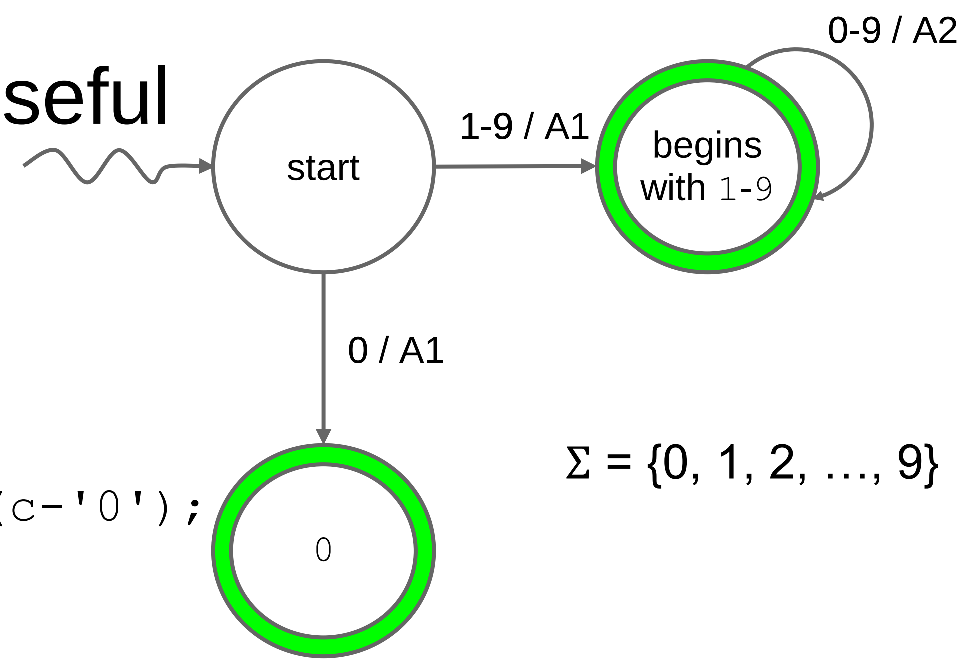
FSM Augmentation: Actions

While following a transition, perform an action

- place actions on transitions following a slash
- should compute a useful property of the word

E.g., What might be a useful property?

- the integer's `value`
- A1: `val = c - '0';`
- A2: `val = 10*val + (c-'0');`



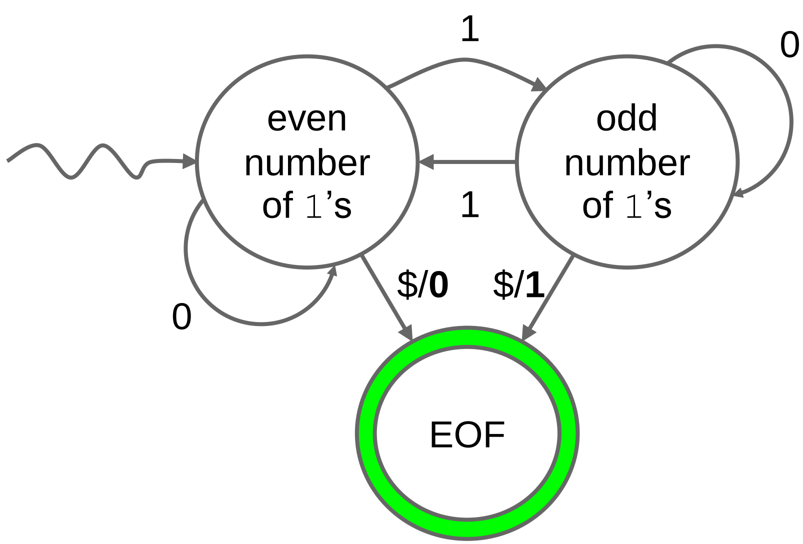
FSM Augmentations: Output

Another possible action: output

- need to add a special symbol for EOF (usually `$`)

Problem: Construct a FSM with output that reports the parity of a sequence of bits

