

# CMPT 882

Special Topics in Artificial Intelligence

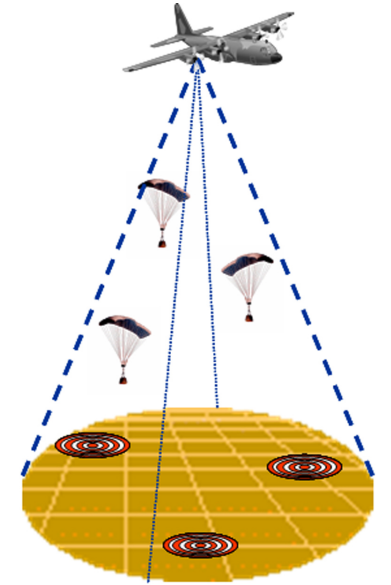
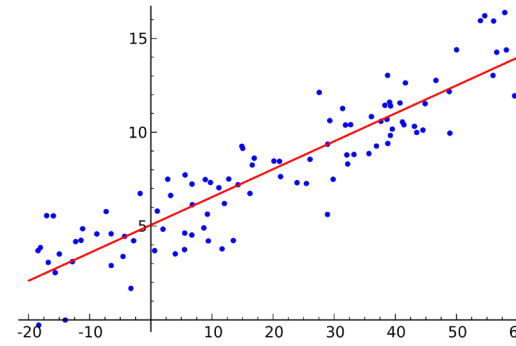
Robotic Decision Making

Mo Chen

<https://www.sfu.ca/~mochen>

# Course Outline

- Overview of algorithms used for robotic decision making
  - Theory-focused
  - Fundamentals for doing many areas of robotics research
- Dynamical systems
- Nonlinear optimization
- Optimal control
- Machine learning in robotics
- Localization and mapping



# Logistics

- Academic Quadrangle 5030, MWF 12:30-13:20
- Office hour: Thursdays 15:00-16:00, TASC 1 8225
- Course website: <https://coursys.sfu.ca/2019sp-cmpt-882-g1/pages/>
- Contact: [mochen@cs.sfu.ca](mailto:mochen@cs.sfu.ca)

# Caveats

- This class is in “experimental mode”
- Slight changes are expected
- Some things may not be super polished
- Please provide feedback and comments



# Grading

- 40% Homework
  - 5 assignments, one on each unit
- 60% Project

# Project suggestions

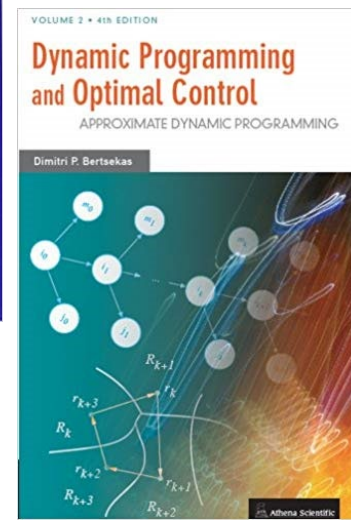
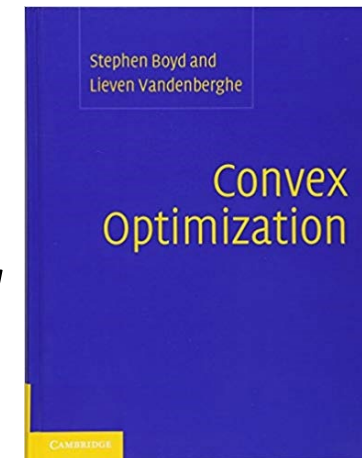
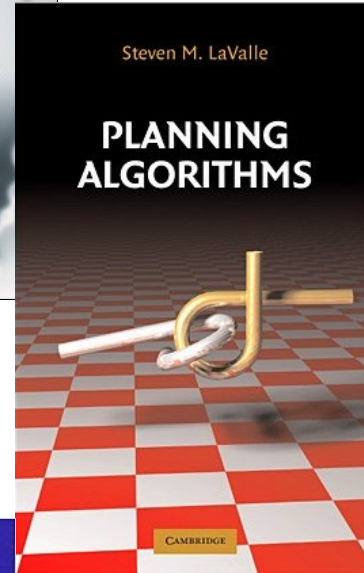
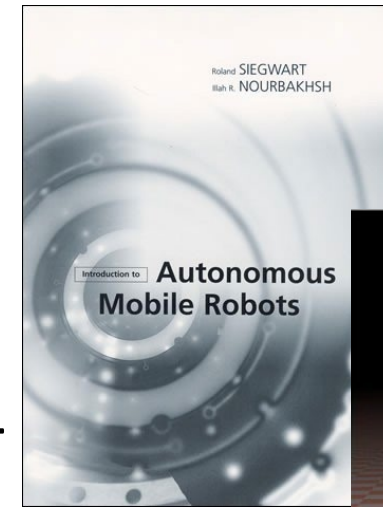
- Thoroughly understand and critically evaluate 3 to 5 papers in an area covered in this course
- Reproduce the results of 1 to 2 papers in an area covered in this course, and suggest or make improvements
- Implement an algorithm covered in or related to the class on a robot
- Mini Research project related to an area covered in this course

# Project timeline

- Proposal (1-2 paragraphs)
  - Due Feb. 18
- Presentation
  - Last three classes of the semester
- Report (5 pages maximum)
  - Due Apr. 20

# Recommended textbooks

- R. Siegwart, I. R. Nourbakhsh, and D. Scaramuzza, *Introduction to Autonomous Mobile Robots*. The MIT Press, 2011, 9780262015356.
- S. M. LaValle, *Planning Algorithms*. Cambridge University Press, 2006, 9780521862059.
- S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2008, 9780521833783.
- D. P. Bertsekas, *Dynamic Programming and Optimal Control*. Athena Scientific, 2017, 1886529434.



