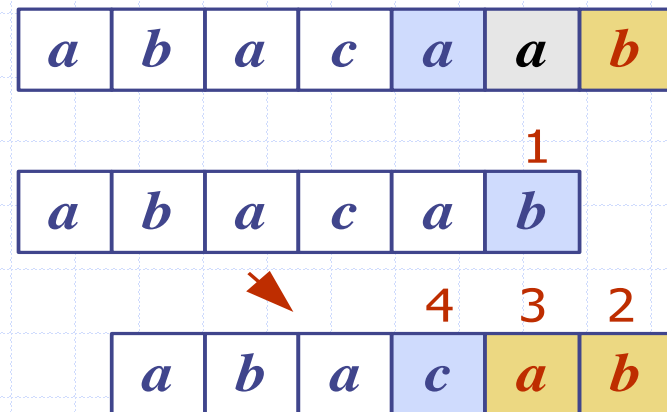


Strings and Pattern Matching

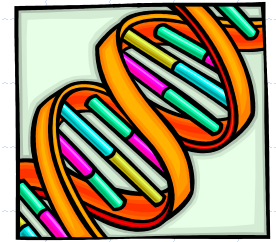
Sections 12.1 and 12.3



Strings



- ◆ A string is a sequence of characters
- ◆ Examples of strings:
 - C++ program
 - HTML document
 - DNA sequence
 - Digitized image
- ◆ An alphabet Σ is the set of possible characters for a family of strings
- ◆ Example of alphabets:
 - ASCII
 - Unicode
 - $\{0, 1\}$
 - $\{A, C, G, T\}$
- ◆ Let P be a string of size m
 - A substring $P[i..j]$ of P is the subsequence of P consisting of the characters with ranks between i and j
 - A prefix of P is a substring of the type $P[0..i]$
 - A suffix of P is a substring of the type $P[i..m-1]$
- ◆ Given strings T (text) and P (pattern), the pattern matching problem consists of finding a substring of T equal to P
- ◆ Applications:
 - Text editors
 - Search engines
 - Biological research



Brute-Force Pattern Matching

- ◆ The brute-force pattern matching algorithm compares the pattern P with the text T for each possible shift of P relative to T , until either
 - a match is found, or
 - all placements of the pattern have been tried
- ◆ Brute-force pattern matching runs in time $O(nm)$
- ◆ Example of worst case:
 - $T = aaa \dots ah$
 - $P = aaah$
 - may occur in images and DNA sequences
 - unlikely in English text

Algorithm *BruteForceMatch*(T, P)

Input text T of size n and pattern P of size m

Output starting index of a substring of T equal to P or -1 if no such substring exists

