# Binary Search

CMPT 125
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SFU Computing Science
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# Lecture 12

Today

- Binary Search

# What if the array was ordered?

Think of searching a dictionary for a word?

- Strategy: *Not* one word at a time in sequential order starting from aardvark, etc.

- Strategy: Jump to where you estimate the word to be based on what you know about the alphabet.

  Refine your jumps + hone in on the correct page quickly.

This is the main idea behind *binary search*.

# Divide and Conquer

Generic Strategy (*Paradigm*):

1. **Divide:** Cut the array into 2 or more roughly equally sized pieces

2. **Conquer:** Use what you know about the pieces to solve the original problem

# Binary Search

Strategy: Divide and Conquer

1. Examine the *middle* element of the array of candidates. This divides the array into two [roughly] equal halves.
2. Compare the middle element with the target.
   - If middle < target then throw out the first half.
   - But if middle > target then throw out second half.
3. Repeat 1-3 until middle == target (found!) or no candidates remain (fail!).

E.g., target = `42`:

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| -8 | -7 | -5 | -2 | 0 | 4 | 6 | 7 | 17 | 20 | 28 | 29 | 42 | 49 | 64 |

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return true
or index=12

# Binary Search

## Requirements (Pre-Conditions):

- Candidate array must be sorted

## How to keep track of the list of candidates?

- Use integers `first` and `last` for remaining candidates `arr[first..last]`
- Initially, `first=0`; `last=len-1`
- Middle element is at index `(first+last)/2`

E.g., target = `42`:

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first = 0
last = 14
mid = 7

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| | | |
|---|---|---|
| first = 0 | | 8 |
| last = 14 | → | 14 |
| mid = 7 | | 11 |

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| | | | | |
|---|---|---|---|---|
| first = | 0 | 8 | 12 | 12 |
| last = | 14 | 14 | 14 | 12 |
| mid = | 7 | 11 | 13 | 12 |

return true
or index=12

# Binary Search Code

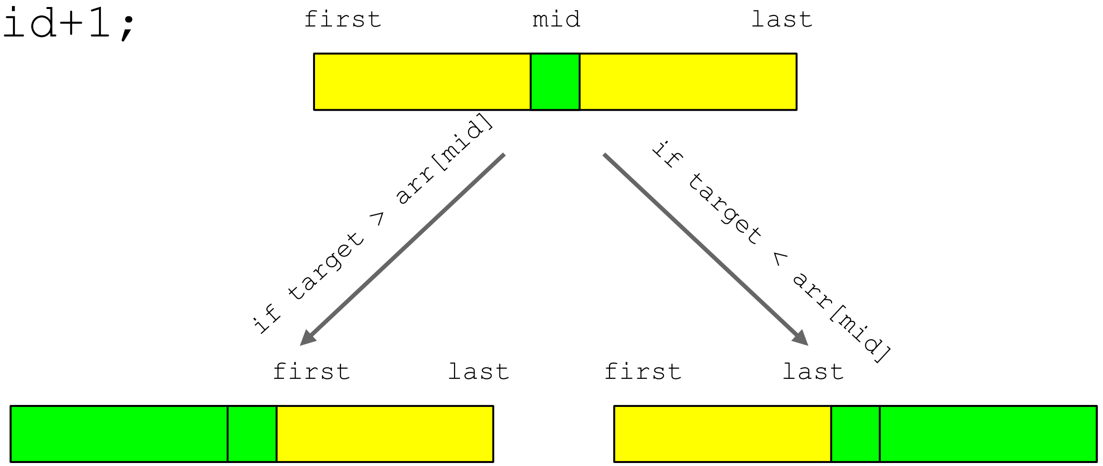
```
int BinarySearch(int arr[], int len, int target) {
```

- Search candidate array `arr[first..last]` while not empty
  - Compare with the middle element
  - Algorithm:
    - found if equal to `target`, so return position
    - throw out second half if greater than `target` OR
    - throw out first half if less than `target`
- No candidates, so return fail

```
}
```

# Binary Search Code

```
int BinarySearch(int arr[], int len, int target) {
    int first = 0;
    int last = len-1;
    while(first <= last) {
       //   Q.  What’s a good assertion this time?
       int mid = (first+last) / 2;
       if (target == arr[mid]) return mid;
       if (target < arr[mid]) last = mid-1;
       else first = mid+1;
    }
    return -1;
}
```