# Dynamical systems

- Mathematical models of robotic systems
  - Deterministic vs. stochastic
  - Continuous vs. discrete time
- Configuration of system described by the state, often denoted  $x$ 
  - State changes, or evolves, according to the model

# Dynamical systems

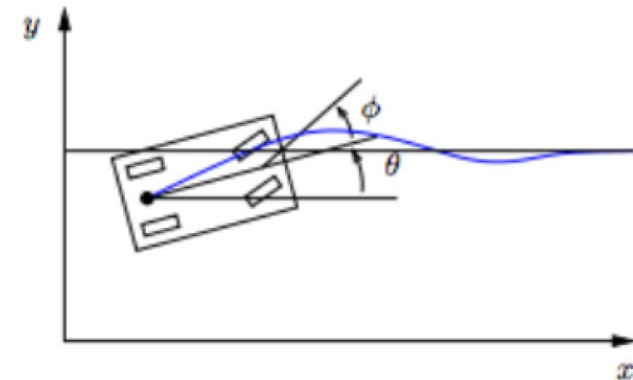
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  - $\frac{dx}{dt} = \dot{x}(t) = f(x, u)$

# Dynamical systems

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  - Continuous vs. discrete time
- Configuration of system described by the state, often denoted  $x$ 
  - State changes, or evolves, according to the model
- Deterministic, continuous time
  - $\frac{dx}{dt} = \dot{x}(t) = f(x, u)$
- Stochastic, discrete time
  - $x_{k+1}$  obtained from the probability distribution  $p(x_{k+1}|x_k, u_k)$

# System State

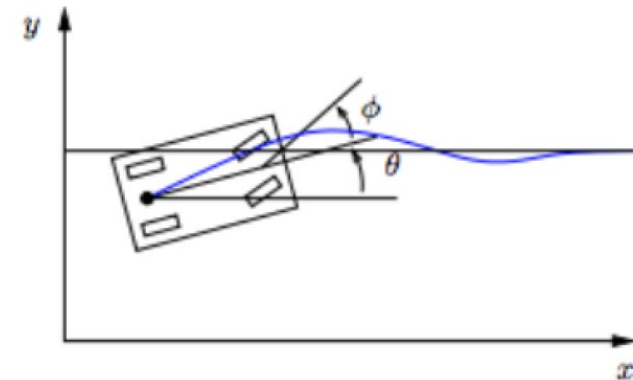
- Defined in terms of any variables of interest
  - Often denoted  $x(t)$  or  $x_k$
- Position
- Heading





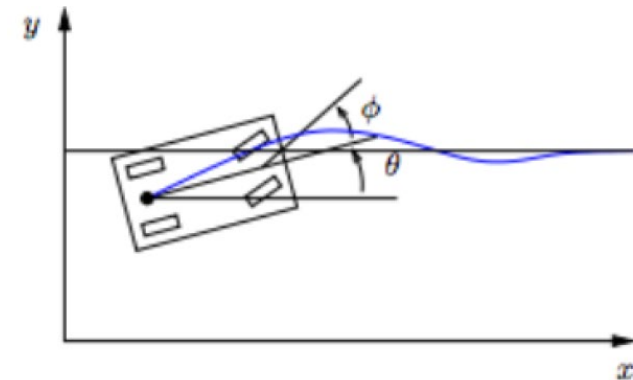
# System State

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- Position
- Heading
- Velocity
- Angular velocity



# System State

- Defined in terms of any variables of interest
  - Often denoted  $x(t)$  or  $x_k$
- Position
- Heading
- Velocity
- Angular velocity
- Voltages, concentrations of chemicals
- Human comfort, degree of trust



# Control and disturbance

- Control/action: usually used to achieve a desired goal
  - Usually denoted  $u(t)$  or  $u_k$
  - Acceleration
  - Turn rate



# Control and disturbance

- Control/action : usually used to achieve a desired goal
  - Usually denoted  $u(t)$  or  $u_k$
  - Acceleration
  - Turn rate
- Gas throttle
- Steering wheel angle