for $i \leftarrow 0$ **to** $n - m$

 { test shift i of the pattern }

$j \leftarrow 0$

while $j < m \wedge T[i + j] = P[j]$

$j \leftarrow j + 1$

if $j = m$

return i { match at i }

return -1 { no match anywhere }

Boyer-Moore Heuristics

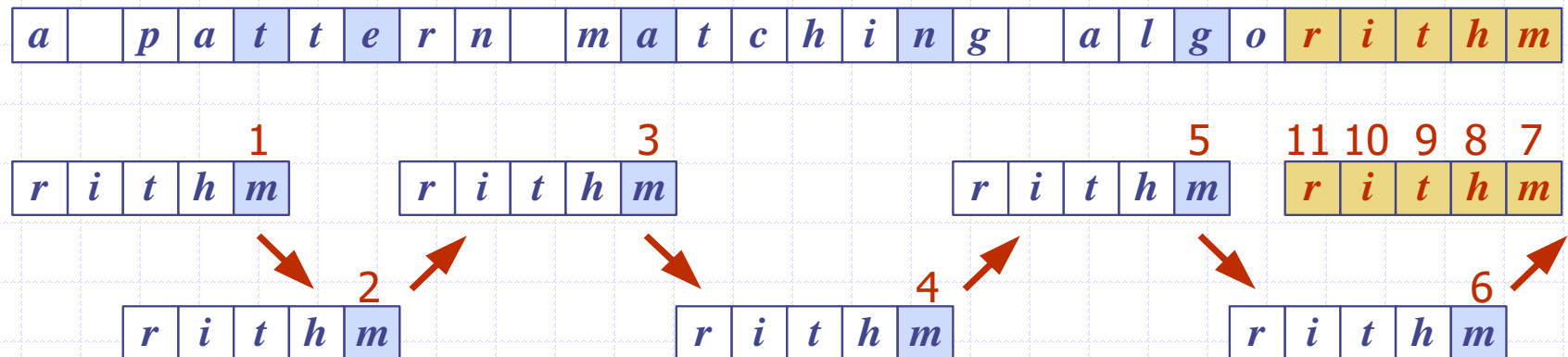
- ◆ The Boyer-Moore's pattern matching algorithm is based on two heuristics

Looking-glass heuristic: Compare P with a subsequence of T moving backwards

Character-jump heuristic: When a mismatch occurs at $T[i] = c$

- If P contains c , shift P to align the last occurrence of c in P with $T[i]$
- Else, shift P to align $P[0]$ with $T[i + 1]$

- ◆ Example



Last-Occurrence Function

- ◆ Boyer-Moore's algorithm preprocesses the pattern P and the alphabet Σ to build the last-occurrence function L mapping Σ to integers, where $L(c)$ is defined as
 - the largest index i such that $P[i] = c$ or
 - -1 if no such index exists

- ◆ Example:

- $\Sigma = \{a, b, c, d\}$
- $P = abacab$

c	a	b	c	d
$L(c)$	4	5	3	-1

- ◆ The last-occurrence function can be represented by an array indexed by the numeric codes of the characters
- ◆ The last-occurrence function can be computed in time $O(m + s)$, where m is the size of P and s is the size of Σ

The Boyer-Moore Algorithm

Algorithm *BoyerMooreMatch*(T, P, Σ)

