1. Express $\sum_{i=0}^{n}(7i^2 - 4i + 3)$ as a polynomial $p(n)$. Then prove that the sum $= p(n)$ by induction. Do not omit any steps of the induction proof.

2. Agrisort is a sorting algorithm.
   Agrisort(A, i, j) // A is array to sort; i and j are start and end indices.
   
   n = j - i + 1
   If (n < 18) {
     sort A[i...j] by insertion-sort
     return
   }
   m1 = i + 2 * n / 3
   m2 = i + n / 3
   Agrisort(A, i, m1)
   Agrisort(A, m2, j)
   Agrisort(A, i, m1)

   a. What is the asymptotic worst-case running time of Agrisort? Show your work.
   b. Prove that Agrisort(A, 1, n) correctly sorts the array A of n elements.

3. Let C be a collection of integers that is represented by two n-element arrays of integers A and B. Each of A and B is sorted from lowest to highest. Give an $O(\log n)$-time algorithm to find the median of C (that is, of the elements of A and B combined). Give pseudocode and analyze it.

4. An Elder Matrix is a $m \times n$ matrix such that each row is sorted in ascending order and each column is sorted in ascending order. Entries in the matrix are allowed to be finite integers or $\infty$. We use $\infty$ for nonexistent entries. An Elder Matrix is therefore a holder for up to $mn$ integers.

   Here’s a sample 5 x 5 Elder Matrix:
1  4  8  21  28
14  17  24  33  45
22  30  42  58  79
37  41  48  ∞  ∞
88  92  ∞  ∞  ∞

(a) give an algorithm to perform an EXTRACT-MIN on a $m$ by $n$ Elder Matrix that is not empty. Your algorithm should run in $O(m+n)$ time. Your algorithm should use a recursive function that solves an $m$ by $n$ problem by recursively solving either a $(m-1)$ by $n$ problem or a $m$ by $(n-1)$ problem. Give pseudocode and analyze it.

(b) give an algorithm to perform and INSERT of an integer into a $m$ by $n$ Elder Matrix in $O(m+n)$ time.

(c) Using no other sorting method as a subroutine, show how to use a $n$ by $n$ Elder Matrix to sort $n^2$ numbers in $O(n^3)$ time. (Suppose we let $k = n^2$. Then this is an $O(k^{1.5})$ sorting algorithm.)