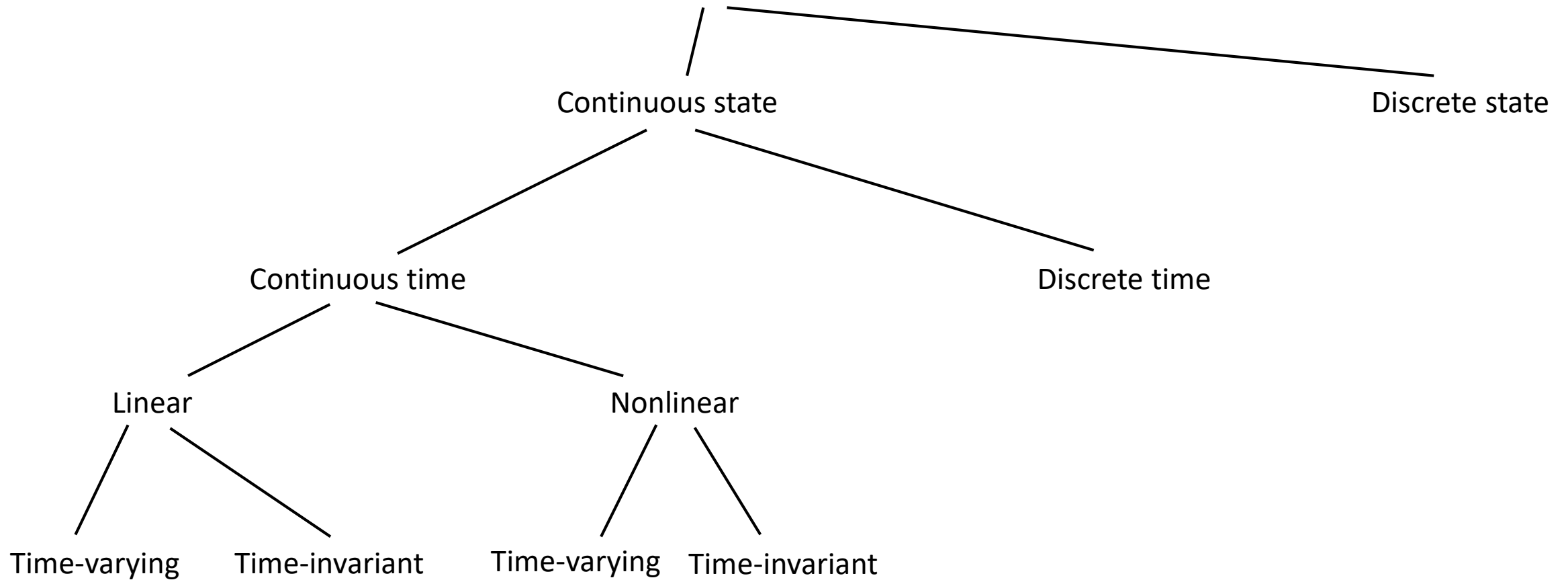


# Control and disturbance

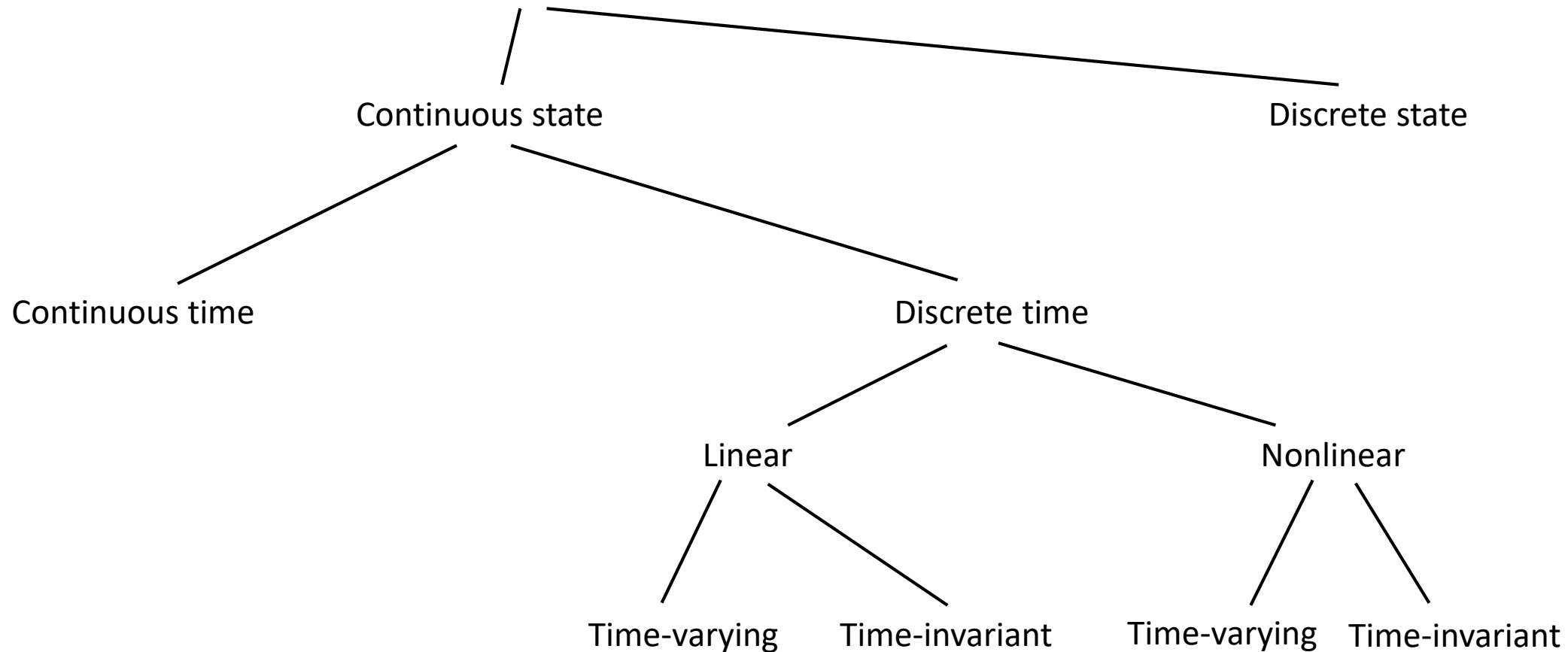
- Control/action : usually used to achieve a desired goal
  - Usually denoted  $u(t)$  or  $u_k$
  - Acceleration
  - Turn rate
  - Gas throttle
  - Steering wheel angle
- Disturbance
  - Usually denoted  $d(t)$  or  $d_k$
  - Bumps on the road
  - Input noise



# Mathematical models of robotic systems



# Mathematical models of robotic systems





# Examples of Robotic Systems

Amazon.com Inc.