$L \leftarrow \text{lastOccurrenceFunction}(P, \Sigma)$

$i \leftarrow m - 1$

$j \leftarrow m - 1$

repeat

if $T[i] = P[j]$

if $j = 0$

return i { match at i }

else

$i \leftarrow i - 1$

$j \leftarrow j - 1$

else

 { character-jump }

$l \leftarrow L[T[i]]$

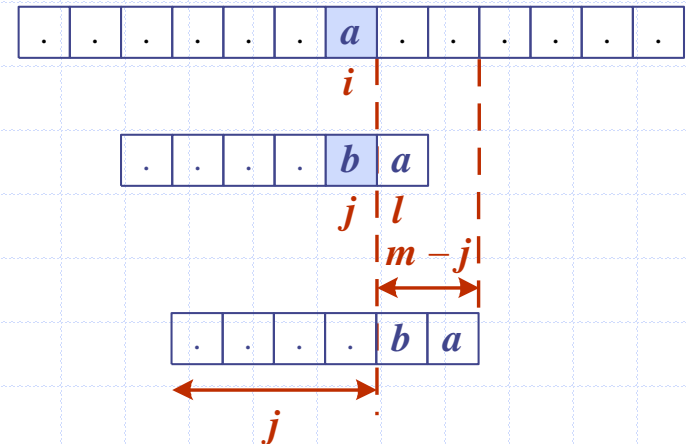
$i \leftarrow i + m - \min(j, 1 + l)$

$j \leftarrow m - 1$

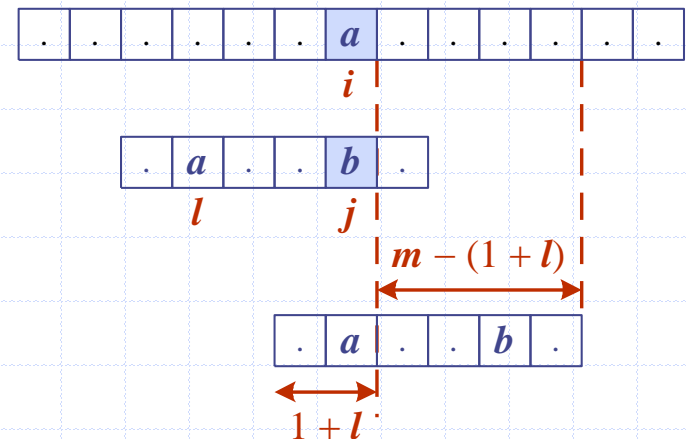
until $i > n - 1$

return -1 { no match }