- E.g., 1011 → 1, 11011 → 0, λ → 0



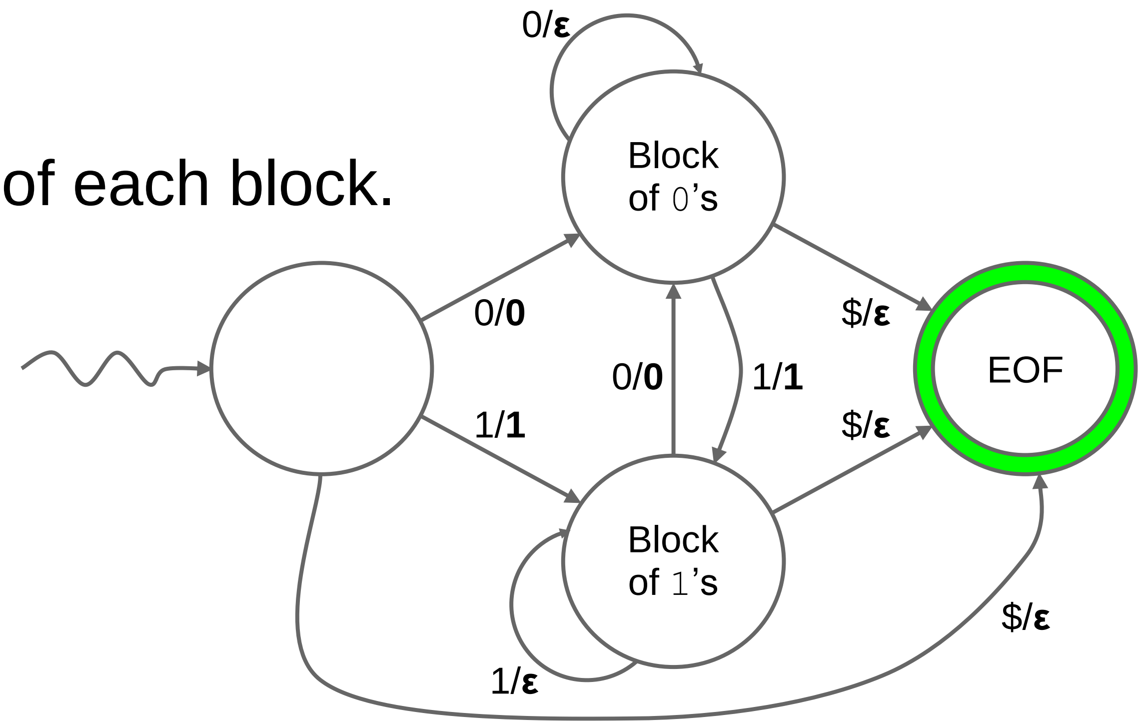
Example: Block Reduction

Problem: Construct a FSM with output that reports the 0/1 blocks of a binary sequence

- E.g., 1110000100111100011 → 1010101

Strategy:

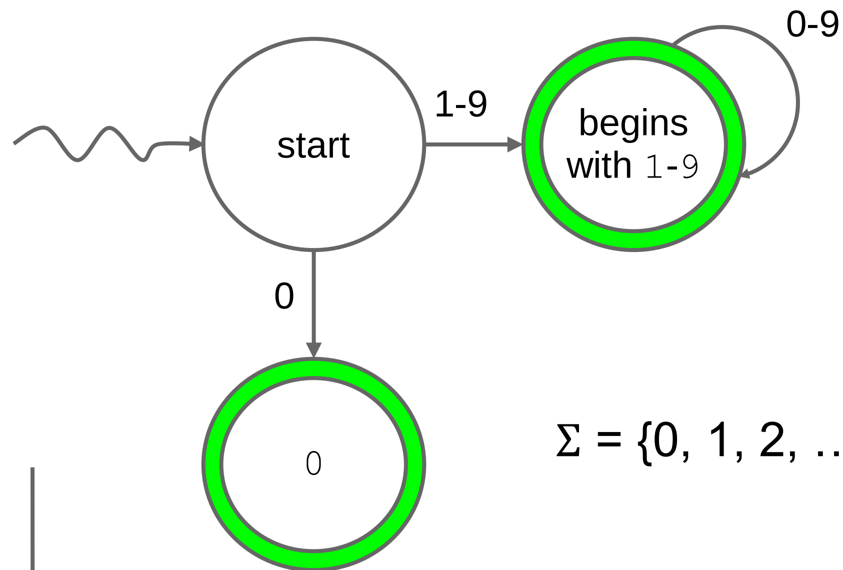
- Output the first of each block.



Case Method

Algorithm:

- Use a large if / else if / . . .
- Use a nested switch / case



Σ = {0, 1, 2, …, 9}

```
switch (state) {
  case Start:
    switch (c) {
      case '0':
        state = BeginWith0;
        break;
      case '1':
      case '2':
      case '3':
      case '4':
      case '5':
      case '6':
      case '7':
      case '8':
      case '9':
        state = BeginWith1to9;
    } break;
```

```
  case BeginWith0:
    state = Dead;
    break;

  case BeginWith1to9:
    break;

  default:
    state = Dead;
}
```