



Introduction to Trees

CMPT 125

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SFU Computing Science

16/3/2020

Lecture 26

Today:

- Graphs and Rooted Trees
- Tree Anatomy
- Expression Trees

Graphs

Also: *nodes*

Also: *arcs*

A *graph* depicts the relationships among a collection

- the collection is known as the **vertices** (depicted by dots or junctions)
- the relations are known as the **edges** (depicted by lines between dots)

What are the vertices and edges in these common graphs?

Graphs

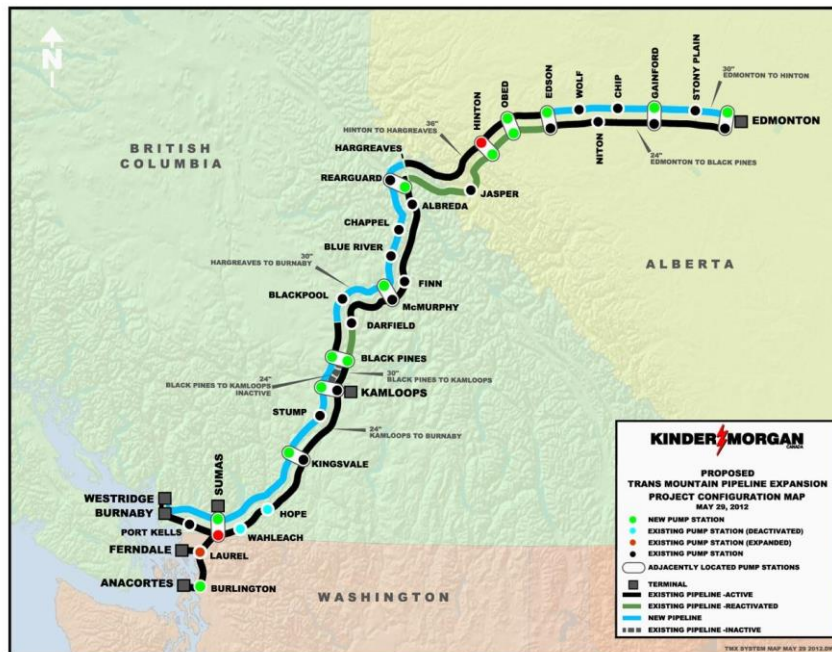
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Gas Pipe System

Vertices?

- Stations

Edges?

- Pipes

Graphs

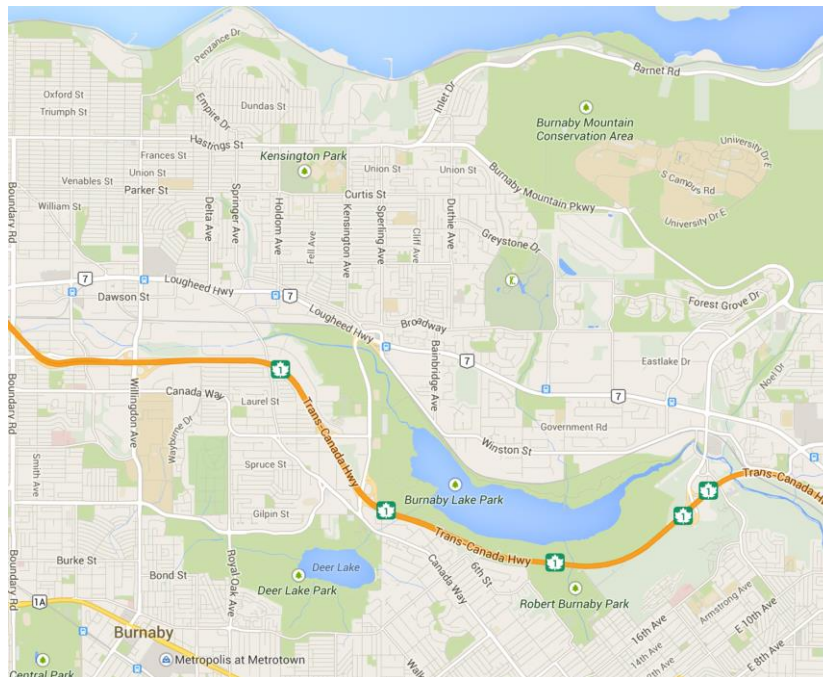
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Road Map

Vertices?

- Intersections

Edges?

