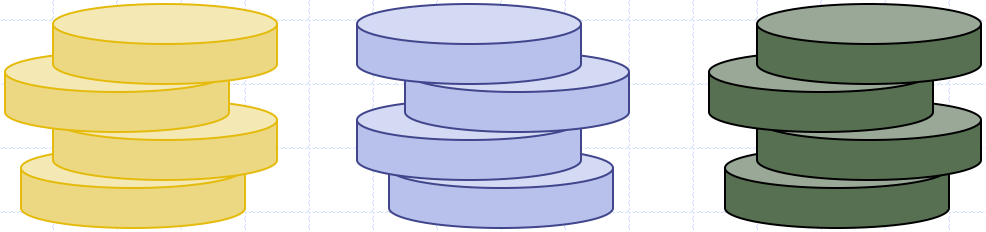
Stacks

Section 5.1



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Abstract Data Types (ADTs)

- An abstract data type (ADT) is an abstraction of a data structure and operations on it.

- An ADT specifies:
  - Data stored
  - Operations on the data
  - Error conditions associated with operations

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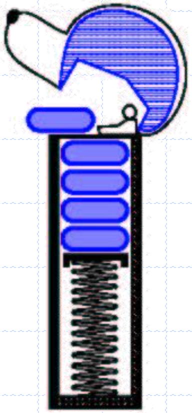
The Stack ADT



- The Stack ADT stores arbitrary objects
- Insertions and deletions follow the last-in first-out (LIFO) scheme
- Main stack operations:
  - void push(object): inserts an element on the top.
  - void pop(): removes the top element. Gives a StackEmpty error if there is no element to remove.
- Auxiliary stack operations:
  - object top(): returns the element at the top without removing it. Gives a StackEmpty error if the stack is empty.
  - integer size(): returns the number of elements stored.
  - boolean empty(): indicates whether no elements are stored.

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- void push(object): inserts an element on the top.
- void pop(): removes the top element.
- object top(): returns the element at the top without removing it.
- integer size(): returns the number of elements stored.
- boolean empty(): indicates whether no elements are stored.

- void push(object): inserts an element on the top.
- object pop(): removes and returns the top element.
- integer size(): returns the number of elements stored.
- boolean empty(): indicates whether no elements are stored.

Stack Interface

- Pseudo-C++ interface corresponding to our Stack ADT
- Uses an exception class StackEmpty
- Different from the built-in C++ STL class stack

```
template <typename E>
class Stack  {
public:
   int size() const;
   bool empty() const;
   const E& top() const
         throw(StackEmpty);
   void push(const E& e);
   void pop() throw(StackEmpty);
}
```

STL stack class

- The Standard Template Library (STL) provides an implementation of a stack.
- To declare a stack of integers:

```
#include <stack>
std::stack<int> myStack;
```

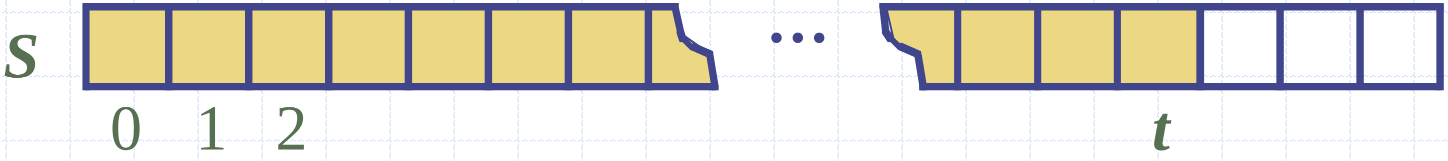
- STL's stack interface is basically the same as the one we just saw, except that executing pop or top on an empty stack results in *undefined behavior*. This generally means your program crashes.

Array-based Stack Implementation

- A simple way of implementing the Stack ADT uses an array
- We add elements from left to right
- A variable *t* keeps track of the index of the top element

```
Algorithm size()
    return $t + 1$

Algorithm pop()
    if empty() then
        throw StackEmpty
    else
        $t \leftarrow t - 1$
        return $S[t + 1]$
```



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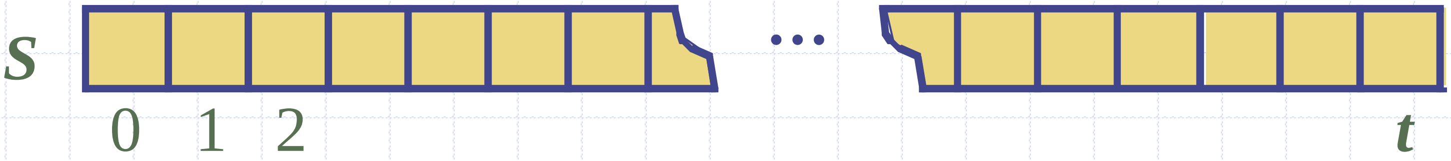
Array-based Stack