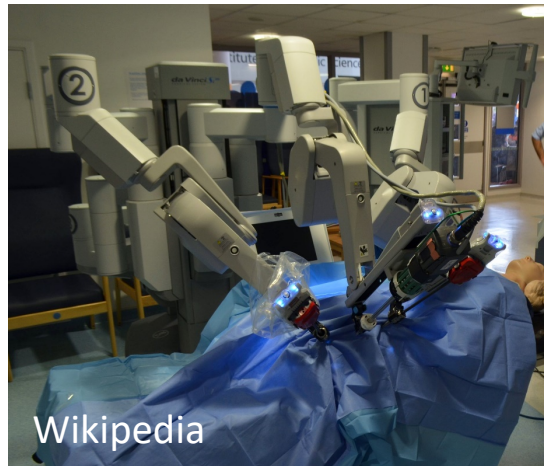


# Examples of Robotic Systems

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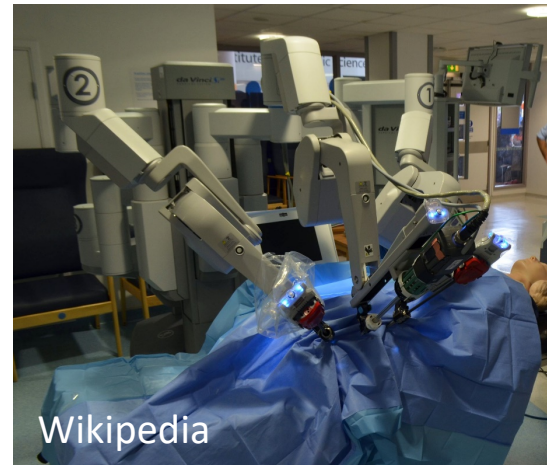


Google Inc.



Wikipedia

# Examples of Robotic Systems



# Car models

1

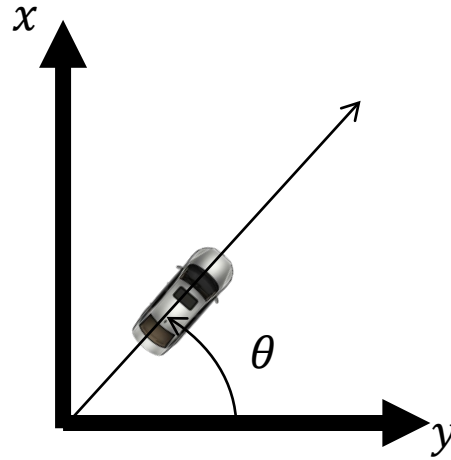
$$\dot{x} = v \cos \theta$$

$$\dot{y} = v \sin \theta$$

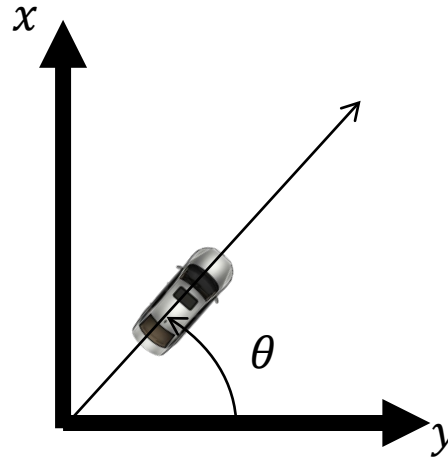
$$\dot{\theta} = \omega$$

States:  $(x, y, \theta)$ ; position and heading

Control:  $\omega$ ; turn rate (angular speed)



# Car models



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$$\dot{x} = v \cos \theta$$

$$\dot{y} = v \sin \theta$$

$$\dot{\theta} = \omega$$

States:  $(x, y, \theta)$ ; position and heading

Control:  $\omega$ ; turn rate (angular speed)