- Roads

Graphs

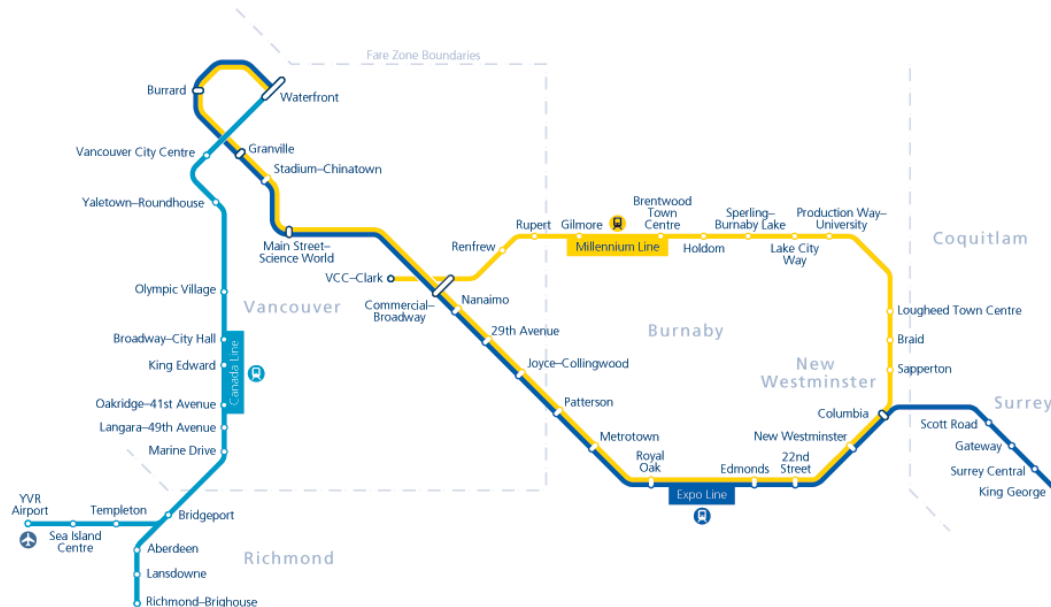
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Skytrain Map

Vertices?

- Stations

Edges?

- Tracks

Graphs

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Social Media

Vertices?

- People

Edges?

- Friends

Graphs

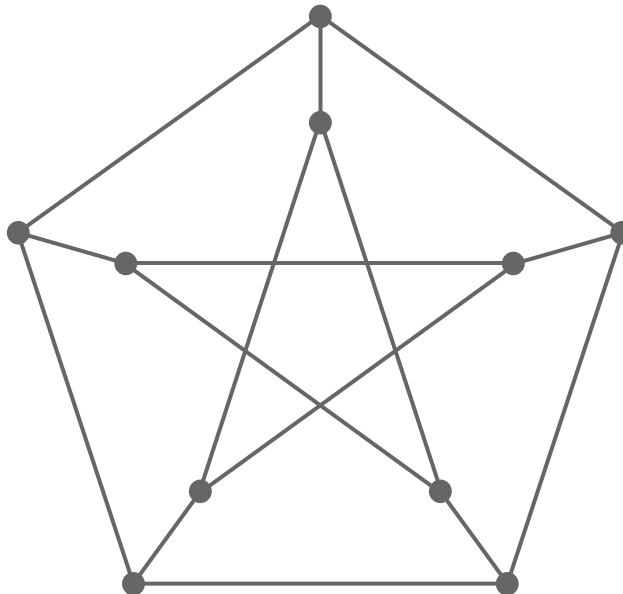
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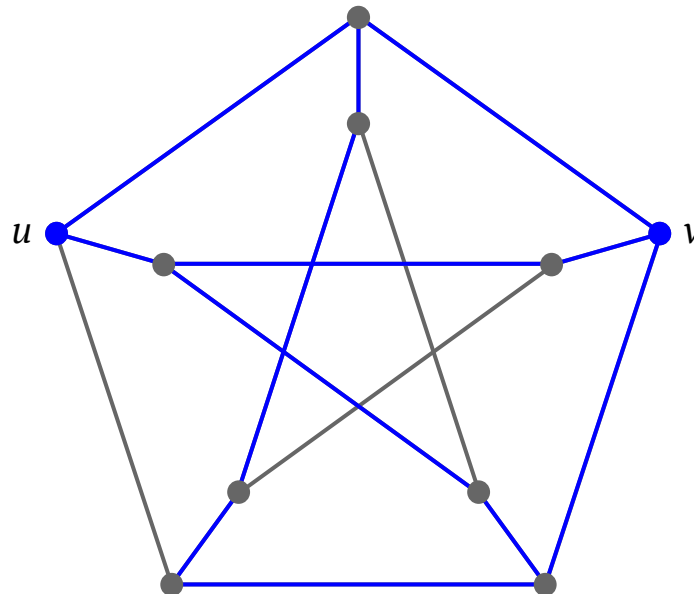


Paths and Connectivity

A *path* from vertex u to vertex v is sequence of edges which connect a sequence of vertices between them.

