

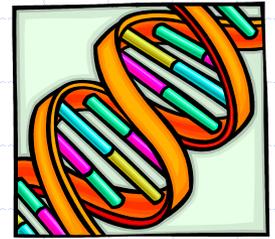


# Strings



- ◆ A string is a sequence of characters
- ◆ Examples of strings:
  - C++ program
  - HTML document
  - DNA sequence
  - Digitized image
- ◆ An alphabet  $\Sigma$  is the set of possible characters for a family of strings
- ◆ Example of alphabets:
  - ASCII
  - Unicode
  - $\{0, 1\}$
  - $\{A, C, G, T\}$
- ◆ Let  $P$  be a string of size  $m$ 
  - A substring  $P[i..j]$  of  $P$  is the subsequence of  $P$  consisting of the characters with ranks between  $i$  and  $j$
  - A prefix of  $P$  is a substring of the type  $P[0..i]$
  - A suffix of  $P$  is a substring of the type  $P[i..m-1]$
- ◆ Given strings  $T$  (text) and  $P$  (pattern), the pattern matching problem consists of finding a substring of  $T$  equal to  $P$
- ◆ Applications:
  - Text editors
  - Search engines
  - Biological research

# Brute-Force Pattern Matching



- ◆ The brute-force pattern matching algorithm compares the pattern  $P$  with the text  $T$  for each possible shift of  $P$  relative to  $T$ , until either
  - a match is found, or
  - all placements of the pattern have been tried
- ◆ Brute-force pattern matching runs in time  $O(nm)$
- ◆ Example of worst case:
  - $T = aaa \dots ah$
  - $P = aaah$
  - may occur in images and DNA sequences
  - unlikely in English text

**Algorithm** *BruteForceMatch*( $T, P$ )

**Input** text  $T$  of size  $n$  and pattern  $P$  of size  $m$

**Output** starting index of a substring of  $T$  equal to  $P$  or  $-1$  if no such substring exists

**for**  $i \leftarrow 0$  **to**  $n - m$

{ test shift  $i$  of the pattern }

$j \leftarrow 0$

**while**  $j < m \wedge T[i + j] = P[j]$

$j \leftarrow j + 1$

**if**  $j = m$

**return**  $i$  { match at  $i$  }

**return**  $-1$  { no match anywhere }

# Boyer-Moore Heuristics

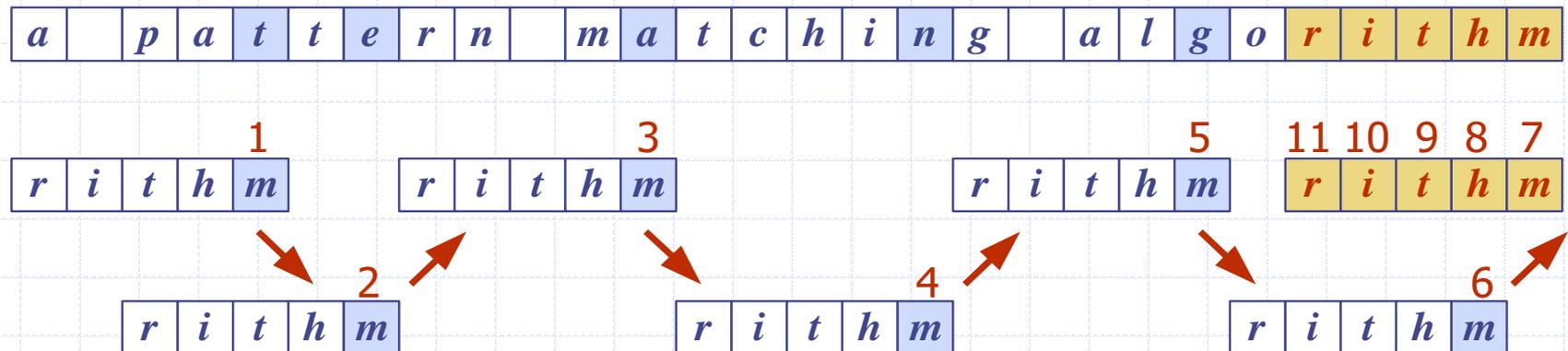
- ◆ The Boyer-Moore's pattern matching algorithm is based on two heuristics