2

$$\dot{x} = v \cos \theta$$

$$\dot{y} = v \sin \theta$$

$$\dot{\theta} = \omega$$

$$\dot{v} = a$$

$$\dot{\omega} = \alpha$$

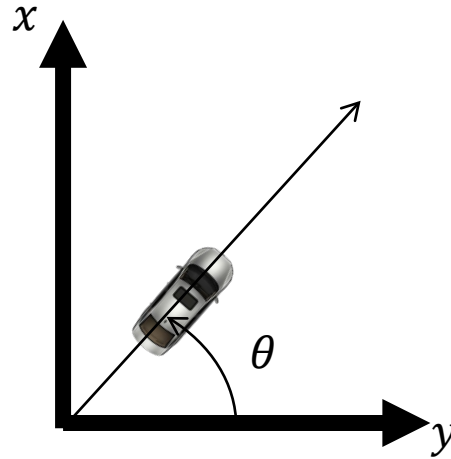
States:  $(x, y, \theta, v, \omega)$ ;

position, heading, speed, turn rate

Control:  $(a, \alpha)$ ;

acceleration, angular acceleration

# Car models



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3

$$\dot{x} = v_x$$

$$\dot{y} = v_y$$

$$\dot{v}_x = \omega v_y + a_{x2}$$

$$\dot{v}_y = -\omega v_x + \frac{2}{m} (F_{c,f} \cos \delta_f + F_{c,r})$$

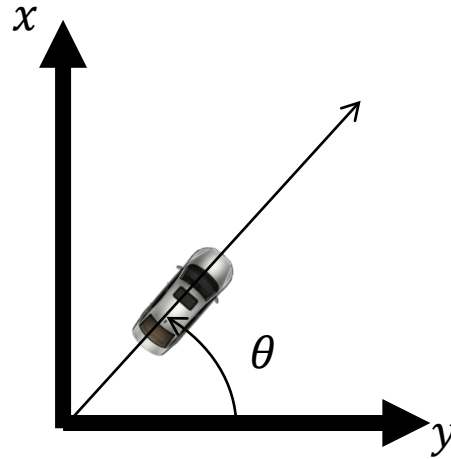
$$\dot{\psi} = \omega$$

$$\dot{\omega} = \frac{2}{I_z} (l_f F_{c,f} - l_r F_{c,r})$$

$$\dot{X} = v_x \cos \psi - v_y \sin \psi$$

$$\dot{Y} = v_x \sin \psi + v_y \cos \psi$$

# Car models



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$$\dot{x} = v \cos \theta$$

$$\dot{y} = v \sin \theta$$

$$\dot{\theta} = \omega$$

States:  $(x, y, \theta)$ ; position and heading

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$$\dot{x} = v \cos \theta$$

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$$\dot{\theta} = \omega$$

$$\dot{v} = a$$

$$\dot{\omega} = \alpha$$

States:  $(x, y, \theta, v, \omega)$ ;

position, heading, speed, turn rate

Control:  $(a, \alpha)$ ;

acceleration, angular acceleration

## 3 Bicycle model

$$\dot{x} = v_x$$

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$$\dot{v}_x = \omega v_y + a_{x2}$$

$$\dot{v}_y = -\omega v_x + \frac{1}{m} (F_{c,f} \cos \delta_f + F_{c,r})$$

$$\dot{\psi} = \omega$$

$$\dot{\omega} = \frac{2}{I_z} (l_f F_{c,f} - l_r F_{c,r})$$

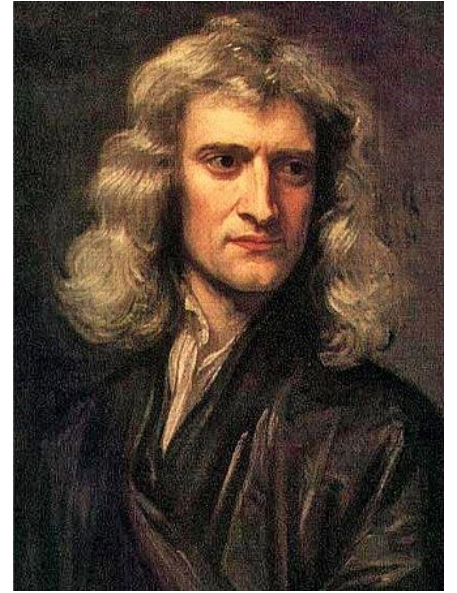
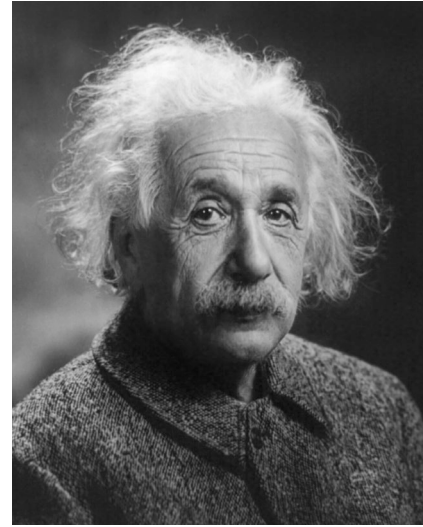
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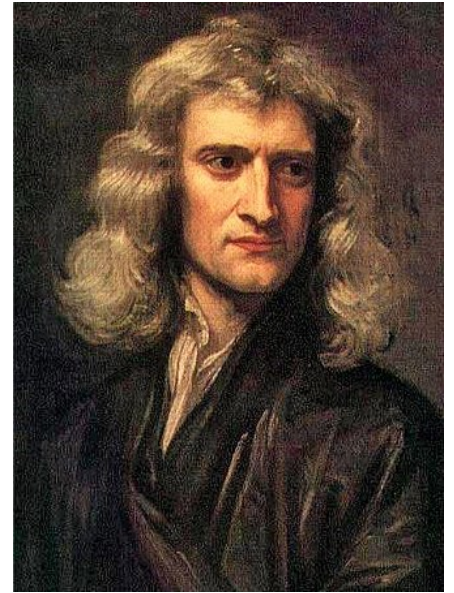
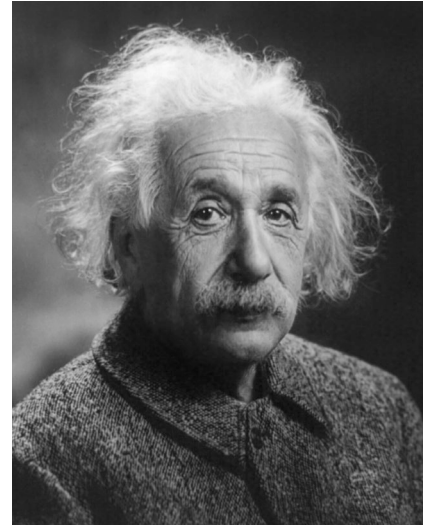
# Models