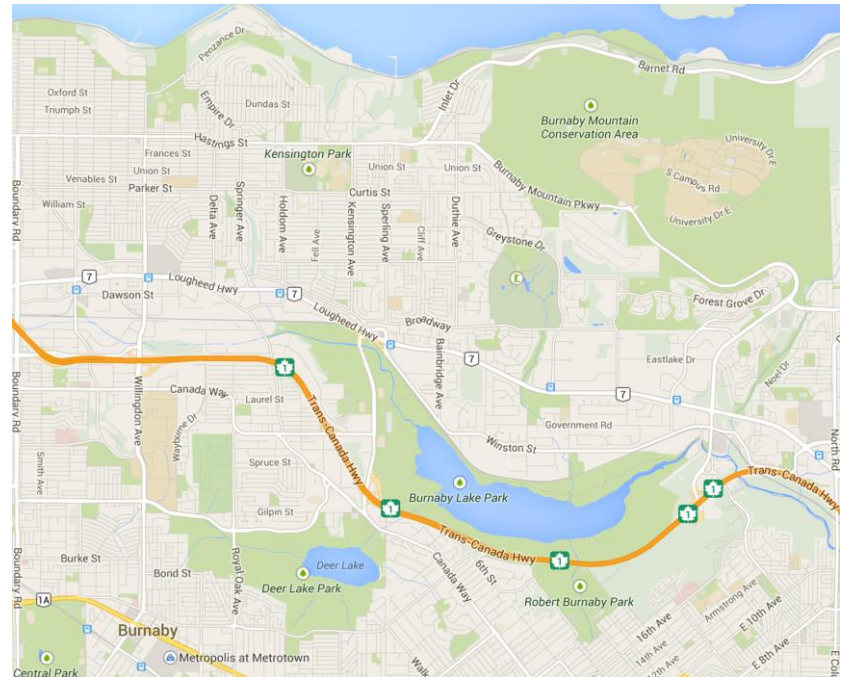
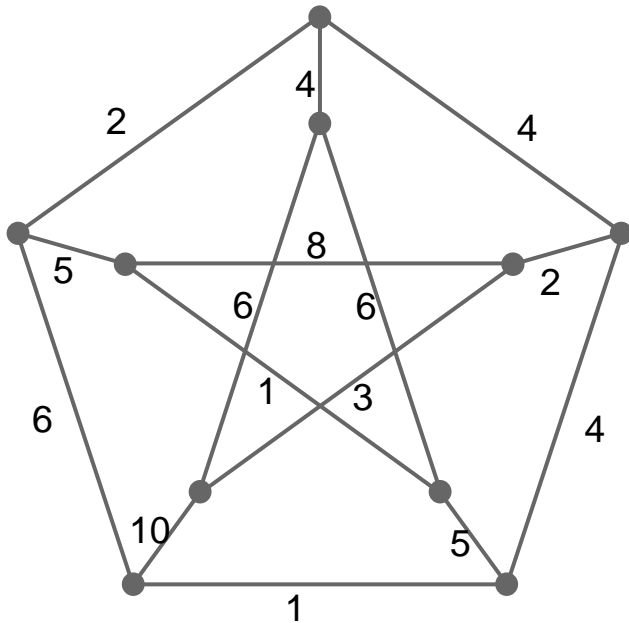
A graph is *connected* if each pair of vertices has a path

Q. When might disconnected graphs arise?



Weighted Graphs

- Each edge has an associated weight
- Weight can represent distance or cost



Representing Graphs

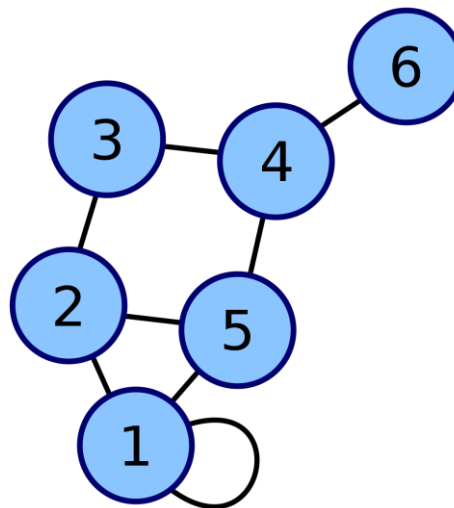
- Graphs as a collection of node objects

```
template <class T>
class node {
private:
    // a node holds data of a generic type
    T data;

    // list of nodes that this node is connected to
    LL<node> * connections;

public:
    ...
    ...
}
```