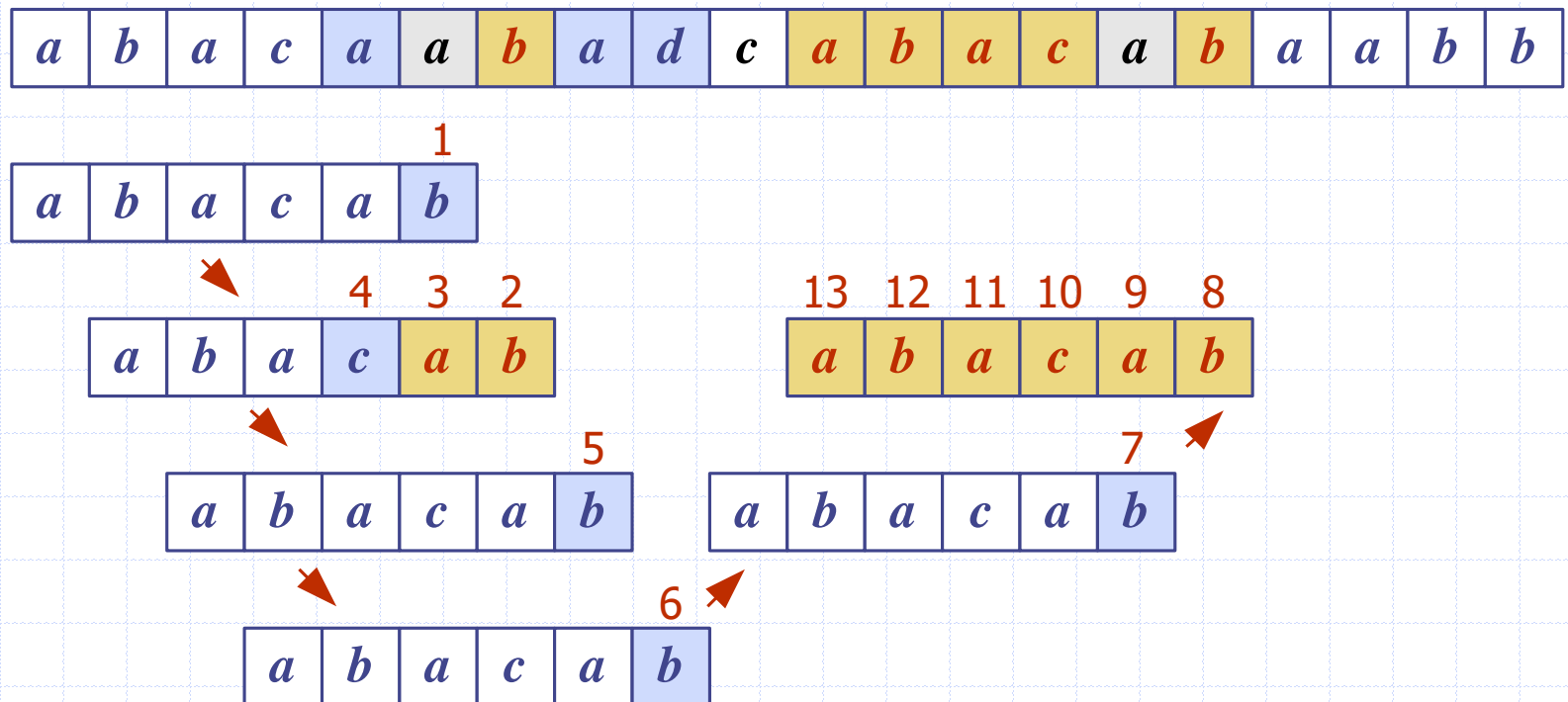
Case 1: $j \leq 1 + l$



Case 2: $1 + l \leq j$

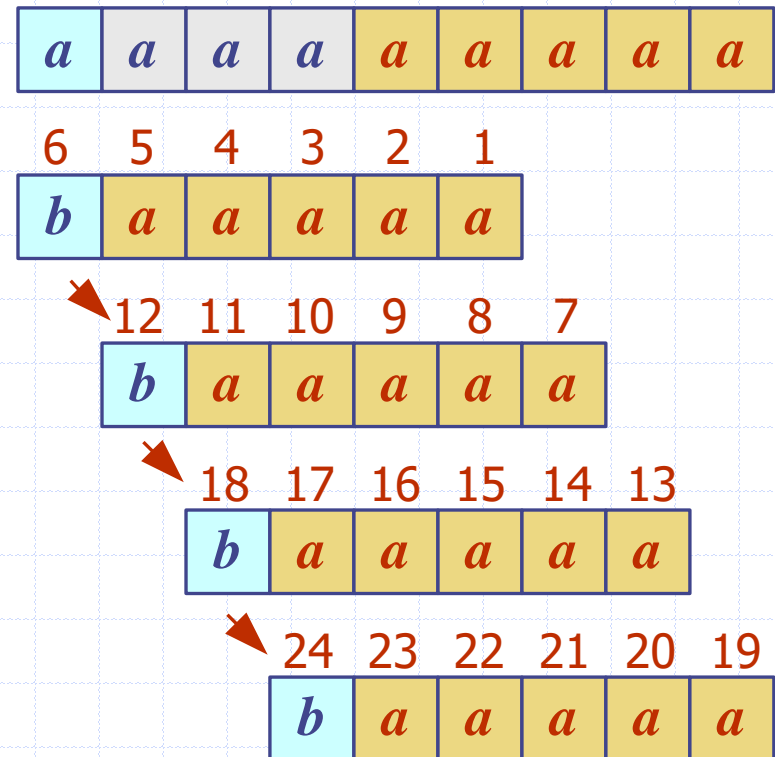


Example



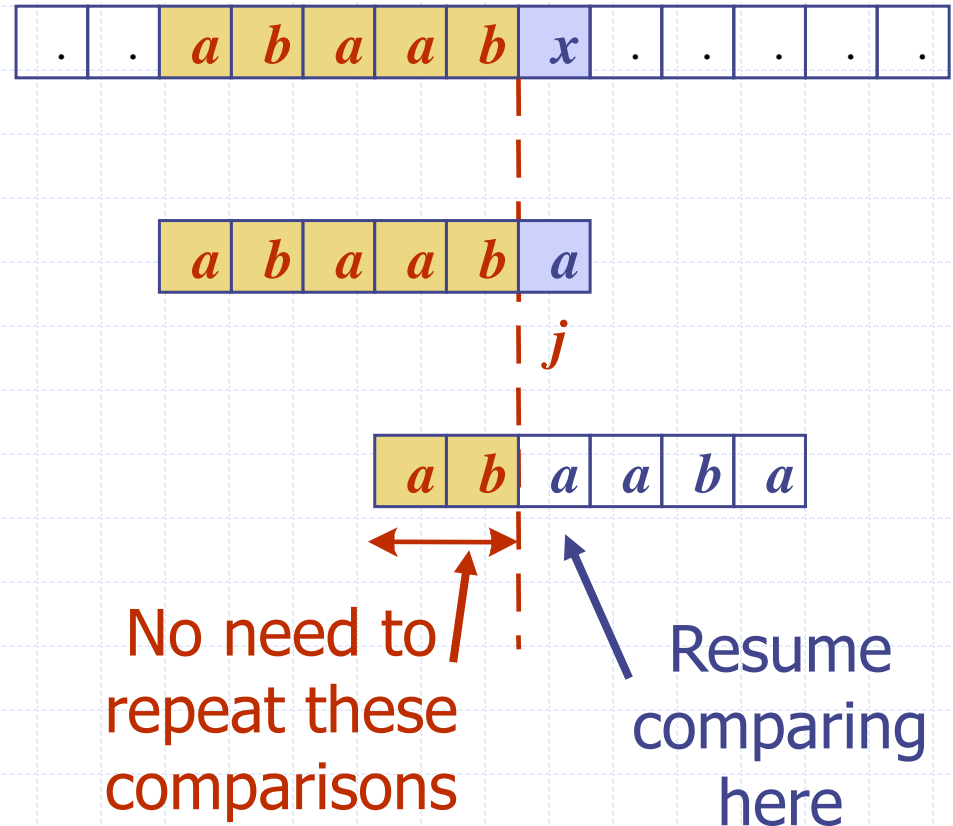
Analysis

- ◆ Boyer-Moore's algorithm runs in time $O(nm + s)$
- ◆ Example of worst case:
 - $T = aaa \dots a$
 - $P = baaa$
- ◆ The worst case may occur in images and DNA sequences but is unlikely in English text
- ◆ Boyer-Moore's algorithm is significantly faster than the brute-force algorithm on English text