**Looking-glass heuristic:** Compare  $P$  with a subsequence of  $T$  moving backwards

**Character-jump heuristic:** When a mismatch occurs at  $T[i] = c$

- If  $P$  contains  $c$ , shift  $P$  to align the last occurrence of  $c$  in  $P$  with  $T[i]$
- Else, shift  $P$  to align  $P[0]$  with  $T[i + 1]$

- ◆ Example



# Last-Occurrence Function

- ◆ Boyer-Moore's algorithm preprocesses the pattern  $P$  and the alphabet  $\Sigma$  to build the last-occurrence function  $L$  mapping  $\Sigma$  to integers, where  $L(c)$  is defined as
  - the largest index  $i$  such that  $P[i] = c$  or
  - $-1$  if no such index exists

- ◆ Example:

- $\Sigma = \{a, b, c, d\}$
- $P = abacab$

$c$	$a$	$b$	$c$	$d$
$L(c)$	4	5	3	-1

- ◆ The last-occurrence function can be represented by an array indexed by the numeric codes of the characters
- ◆ The last-occurrence function can be computed in time  $O(m + s)$ , where  $m$  is the size of  $P$  and  $s$  is the size of  $\Sigma$

# The Boyer-Moore Algorithm

**Algorithm** *BoyerMooreMatch*( $T, P, \Sigma$ )

$L \leftarrow \text{lastOccurrenceFunction}(P, \Sigma)$

$i \leftarrow m - 1$

$j \leftarrow m - 1$

**repeat**

**if**  $T[i] = P[j]$

**if**  $j = 0$

**return**  $i$  { match at  $i$  }

**else**

$i \leftarrow i - 1$

$j \leftarrow j - 1$

**else**

    { character-jump }

$l \leftarrow L[T[i]]$

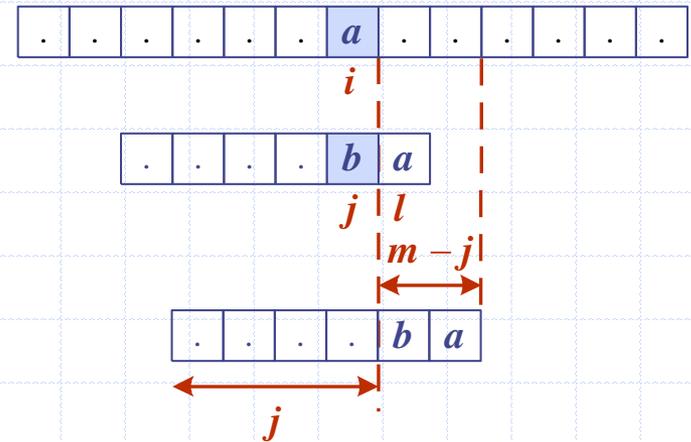
$i \leftarrow i + m - \min(j, 1 + l)$

$j \leftarrow m - 1$

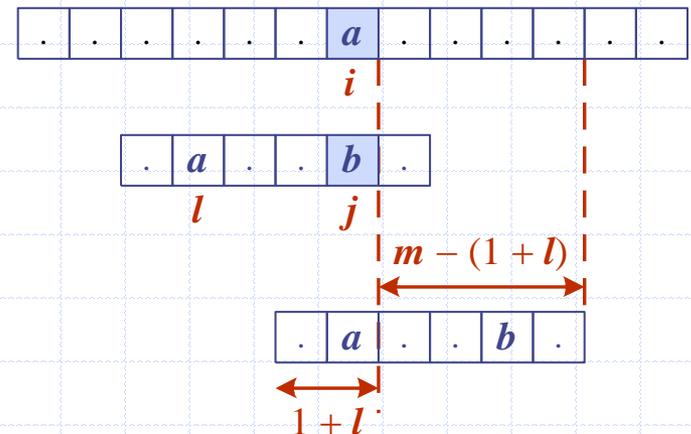
**until**  $i > n - 1$

**return**  $-1$  { no match }

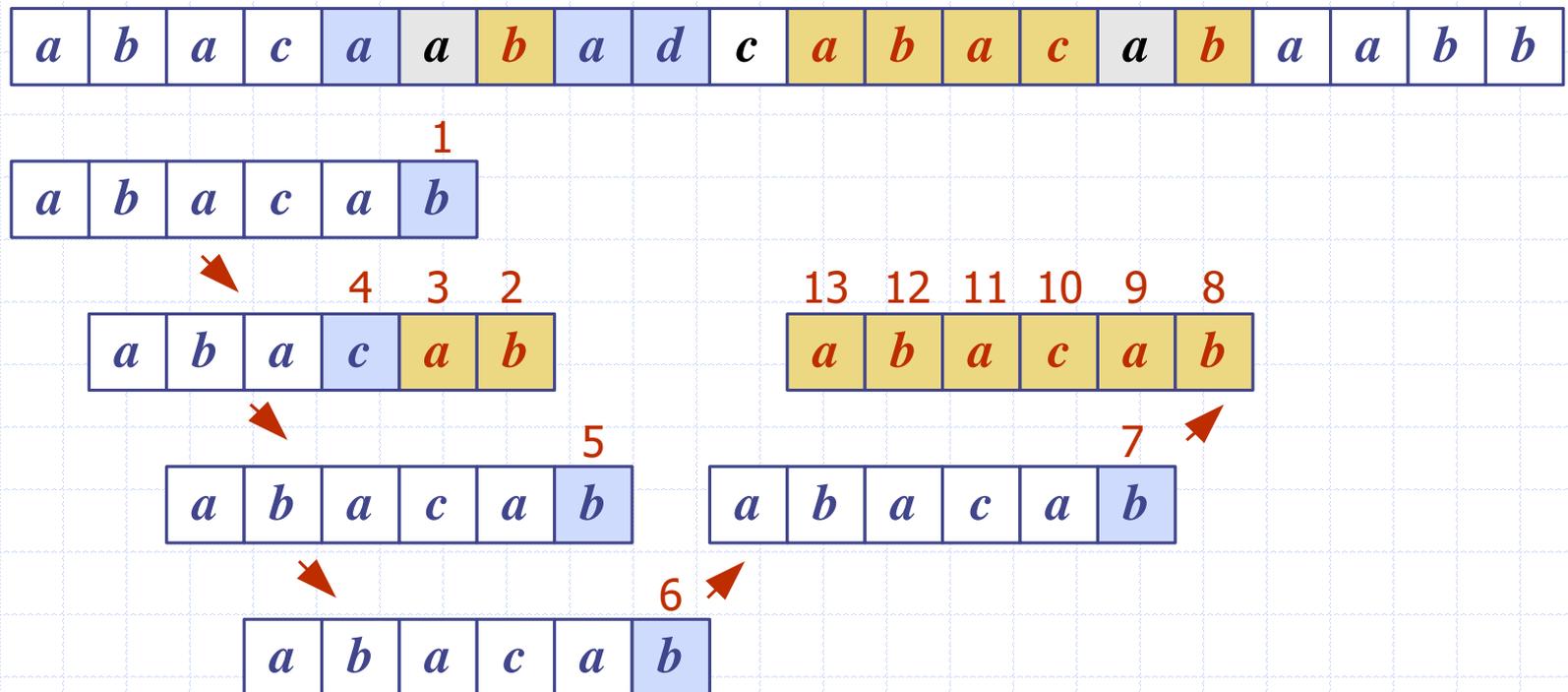