- All models are wrong; some are useful



# Models

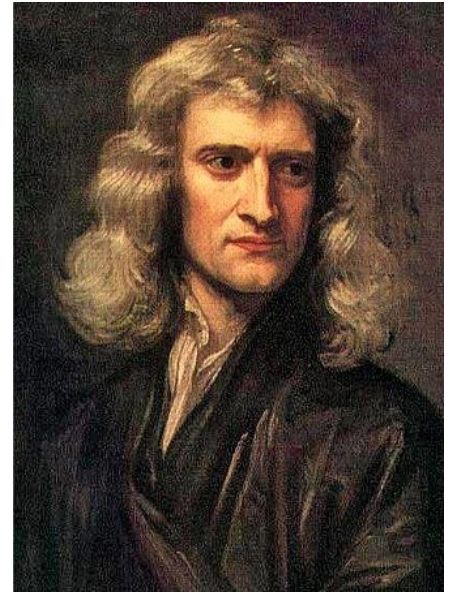
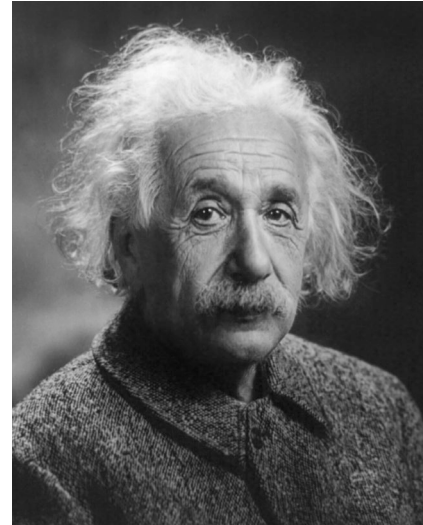
- All models are wrong; some are useful
- Definition of “useful” depends on situation
  - Simulation
  - Analysis and control
  - Verification





# Models

- All models are wrong; some are useful
- Definition of “useful” depends on situation
  - Simulation
  - Analysis and control
  - Verification
- Considerations
  - Does the model capture the desired system behaviours
  - Is the model amenable to tractable computation



# Nonlinear Optimization

- Choose  $x$  to minimize some cost, subject to constraints

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minimize  $f(x)$

Fuel cost, distance to obstacles, distance from goal, prediction error in machine learning

subject to  $g_i(x) \leq 0, i = 1, \dots, n$   
 $h_j(x) = 0, j = 1, \dots, m$

System dynamics, obstacle avoidance, goal reaching

- Equivalently, maximize  $-f(x)$ : Maximize reward, maximize profit

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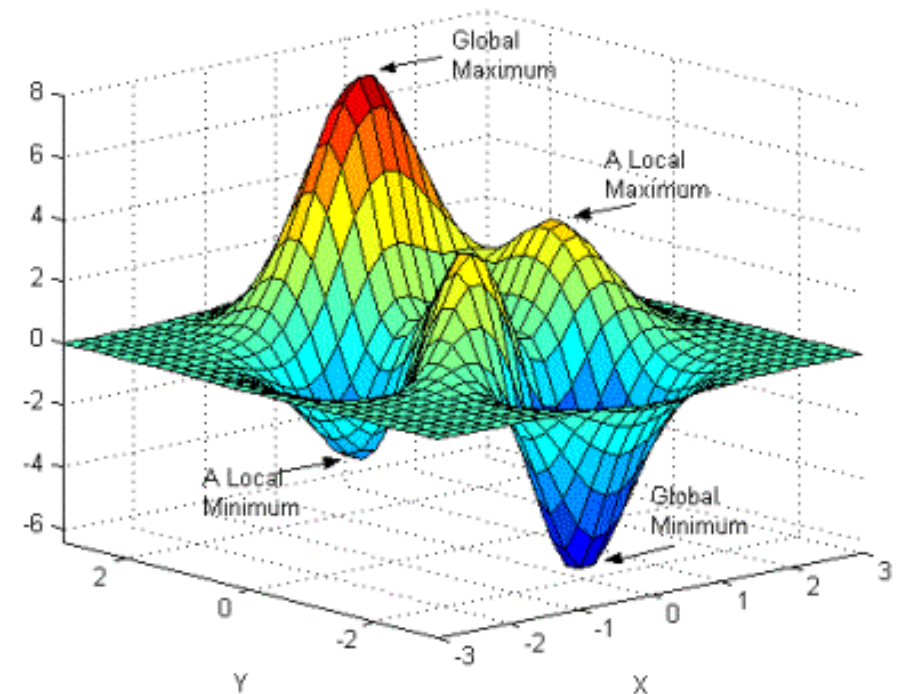
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System dynamics, obstacle avoidance, goal reaching