The KMP Algorithm

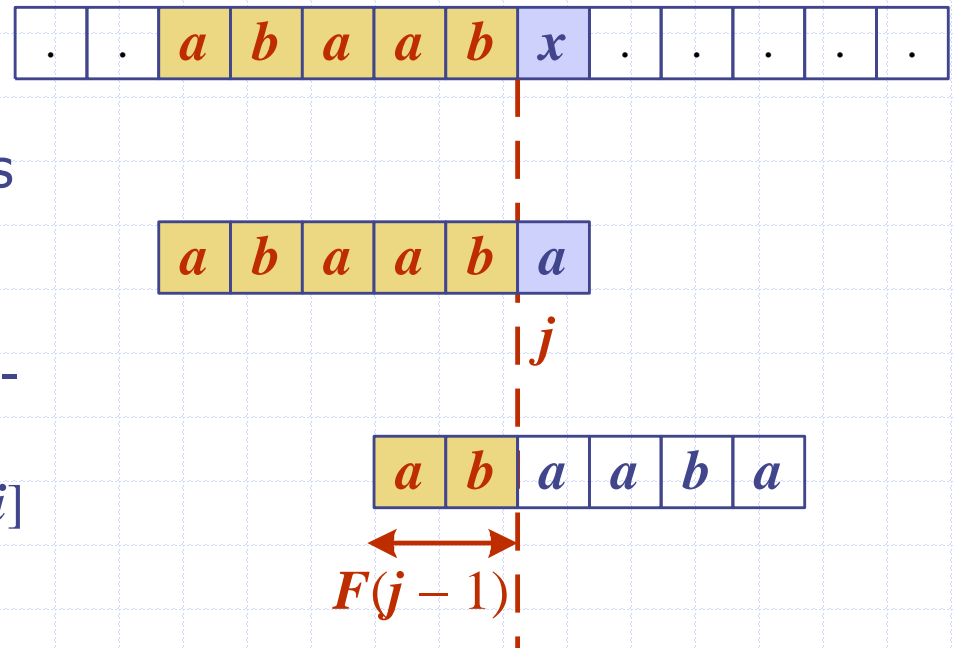
- ◆ Knuth-Morris-Pratt's algorithm compares the pattern to the text in **left-to-right**, but shifts the pattern more intelligently than the brute-force algorithm.
- ◆ When a mismatch occurs, what is the **most** we can shift the pattern so as to avoid redundant comparisons?
- ◆ Answer: the largest prefix of $P[0..j]$ that is a suffix of $P[1..j]$