- Adjacency matrix

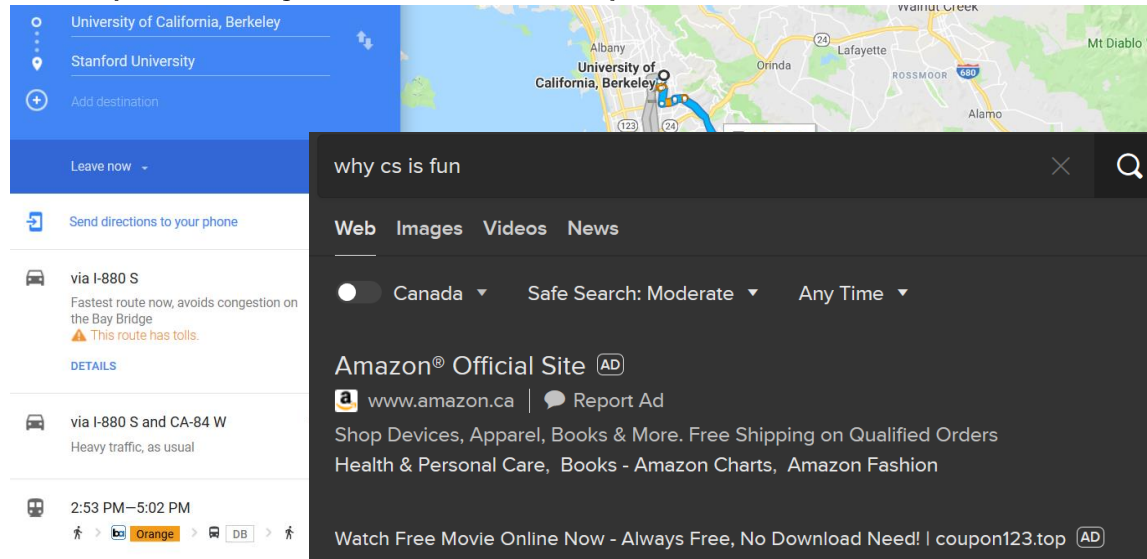


$$\begin{pmatrix} 2 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

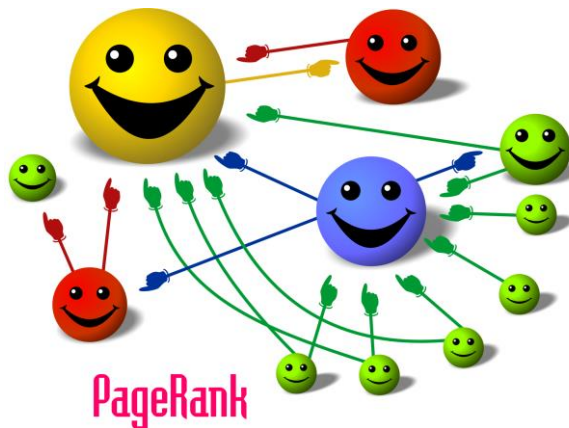
Two (Extremely) Useful Classes of Algorithms

- Shortest Path Algorithms (A*, Dijkstras, etc.)

50	51	54	57	65	69
48	52	51	58	64	64
47	53	52	54	60	63
45	48	49	56	64	61
44	45	51	57	58	60
42	46	50	52	58	59



- Search engines (e.g. PageRank)



why cs is fun

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Why It's Fun: Counter-Strike | PC Gamer

https://www.pcgamer.com/why-its-fun-counter-strike/

On the eve of Valve hitting go on Counter-Strike: GO, I thought it'd be useful to revisit why the once-mod continues to have its hooks in so many of us. Just like true love or a really outstanding ...

Why Computer Science? | UCSB Computer Science

https://www.cs.ucsb.edu/about/cs

Computer scientists are involved in creating technology and systems that are used in a wide range of industries, including medicine, communications, entertainment, manufacturing, business, and science. CS research pushes the state-of-the-art in computing theory and practice, and it leads to new technologies...