- The array storing the stack elements may become full
- A push operation will then throw a StackFull exception
  - Limitation of the simple array-based implementation
  - Not intrinsic to the Stack ADT

```
Algorithm push(o)
  if $t = capacity - 1$ then
    throw StackFull
  else
    $t \leftarrow t + 1$
    $S[t] \leftarrow o$
```



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Performance and Limitations

- Performance
  - Let n be the number of elements in the stack
  - The space used is at least n
  - Each operation runs in time $O(1)$
- Limitations
  - The maximum size of the stack must be defined a priori and cannot be changed
  - Trying to push a new element into a full stack causes an implementation-specific exception

Array-based Stack in C++

```
template <typename E>
class ArrayStack {
private:
    E* S; // array holding the stack
    int cap; // capacity
    int t; // index of top element
public:
    // constructor given capacity
    ArrayStack( int c) :
        S(new E[c]), cap(c), t(-1) { }
```

```
    void pop() {
      if (empty()) throw StackEmpty
               (“Pop from empty stack”);
        t--;
     }
    void push(const E& e) {
        if (size() == cap) throw
            StackFull(“Push to full stack”);
        S[++ t] = e;
    }
```

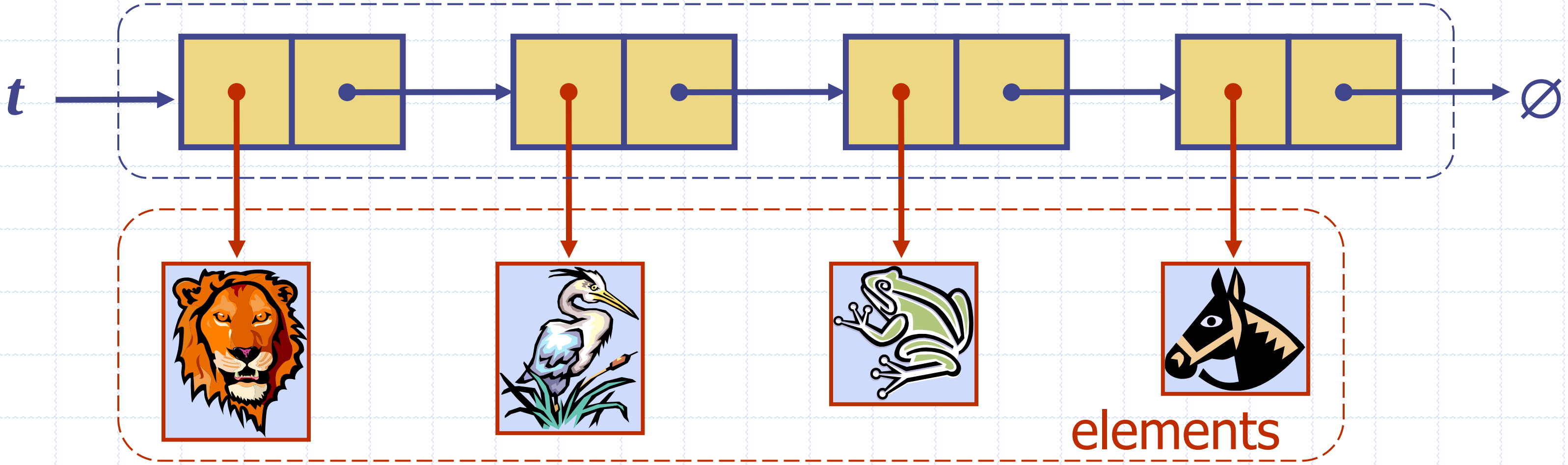
Array-based Stack in C++

```
    const E& top() {
      if (empty()) throw StackEmpty
            ("Top from empty stack");
      return S[t];
    }
    int size() {
      return t+1;
    }
    bool empty() {
      return t < 0;
    }
}  // end of class body
```

Book shows this implementation outside of the class body, like in a .cpp file. It also includes proper templating for that case, and the proper throw declarations.

Linked List-based Stack

- We can implement a stack with a singly linked list
- The top element is stored at the first node of the list
- The space used is $O(n)$ and each operation of the Stack ADT takes $O(1)$ time
- No restrictions on the number of elements



Example use in C++

* indicates top

```
ArrayStack<int> A;                        // A = [ ], size = 0
A.push(7);                                // A = [7*], size = 1
A.push(13);                               // A = [7, 13*], size = 2
cout << A.top() << endl; A.pop();         // A = [7*], outputs: 13
A.push(9);                                // A = [7, 9*], size = 2
cout << A.top() << endl;                  // A = [7, 9*], outputs: 9
cout << A.top() << endl; A.pop();         // A = [7*], outputs: 9

ArrayStack<string> B(10);                 // B = [ ], size = 0
B.push("Bob");                            // B = [Bob*], size = 1
B.push("Alice");                          // B = [Bob, Alice*], size = 2
cout << B.top() << endl; B.pop();         // B = [Bob*], outputs: Alice
B.push("Eve");                            // B = [Bob, Eve*], size = 2
```