Case 1:  $j \leq 1 + l$



Case 2:  $1 + l \leq j$

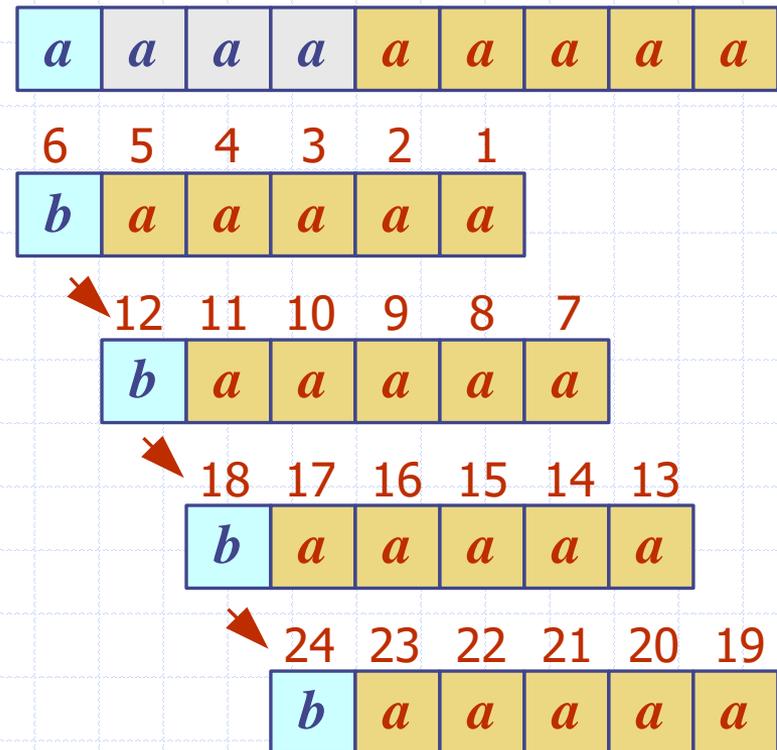


# Example



# Analysis

- ◆ Boyer-Moore's algorithm runs in time  $O(nm + s)$
- ◆ Example of worst case:
  - $T = aaa \dots a$
  - $P = baaa$
- ◆ The worst case may occur in images and DNA sequences but is unlikely in English text
- ◆ Boyer-Moore's algorithm is significantly faster than the brute-force algorithm on English text



# The KMP Algorithm

- ◆ Knuth-Morris-Pratt's algorithm compares the pattern to the text in **left-to-right**, but shifts the pattern more intelligently than the brute-force algorithm.
- ◆ When a mismatch occurs, what is the **most** we can shift the pattern so as to avoid redundant comparisons?
- ◆ Answer: the largest prefix of  $P[0..j]$  that is a suffix of  $P[1..j]$