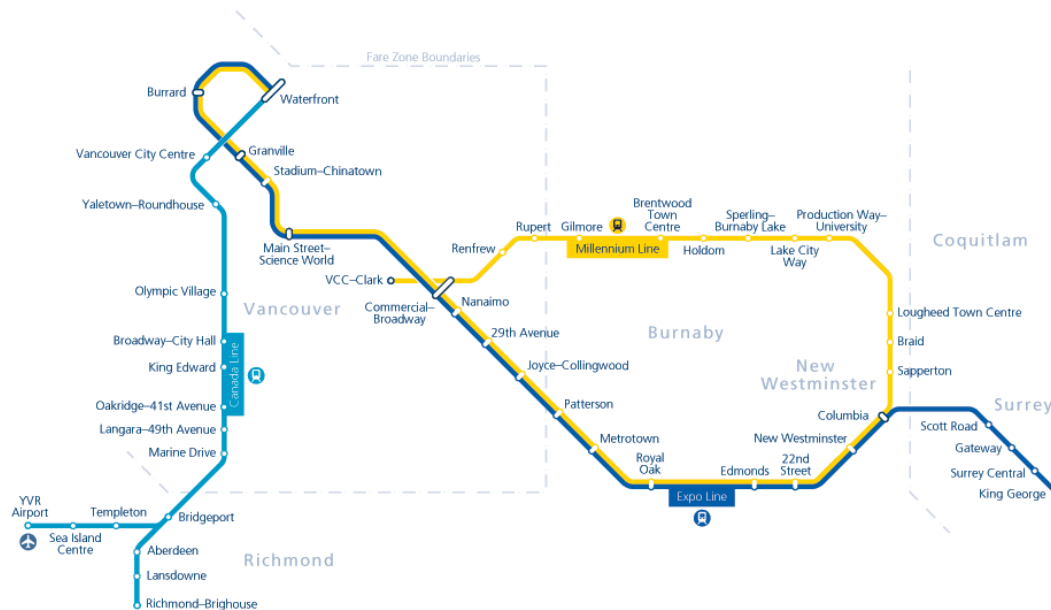
Trees

A *tree* is a minimally connected graph

- all vertices connected
- no cycles

E.g., Skytrain is almost a tree.

- Q. How can we make it into a tree?



Example: Building a Network

The Problem: Join a community with the least amount of infrastructure.

- roads
- train tracks
- ethernet trunk

Strategy:

- Use the smallest tree, a *minimum spanning tree*

Algorithm:

- Repeatedly select the smallest connection that doesn't form a cycle.
- Kruskal's Algorithm



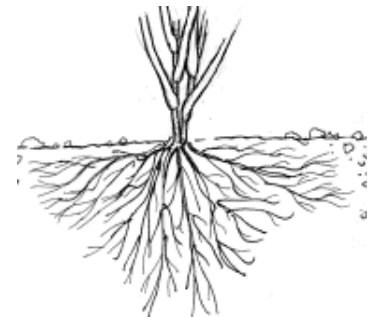
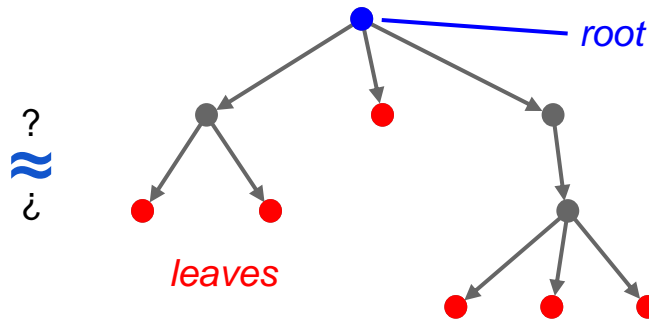
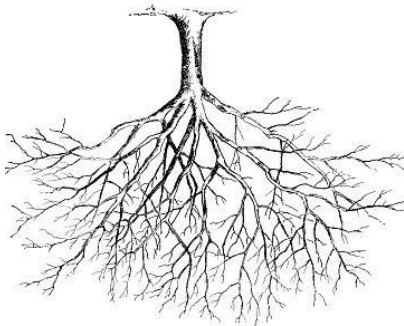
Directed Graphs and Rooted Trees

Sometimes denote a *direction* on an edge:

- one way traffic
- pipeline flow
- web links
- dependency
- A “likes” B

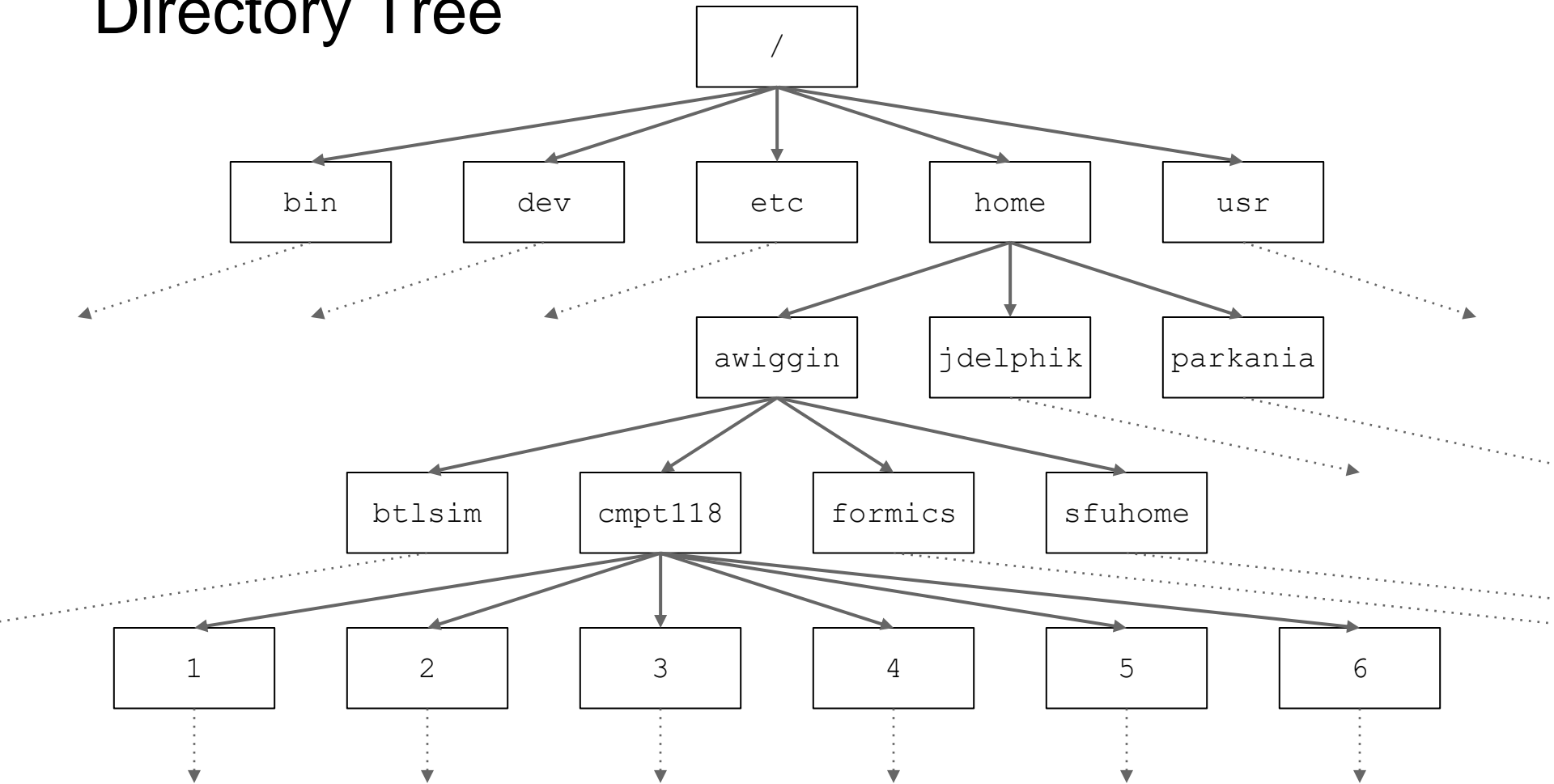


A *rooted tree* is a directed version of the tree where exactly one vertex, called the *root*, has no inbound edges, but all other vertices have exactly one inbound edge.