KMP Failure Function

- ◆ Knuth-Morris-Pratt's algorithm preprocesses the pattern to find matches of prefixes of the pattern with the pattern itself
- ◆ The **failure function** $F(j)$ is defined as the size of the largest prefix of $P[0..j]$ that is also a suffix of $P[1..j]$
- ◆ Knuth-Morris-Pratt's algorithm modifies the brute-force algorithm so that if a mismatch occurs at $P[j] \neq T[i]$ we set $j \leftarrow F(j - 1)$

j	0	1	2	3	4	5
$P[j]$	a	b	a	a	b	a
$F(j)$	0	0	1	1	2	3



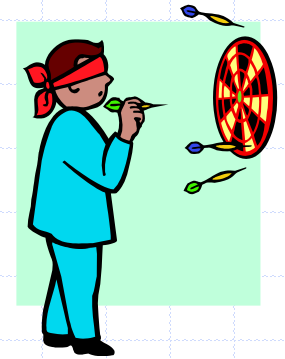
The KMP Algorithm

- ◆ The failure function can be represented by an array and can be computed in $O(m)$ time
- ◆ At each iteration of the while-loop, either
 - i increases by one, or
 - the shift amount $i - j$ increases by at least one (observe that $F(j - 1) < j$)
- ◆ Hence, there are no more than $2n$ iterations of the while-loop
- ◆ Thus, KMP's algorithm runs in optimal time $O(m + n)$

Algorithm *KMPMatch*(T, P)

```
 $F \leftarrow \text{failureFunction}(P)$   
 $i \leftarrow 0$   
 $j \leftarrow 0$   
while  $i < n$   
    if  $T[i] = P[j]$   
        if  $j = m - 1$   
            return  $i - j$  { match }  
        else  
             $i \leftarrow i + 1$   
             $j \leftarrow j + 1$   
    else  
        if  $j > 0$   
             $j \leftarrow F[j - 1]$   
        else  
             $i \leftarrow i + 1$   
return  $-1$  { no match }
```

Computing the Failure Function



- ◆ The failure function can be represented by an array and can be computed in $O(m)$ time
- ◆ The construction is similar to the KMP algorithm itself
- ◆ At each iteration of the while-loop, either
 - i increases by one, or
 - the shift amount $i - j$ increases by at least one (observe that $F(j - 1) < j$)
- ◆ Hence, there are no more than $2m$ iterations of the while-loop

Algorithm *failureFunction*(P)

```
 $F[0] \leftarrow 0$ 
 $i \leftarrow 1$ 
 $j \leftarrow 0$ 
while  $i < m$ 
    if  $P[i] = P[j]$ 
        { we have matched  $j + 1$  chars }
         $F[i] \leftarrow j + 1$ 
         $i \leftarrow i + 1$ 
         $j \leftarrow j + 1$ 
    else if  $j > 0$  then
        { use failure function to shift  $P$  }
         $j \leftarrow F[j - 1]$ 
    else
         $F[i] \leftarrow 0$  { no match }
         $i \leftarrow i + 1$ 
```

Example

a b a c a a b a c c a b a c a b a a b b

1 2 3 4 5 6
a b a c a b

7
a b a c a b

8 9 10 11 12
a b a c a b

13
a b a c a b

14 15 16 17 18 19
a b a c a b

<i>j</i>	0	1	2	3	4	5
<i>P[j]</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>c</i>	<i>a</i>	<i>b</i>
<i>F(j)</i>	0	0	1	0	1	2