- Equivalently, maximize  $-f(x)$ : Maximize reward, maximize profit
- Robotics spans many fields
  - Many conventions
  - Many notations clashes

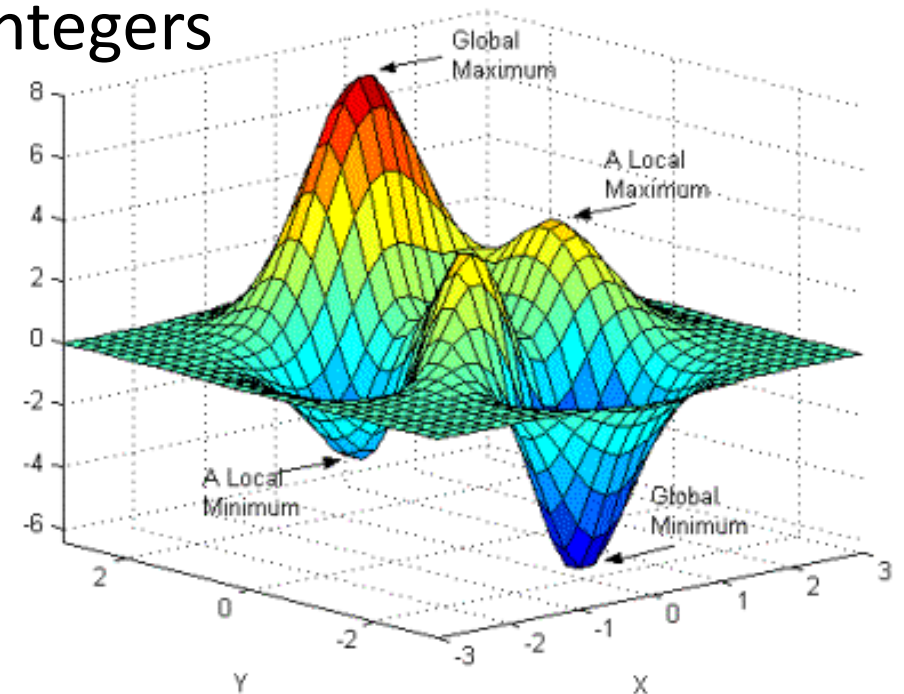
# Nonlinear Optimization

- A very difficult problem in general for  $x \in \mathbb{R}^n$  where  $n$  is large
  - Calculus facts: necessary and sufficient conditions
  - Rely on gradients (if possible)



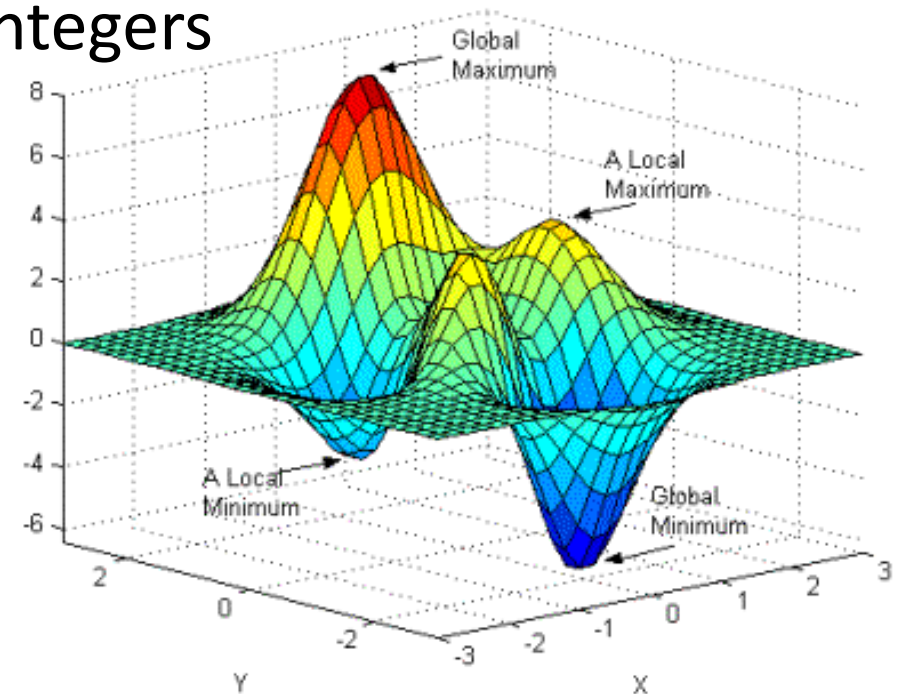
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  - Can we do better than brute force?