Rooted Tree Examples

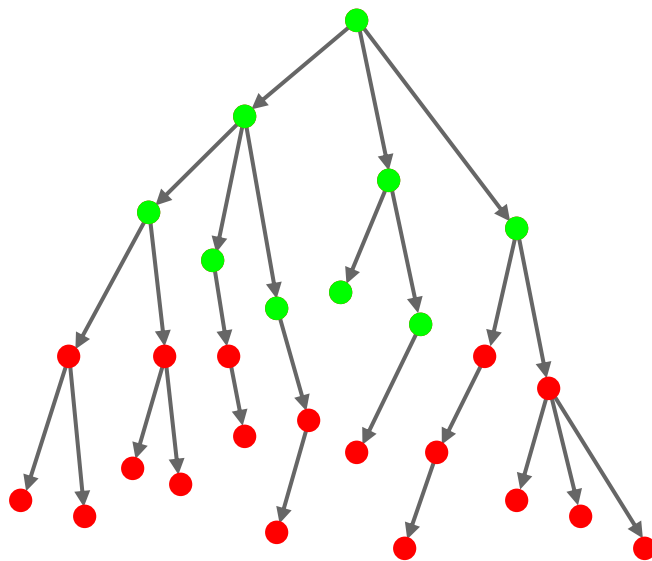
Directory Tree



Rooted Tree Examples

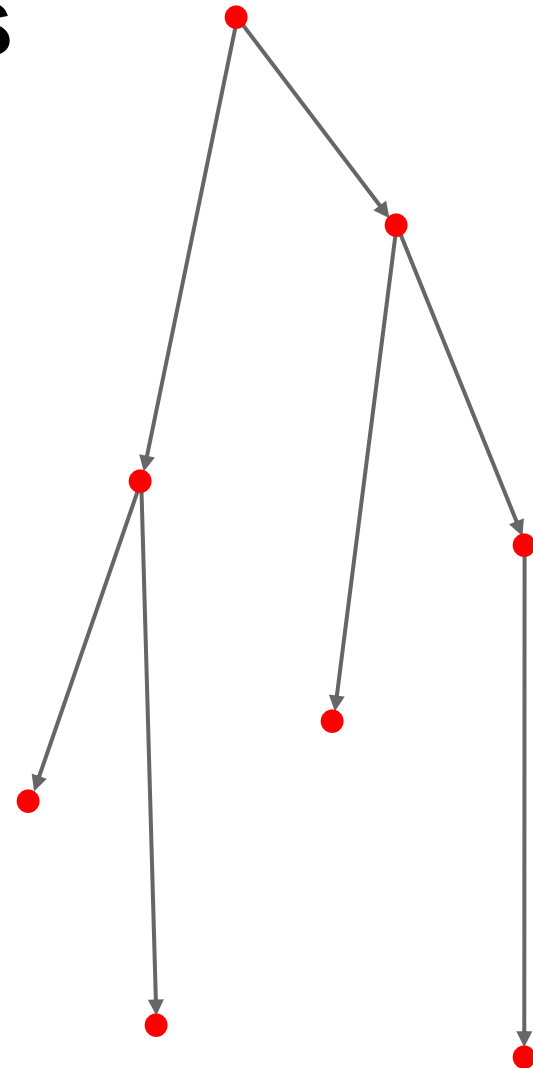
Infection Trees

- communicable period vs . . .
- infection rate



Short communicable period
High infection rate

time

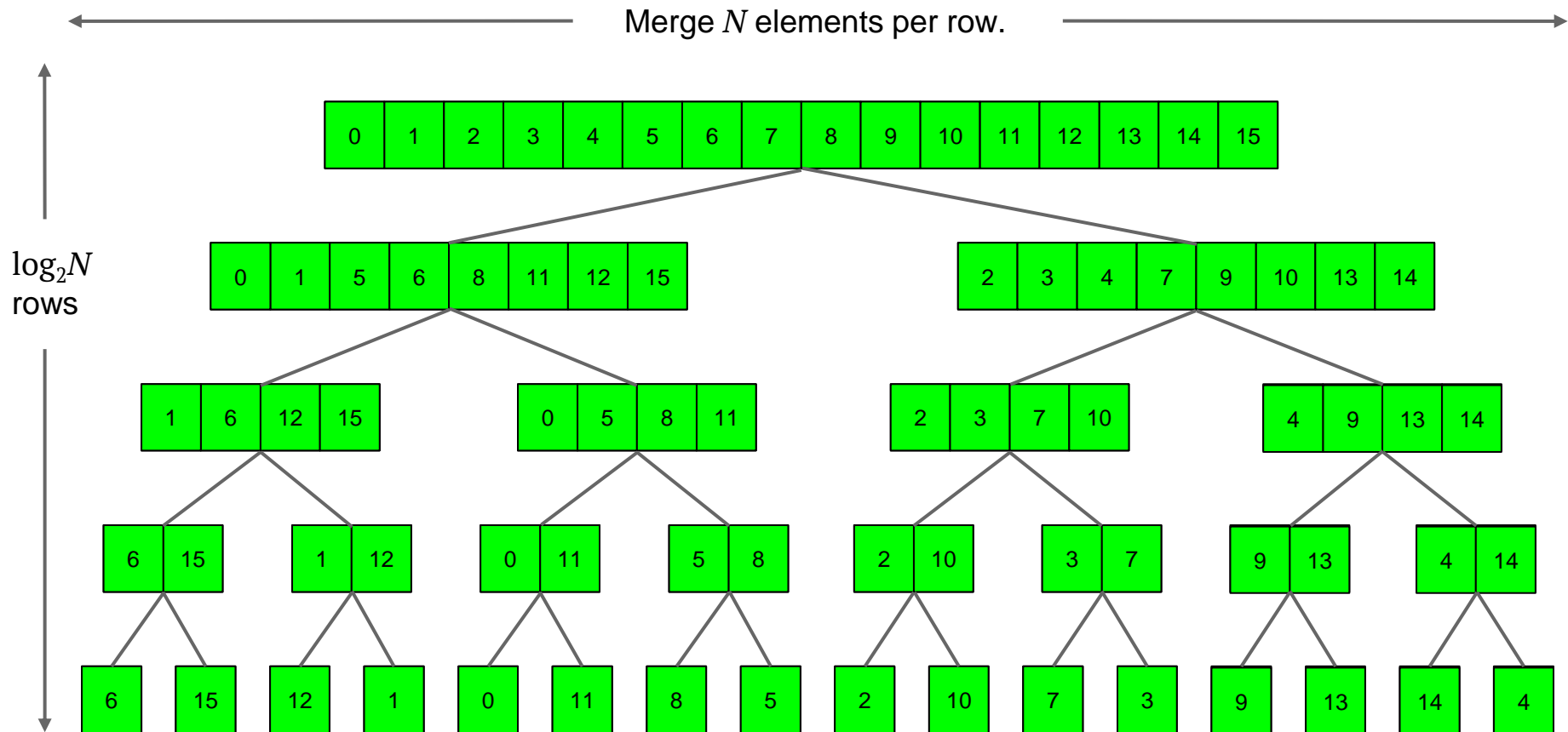


Long communicable period
Low infection rate

Mergesort

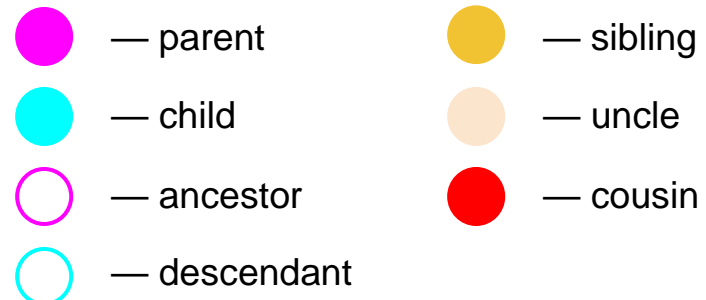
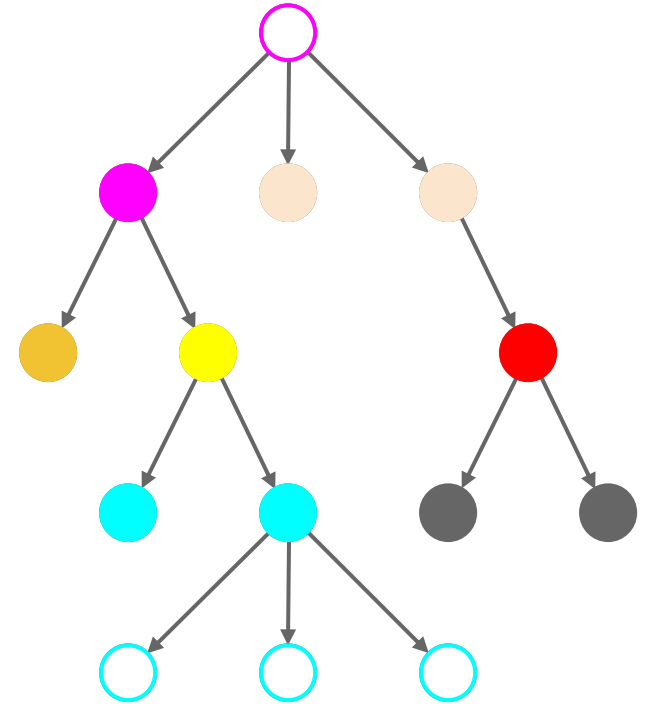
Recursion tree:

- $O(N)$ work per row
 - $O(\log N)$ rows
- $\Rightarrow O(N \log N)$ running time



Tree Terminology

- Inbound edge: *parent*
- Outbound edge: *child*
- Common parent: *sibling*
- On path to root: *ancestor*
- Path away from root: *descendant*
- No child: *leaf*
- At least one child: *internal node*



Binary Trees

An *m*-ary tree is a tree in which each vertex has at most *m* children.

The common name for a 2-ary tree is *binary tree*.

- denote its children as the *left child* and *right child*

Any algebraic expression can be represented as a binary tree.

Represents the expression:

$$(4 * (x / 7)) + (x - (y / 3))$$

Q. How do you evaluate an expression tree?

