

CMPT 307

Fall 2020

Assignment 1

Due Friday October 2 at 23:59

Submit on CS Submission Server/CourSys.

4 problems; 10 points each.

1. Express $\sum_{i=0}^n (7i^2 - 4i + 3)$ as a polynomial $p(n)$. Then prove that the sum = $p(n)$ **by induction**. Do not omit any steps of the induction proof.

2. *Agrisort* is a sorting algorithm.

Agrisort(A, i, j) // A is array to sort; i and j are start and end indices.

$n = j - i + 1$

If ($n < 18$) {

sort A[i...j] by insertion-sort

return

}

$m_1 = i + 2 * n / 3$

$m_2 = i + n / 3$

Agrisort(A, i, m_1)

Agrisort(A, m_2 , j)

Agrisort(A, i, m_1)

- a. What is the asymptotic worst-case running time of Agrisort? Show your work.
 - b. Prove that Agrisort(A, 1, n) correctly sorts the array A of n elements.
3. Let C be a collection of integers that is represented by two n-element arrays of integers A and B. Each of A and B is sorted from lowest to highest. Give an $O(\log n)$ -time algorithm to find the median of C (that is, of the elements of A and B combined). Give pseudocode and analyze it.
 4. An *Elder Matrix* is a m by n matrix such that each row is sorted in ascending order and each column is sorted in ascending order. Entries in the matrix are allowed to be finite integers or ∞ . We use ∞ for nonexistent entries. An Elder Matrix is therefore a holder for up to mn integers.

Here's a sample 5 x 5 Elder Matrix:

1	4	8	21	28
14	17	24	33	45
22	30	42	58	79
37	41	48	∞	∞
88	92	∞	∞	∞

- (a) give an algorithm to perform an EXTRACT-MIN on a m by n Elder Matrix that is not empty. Your algorithm should run in $O(m+n)$ time. Your algorithm should use a recursive function that solves an m by n problem by *recursively* solving either a $(m-1)$ by n problem or a m by $(n-1)$ problem. Give pseudocode and analyze it.
- (b) give an algorithm to perform an INSERT of an integer into a m by n Elder Matrix in $O(m+n)$ time.
- (c) Using no other sorting method as a subroutine, show how to use a n by n Elder Matrix to sort n^2 numbers in $O(n^3)$ time. (Suppose we let $k = n^2$. Then this is an $O(k^{1.5})$ sorting algorithm.)