Introduction to Trees

CMPT 125
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Lecture 26

Today:

- Graphs and Rooted Trees
- Tree Anatomy
- Expression Trees
A graph depicts the relationships among a collection
- the collection is known as the **vertices** (depicted by dots or junctions)
- the relations are known as the **edges** (depicted by lines between dots)

What are the vertices and edges in these common graphs?
A *graph* depicts the relationships among a collection

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What are the vertices and edges in these common graphs?

**Gas Pipe System**

**Vertices?**
- Stations

**Edges?**
- Pipes
A **graph** depicts the relationships among a collection

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What are the vertices and edges in these common graphs?

**Road Map**

**Vertices?**
- Intersections

**Edges?**
- Roads
Graphs

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What are the vertices and edges in these common graphs?

Skytrain Map

**Vertices?**
- Stations

**Edges?**
- Tracks
Graphs

A graph depicts the relationships among a collection

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- the relations are known as the **edges** (depicted by lines between dots)

What are the vertices and edges in these common graphs?

Social Media

Vertices?
- People

Edges?
- Friends
A graph depicts the relationships among a collection

- the collection is known as the vertices (depicted by dots or junctions)
- the relations are known as the edges (depicted by lines between dots)

What are the vertices and edges in these common graphs?
Paths and Connectivity

A *path* from vertex $u$ to vertex $v$ is sequence of edges which connect a sequence of vertices between them.

A graph is *connected* if each pair of vertices has a path.

Q. When might disconnected graphs arise?
Weighted Graphs

- Each edge has an associated weight
- Weight can represent distance or cost
Representing Graphs

- Graphs as a collection of node objects
  ```cpp
template <class T>
class node {
  private:
    // a node holds data of a generic type
    T data;

    // list of nodes that this node is connected to
    LL<node> * connections;

  public:
    ...;
    ...
};
```

- Adjacency matrix
Two (Extremely) Useful Classes of Algorithms

- Shortest Path Algorithms (A*, Dijkstra's, etc.)
- Search engines (e.g. PageRank)
A tree is a minimally connected graph

- all vertices connected
- no cycles

E.g., Skytrain is almost a tree.

- Q. How can we make it into a tree?
Example: Building a Network

The Problem: Join a community with the least amount of infrastructure.

- roads
- train tracks
- ethernet trunk

Strategy:
- Use the smallest tree, a minimum spanning tree

Algorithm:
- Repeatedly select the smallest connection that doesn’t form a cycle.
- Kruskal’s Algorithm
Directed Graphs and Rooted Trees

Sometimes denote a *direction* on an edge:

- one way traffic
- pipeline flow
- web links
- dependency
- A “likes” B

A *rooted tree* is a directed version of the tree where exactly one vertex, called the *root*, has no inbound edges, but all other vertices have exactly one inbound edge.
Rooted Tree Examples

Directory Tree

/  
  ├── bin
  │    ├── btlmem
  │    │    └── 1
  │    └── 2
  ├── dev
  │    └── 3
  ├── etc
  │    └── 4
  ├── home
  │    ├── jdelphik
  │    │    └── 5
  │    └── parkania
  │         └── 6
  └── usr
       ├── formics
       │    └── 4
       └── cmpt118
            └── 3
Rooted Tree Examples

Infection Trees

- communicable period vs...
- infection rate

Short communicable period
High infection rate

Long communicable period
Low infection rate
Mergesort

Recursion tree:

- $O(N)$ work per row
- $O(\log N)$ rows

$\Rightarrow O(N \log N)$ running time
Tree Terminology

- Inbound edge: *parent*
- Outbound edge: *child*
- Common parent: *sibling*
- On path to root: *ancestor*
- Path away from root: *descendant*
- No child: *leaf*
- At least one child: *internal node*
Binary Trees

An $m$-ary tree is a tree in which each vertex has at most $m$ children.

The common name for a 2-ary tree is binary tree.

- denote its children as the *left child* and *right child*

Any algebraic expression can be represented as a binary tree.

Represents the expression:

$$(4 \times (x / 7)) + (x - (y / 3))$$

Q. How do you evaluate an expression tree?