Introduction to Formal Languages

CMPT 125
Mo Chen
SFU Computing Science
23/3/2020

Lecture 29

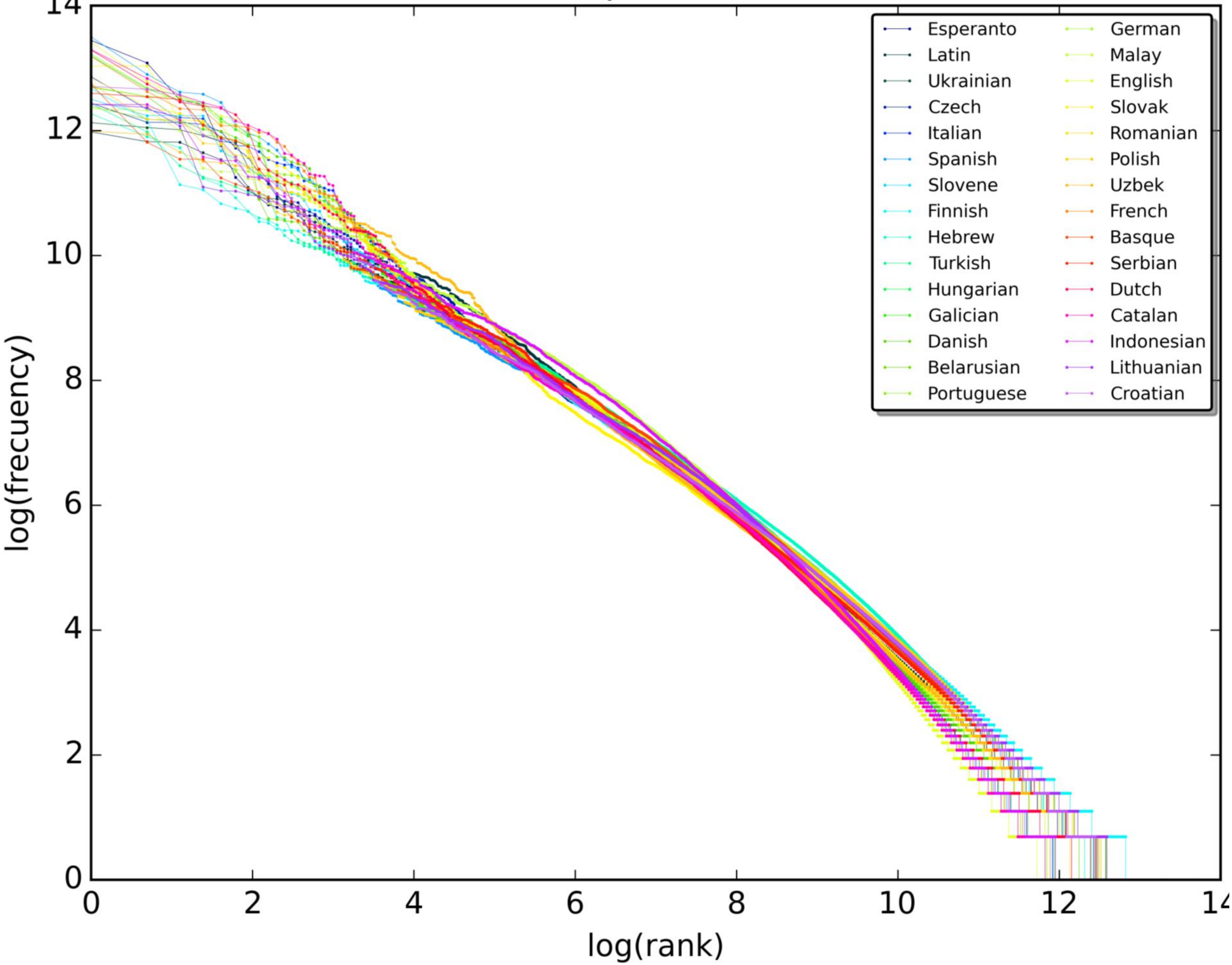
Today:

- Formal Languages
- Finite State Machines

Natural Languages

A *natural language* is used for the purposes of human communication

- spoken, written, or gestured
- E.g., English, French, Mandarin
- Time-intensive for us → universally, the (k+1)th most frequently used word/character occurs half as often as the kth word/character



Esperanto
Latin
Ukrainian
Czech
Italian
Spanish
Slovene
Finnish
Hebrew
Turkish
Hungarian
Galician
Danish
Belarusian
Portuguese
German
Malay
English
Slovak
Romanian
Polish
Uzbek
French
Basque
Serbian
Dutch
Catalan
Indonesian
Lithuanian
Croatian
log(frecuency)
log(rank)

Natural Languages

A *natural language* is used for the purposes of human communication

- spoken, written, or gestured
- E.g., English, French, Mandarin
- Time-intensive for us → universally, the (k+1)th most frequently used word/character occurs half as often as the kth word/character

There are some rules:

- valid characters (alphabet)
- valid words (spelling)
- valid sentences, punctuation (grammar)
- acceptable idioms

Formal Languages

A *formal language* is used to distinguish precisely what is allowed from what is not.