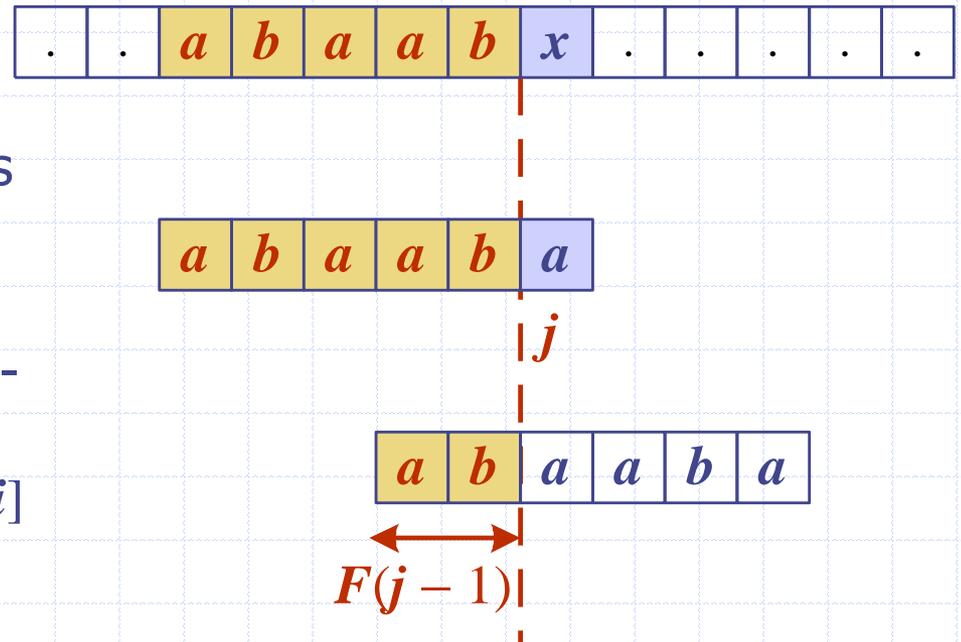
No need to repeat these comparisons

Resume comparing here

# KMP Failure Function

- ◆ Knuth-Morris-Pratt's algorithm preprocesses the pattern to find matches of prefixes of the pattern with the pattern itself
- ◆ The **failure function**  $F(j)$  is defined as the size of the largest prefix of  $P[0..j]$  that is also a suffix of  $P[1..j]$
- ◆ Knuth-Morris-Pratt's algorithm modifies the brute-force algorithm so that if a mismatch occurs at  $P[j] \neq T[i]$  we set  $j \leftarrow F(j - 1)$

$j$	0	1	2	3	4	5
$P[j]$	$a$	$b$	$a$	$a$	$b$	$a$
$F(j)$	0	0	1	1	2	3



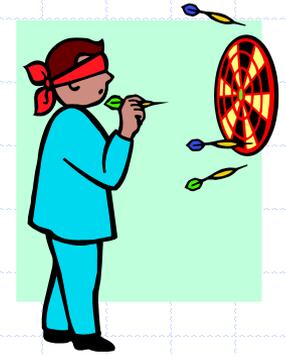
# The KMP Algorithm

- ◆ The failure function can be represented by an array and can be computed in  $O(m)$  time
- ◆ At each iteration of the while-loop, either
  - $i$  increases by one, or
  - the shift amount  $i - j$  increases by at least one (observe that  $F(j - 1) < j$ )
- ◆ Hence, there are no more than  $2n$  iterations of the while-loop
- ◆ Thus, KMP's algorithm runs in optimal time  $O(m + n)$

**Algorithm** *KMPMatch*( $T, P$ )

```
 $F \leftarrow \text{failureFunction}(P)$   
 $i \leftarrow 0$   
 $j \leftarrow 0$   
while  $i < n$   
  if  $T[i] = P[j]$   
    if  $j = m - 1$   
      return  $i - j$  { match }  
    else  
       $i \leftarrow i + 1$   
       $j \leftarrow j + 1$   
  else  
    if  $j > 0$   
       $j \leftarrow F[j - 1]$   
    else  
       $i \leftarrow i + 1$   
return  $-1$  { no match }
```

# Computing the Failure Function



- ◆ The failure function can be represented by an array and can be computed in  $O(m)$  time
- ◆ The construction is similar to the KMP algorithm itself
- ◆ At each iteration of the while-loop, either
  - $i$  increases by one, or
  - the shift amount  $i - j$  increases by at least one (observe that  $F(j - 1) < j$ )
- ◆ Hence, there are no more than  $2m$  iterations of the while-loop

**Algorithm** *failureFunction*( $P$ )

```
 $F[0] \leftarrow 0$   
 $i \leftarrow 1$   
 $j \leftarrow 0$   
while  $i < m$   
    if  $P[i] = P[j]$   
        { we have matched  $j + 1$  chars }  
         $F[i] \leftarrow j + 1$   
         $i \leftarrow i + 1$   
         $j \leftarrow j + 1$   
    else if  $j > 0$  then  
        { use failure function to shift  $P$  }  
         $j \leftarrow F[j - 1]$   
    else  
         $F[i] \leftarrow 0$  { no match }  
         $i \leftarrow i + 1$ 
```

# Example

*a* *b* *a* *c* *a* *a* *b* *a* *c* *c* *a* *b* *a* *c* *a* *b* *a* *a* *b* *b*

1 2 3 4 5 6  
*a* *b* *a* *c* *a* *b*

7  
*a* *b* *a* *c* *a* *b*

8 9 10 11 12  
*a* *b* *a* *c* *a* *b*

13  
*a* *b* *a* *c* *a* *b*

14 15 16 17 18 19  
*a* *b* *a* *c* *a* *b*

<i>j</i>	0	1	2	3	4	5
<i>P</i> [ <i>j</i> ]	<i>a</i>	<i>b</i>	<i>a</i>	<i>c</i>	<i>a</i>	<i>b</i>
<i>F</i> ( <i>j</i> )	0	0	1	0	1	2