# Binary Search - Loop Free Version

```
int BinarySearch(int arr[], int len, int target) {
```

- Search candidate array `arr[0..len-1]`
- Algorithm:
  - return fail if empty

- Compare with the middle element + re-search
- Algorithm:
  - found if equal to `target`, so return true
  - throw out second half if greater than `target` OR
  - throw out first half if less than `target`

```
}
```

# Binary Search - Loop Free Version

```
int BinarySearch(int arr[], int len, int target) {
    if (len <= 0) {
        return 0;
    }
```

- Compare with the middle element + re-search
- Algorithm:
  - found if equal to `target`, so return true
  - throw out second half if greater than `target` OR
  - throw out first half if less than `target`

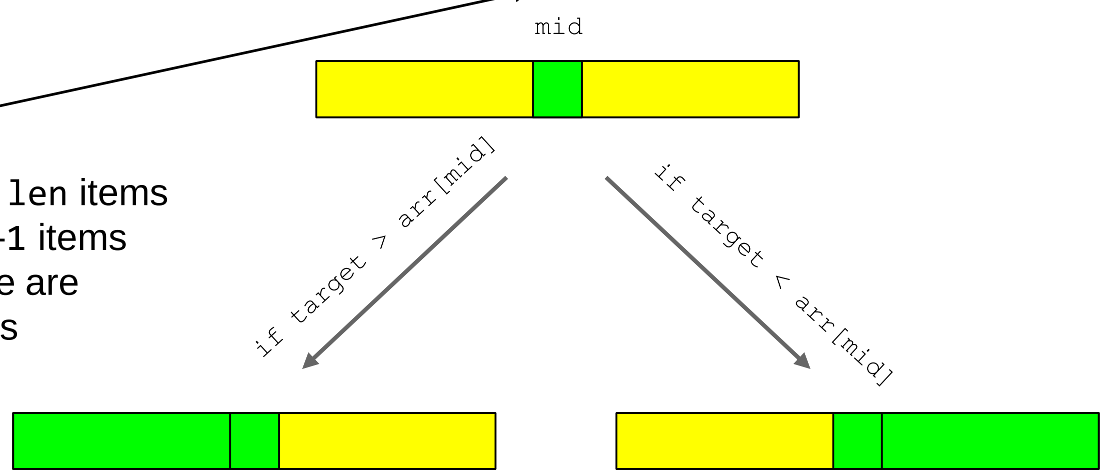
```
}
```

# Binary Search - Loop Free Version

```
int BinarySearch(int arr[], int len, int target) {
    if (len <= 0) {
        return 0;
    }
    int mid = len/2;
    if (target == arr[mid]) return 1;
    if (target < arr[mid]) return BinarySearch(arr,mid,target);
    else return BinarySearch(arr+mid+1,len-mid-1,target);

}
```

If we go from index `0` to `len-1`, there are `len` items
If we go from index `a` to `b`, there are `b-a+1` items
If we go from index `mid+1` to `len-1`, there are
`(len-1) - (mid+1) + 1` items



# Analysis of Binary Search

What's the worst case on an array of length $N$?

- After one iteration, the possible candidates are [roughly] cut in half.

After $k$ iterations, how many candidates remain?

- Roughly $N / 2^k$

When do you run out of candidates?

- when $2^k \geqslant N$
- i.e., after $k \geqslant \log_2 N$ iterations

Thus binary search runs in $O(\log N)$.

# Linear Search vs Binary Search

Even though the inner loop of binary search is more complex than linear search, we expect $O(\log N)$ to outperform $O(N)$ as $N$ gets large.

| $N$ | Linear Search $(3 + 4N)$ | Binary Search $(4 + 12 \log_2(N+1))$ |
|---|---|---|
| 1 | 7 | 16 |
| 3 | 15 | 28 |
| 7 | 31 | 40 |
| 15 | 63 | 52 |
| 100 | 403 | 88 |
| 1000 | 4003 | 124 |
| $10^6$ | 4000003 | 244 |
| $10^9$ | $4 \times 10^9$ | 364 |

# Linear Search vs Binary Search

- Binary search has a fast running time.
- Disadvantages?
  - Harder to code
  - Requires the array be sorted
- Keeping the array sorted can be expensive!
  - Significantly more searching than update? Keep list sorted (slow) and use (fast) binary search
  - Significantly more update than search? Keep array unsorted (fast) and use (slow) linear search