- expressed mathematically, often with recursion
- E.g., valid postfix expressions, valid C++

Similar to natural languages, there are:

- alphabets
- words
- grammars
- but no idioms

Alphabets and Words

An *alphabet* is a finite collection of symbols

- E.g., $\Sigma = \{a, b, c, \ldots, z\}$ — letters of the alphabet
- E.g., $\Sigma = \{0, 1, 2, \ldots, 9\}$ — base ten digits
- E.g., $\Sigma = \{0, 1\}$ — binary digits

A *word* is a finite sequence of alphabet symbols

- symbols may be repeated, e.g., *baa*, *100*, *sheep*
- order matters, e.g., *stressed* vs *desserts*
- word of length 0 is special, i.e., the *empty string* (λ, ε)

Distinguish which words are valid vs invalid.

Languages

A [*formal*] *language* is a set of words.

- can be finite, e.g., L = {all valid English words}
- can be infinite, e.g., L = {all valid integers}
- remember that words are always of finite length

E.g., Let L = {all valid C++ programs}

- Q. What's the alphabet?
- Q. What are the words?
- Q. Is L finite or infinite?
- Q. What does it mean to have an infinite length word?

- ASCII?
- Unicode?
- Tokens?

Specifying Formal Languages

Just like in natural language, use a *grammar*

- describes the symbols allowed and the order that they should appear
- usually specified recursively
- E.g., A valid *sentence* is a *noun phrase* followed by a *verb phrase* followed by a *subordinate clause*.

  A *subordinate clause* may be composed of the symbol “where” followed by a valid sentence.

- E.g., A *valid postfix expression* is either: a single number OR

  two *valid postfix expressions* followed by an operator.

Can represent a grammar using *production rules*:

- E.g., Grammar for postfix: $E \rightarrow$ number

  $E \rightarrow E\ E$ operator

Parse

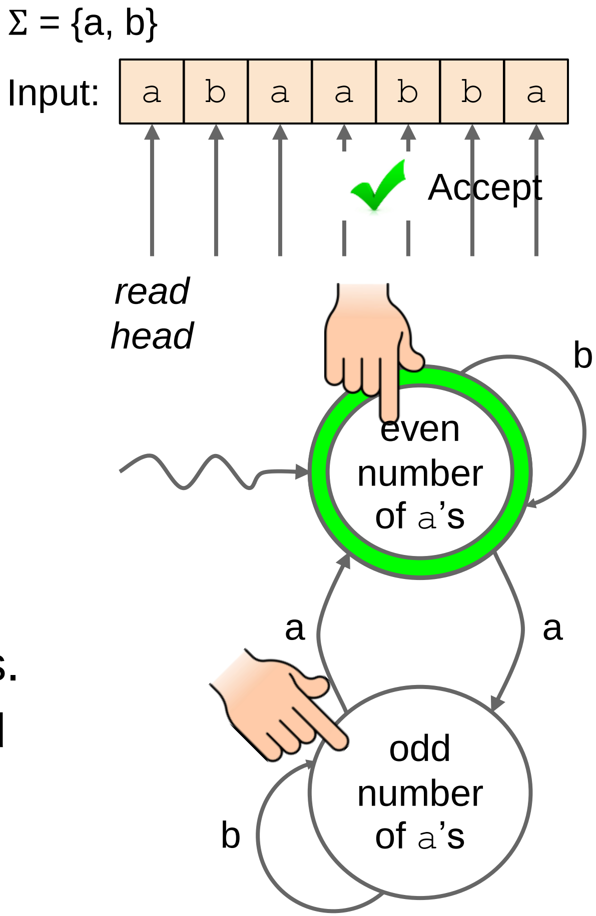
Write algorithms to decide inclusion/exclusion in the language

Modelling Computation

To decide a language, try a *finite state machine* (FSM).