Applications of Stacks

- Direct applications
  - Page-visited history in a Web browser
  - Undo sequence in a text editor
  - Chain of method calls in the C++ run-time system
- Indirect applications
  - Auxiliary data structure for algorithms
  - Component of other data structures

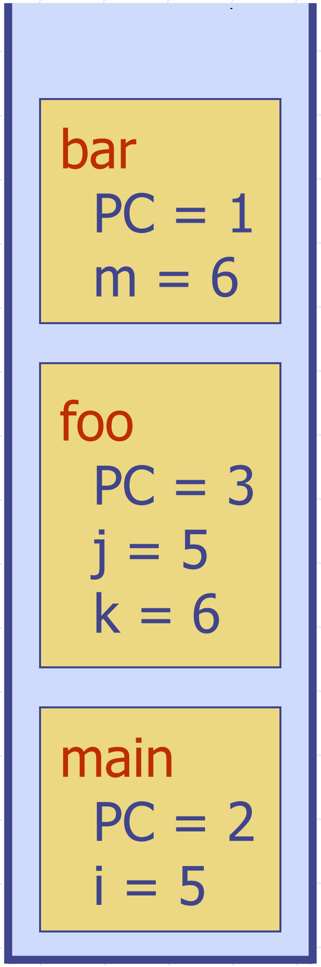
C++ Run-Time Stack

- The C++ run-time system keeps track of the chain of active functions with a stack
- When a function is called, the system pushes on the stack a frame containing
  - Local variables and return value
  - Program counter, keeping track of the statement being executed
- When the function ends, its frame is popped from the stack and control is passed to the function on top of the stack
- Allows for recursion

```
main() {
   int i = 5;
   foo(i);
}

foo(int j) {
   int k;
   k = j+1;
   bar(k);
}

bar(int m) {
   …
}
```



bar
PC = 1
m = 6

foo
PC = 3
j = 5
k = 6

main
PC = 2
i = 5

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Parentheses Matching

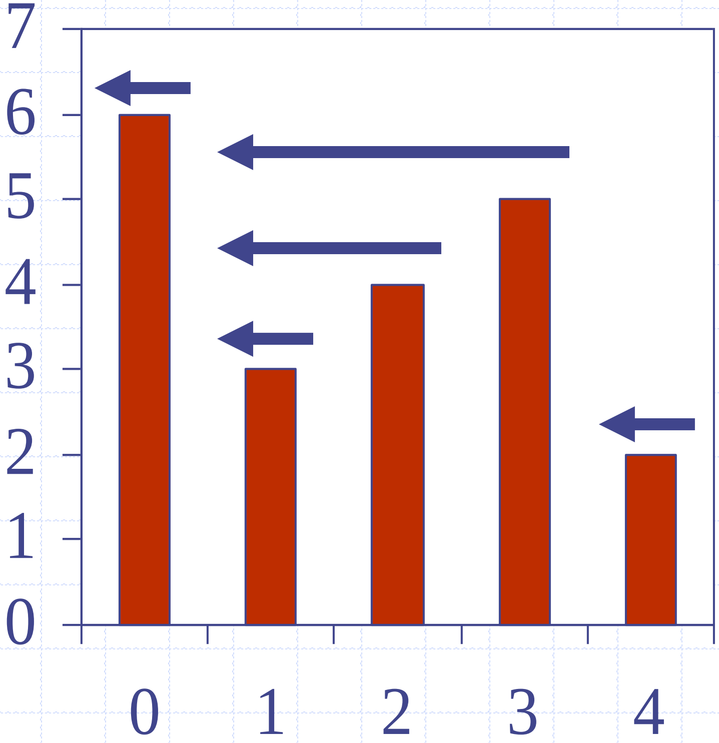
- Each “(”, “{”, or “[” must be paired with a matching “)”, “}”, or “]”
  - correct: ( )(( )){([( )])}
  - correct: ((( )(( )){([( )])}
  - incorrect:)(( )){([( )])}
  - incorrect: ({[ ])}
  - incorrect: (

Parentheses Matching Algorithm

```
Algorithm ParenMatch(X,n):
Input: An array X of n tokens, each of which is either a grouping symbol, a
variable, an arithmetic operator, or a number
Output: true if and only if all the grouping symbols in X match
Let S be an empty stack
for i=0 to n-1 do
    if X[i] is an opening grouping symbol then
        S.push(X[i])
    else if X[i] is a closing grouping symbol then
        if S.empty() then
            return false {nothing to match with}
        if S.pop() does not match the type of X[i] then
            return false {wrong type}
if S.empty() then
    return true {every symbol matched}
else return false {some symbols were never matched}
```