Rules of the Game:

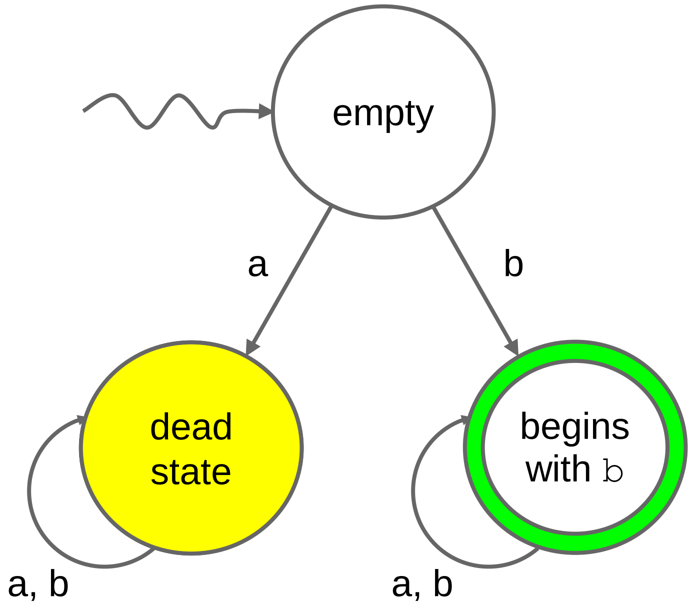
- Finite number of states.
- The FSM reads one character at a time.
- The *next state* is determined by examining the *current state* and the next input character, and nothing else.
- Each state has at most one *transition* on any given character.
- One state is identified as the *Start* state
- One or more states are designated *Final* states.
- Under no circumstances may a previously read character be examined again.
- If the last state is a final state: *Accept*
- If not: *Reject*



Two Puzzles for You

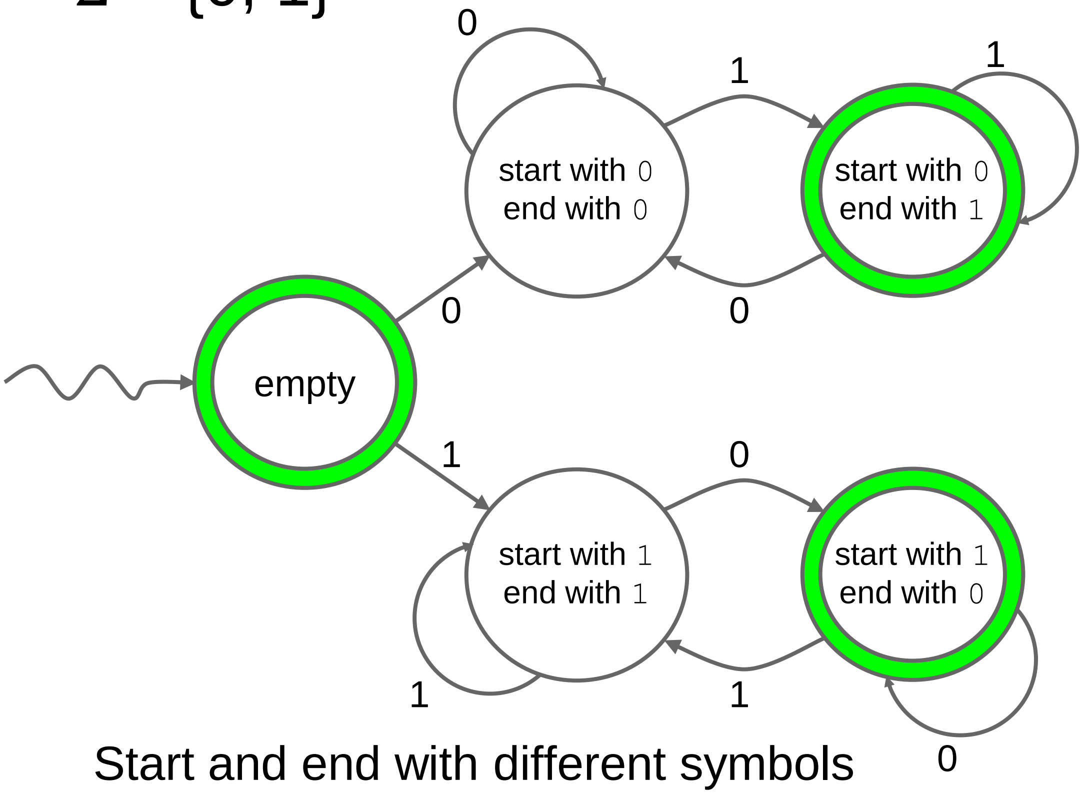
Q. What languages do these FSMs represent?

$\Sigma = \{a, b\}$



Begins with b

$\Sigma = \{0, 1\}$



Start and end with different symbols

Using The Dead State

Default transition to the dead state if no transition present.

Q. Build a FSM that accepts all words of length 3. $\Sigma = \{a, b\}$.

Q. Build a FSM that accepts all decimal integers.

Leading zeroes are disallowed. $\Sigma = \{0, 1, 2, \ldots, 9\}$