Computing Spans (not in book)

- Using a stack as an auxiliary data structure in an algorithm
- Given an an array X, the span $S[i]$ of $X[i]$ is the maximum number of consecutive elements $X[j]$ immediately preceding $X[i]$ and such that $X[j] \leq X[i]$
- Spans have applications to financial analysis
    - E.g., stock at 52-week high



X	6	3	4	5	2
S	1	1	2	3	1

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Quadratic Algorithm

```
Algorithm spans1($X$, $n$)
    Input array $X$ of $n$ integers
    Output array $S$ of spans of $X$                      #
    $S \leftarrow$ new array of $n$ integers              $n$
    for $i \leftarrow 0$ to $n - 1$ do                    $n$
        $s \leftarrow 1$                                  $n$
        while $s \leq i \wedge X[i - s] \leq X[i]$        $1 + 2 + \ldots + (n - 1)$
            $s \leftarrow s + 1$                          $1 + 2 + \ldots + (n - 1)$
        $S[i] \leftarrow s$                               $n$
    return $S$                                            $1$
```

- Algorithm ***spans1*** runs in $O(n^2)$ time

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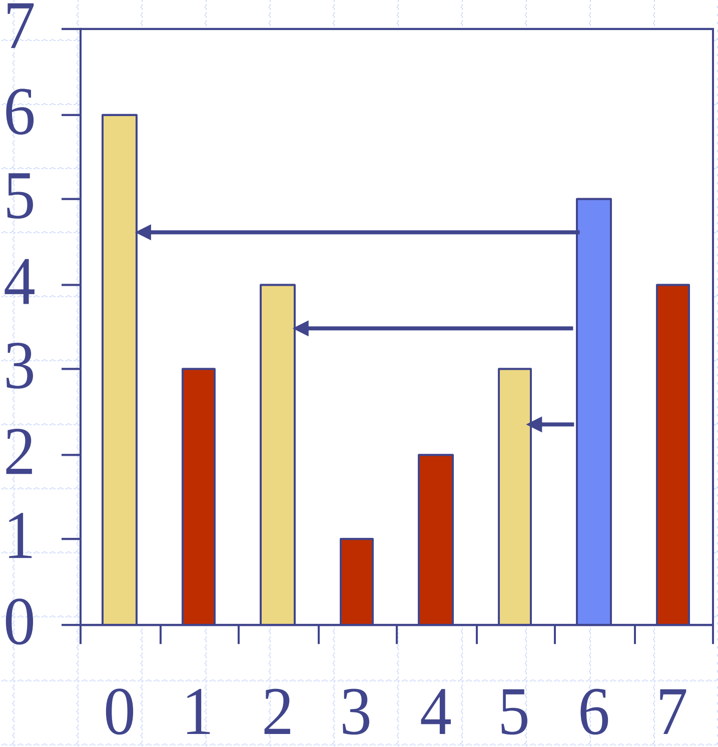
Computing Spans with a Stack

- We keep in a stack the indices of the elements visible when "looking back"
- We scan the array from left to right
  - Let i be the current index
  - We pop indices from the stack until we find index j such that $X[i] < X[j]$
  - We set $S[i] \leftarrow i - j$
  - We push i onto the stack



Linear Algorithm

- Each index of the array
  - Is pushed into the stack exactly once
  - Is popped from the stack at most once
- The statements in the while-loop are executed at most *n* times overall
- Algorithm ***spans2*** runs in $O(n)$ time

```
Algorithm spans2(X, n)                                      #
    S ← new array of n integers                             n
    A ← new empty stack                                     1
    for $i \leftarrow 0$ to $n - 1$ do                      n
        while ($\neg A.empty() \wedge$
                $X[A.top()] \leq X[i]$ ) do                 n
            A.pop()                                         n
        if A.empty() then                                   n
            $S[i] \leftarrow i + 1$                         n
        else
            $S[i] \leftarrow i - A.top()$                   n
        A.push(i)                                           n
    return S                                                1
```

Linear Algorithm

```
Algorithm spans2(X, n)
        S ← new array of n integers   O(1)
        A ← new empty stack           O(1)
    for i ← 0 to n – 1 do              n
        while (¬A.empty() ∧            $w_i$
              X[A.top()] ≤ X[i] ) do
            A.pop()                    O(1)
        if A.empty() then              O(1)
            S[i] ← i + 1               O(1)
        else
            S[i] ← i – A.top()         O(1)
        A.push(i)                      O(1)
    return S                           O(1)
```

w_i * O(1)

O(1)

n * O(1) + $\sum_{i=1..n} w_i$ =

O(n) + n = O(n)

O(n)