# Nonlinear Optimization

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  - Calculus facts: necessary and sufficient conditions
  - Rely on gradients (if possible)
- Sometimes, some components of  $x$  may be integers
  - Can we do better than brute force?
- Simpler cases
  - Differentiable functions
  - Linear, convex, quasiconvex
  - Unconstrained problems

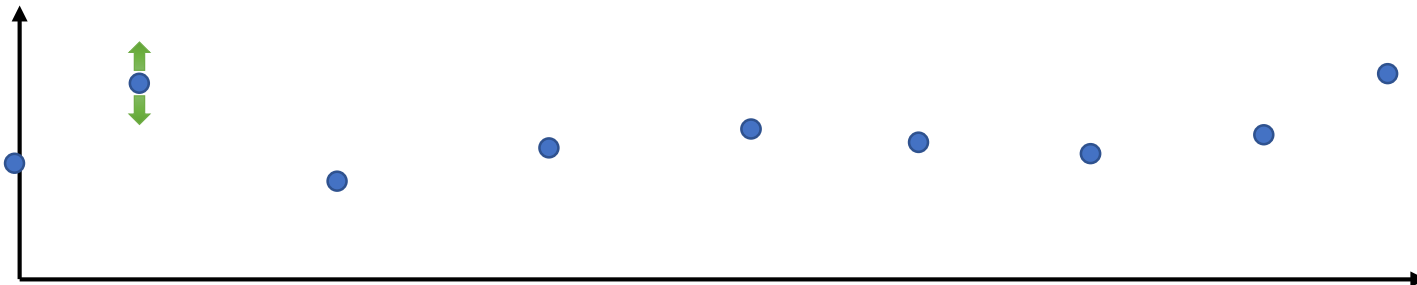


# Nonlinear Optimization

minimize  $f(x)$

subject to  $g_i(x) \leq 0, i = 1, \dots, n$   
 $h_j(x) = 0, j = 1, \dots, m$

- Nonlinear optimization:
  - Decision variable is  $x \in \mathbb{R}^n$





# Optimal Control

$$\begin{aligned} & \underset{u(\cdot)}{\text{minimize}} \quad \overbrace{l(x(t_f), t_f)}^{\text{Final cost}} + \overbrace{\int_0^{t_f} c(x(t), u(t), t) dt}^{\text{Running cost}} \\ & \text{subject to } \dot{x}(t) = f(x(t), u(t)) \\ & \quad \quad \quad g(x(t), u(t)) \geq 0 \\ & \quad \quad \quad x(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^m, x(0) = x_0 \end{aligned}$$

Cost functional,  $J(x(\cdot), u(\cdot))$

Dynamic model

Additional constraints

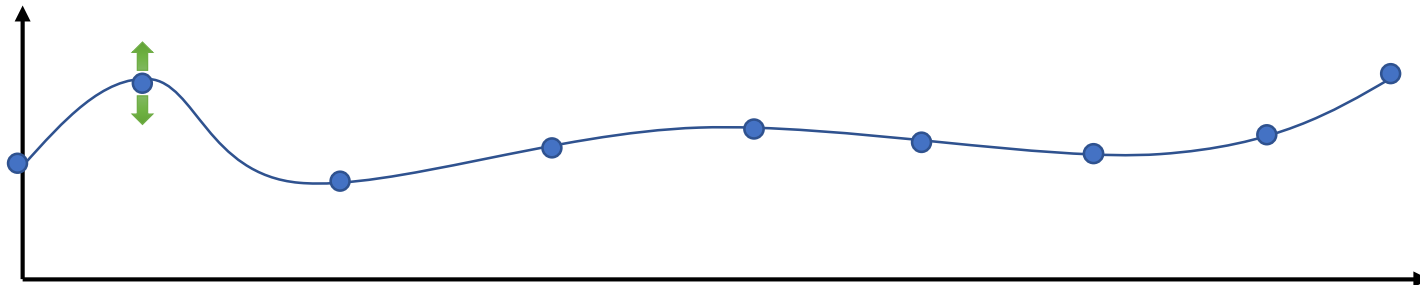
- Eg. actuation limits

- Nonlinear optimization:

- Decision variable is  $x \in \mathbb{R}^n$

- Optimal control:

- Decision variable is a function  $u(\cdot)$



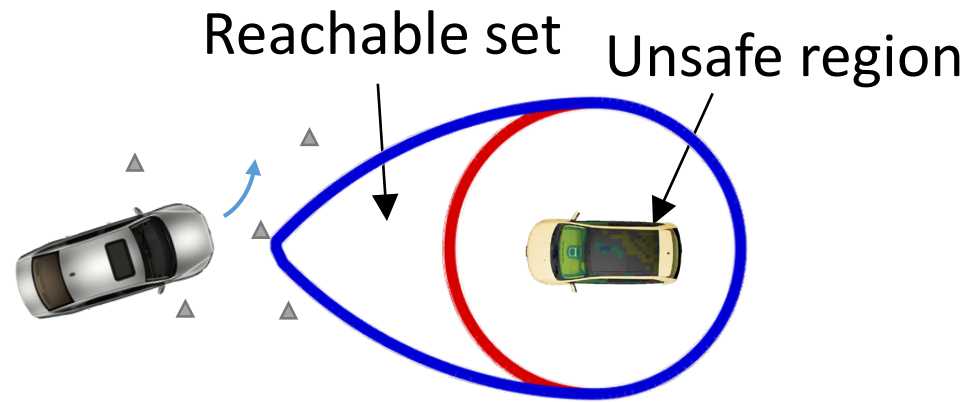
# Robotic Safety

- Verification methods



- Considers all possible system behaviours, given assumptions
- Can be written as an optimal control problem

# Reachability Analysis



- Model of robot
- Unsafe region

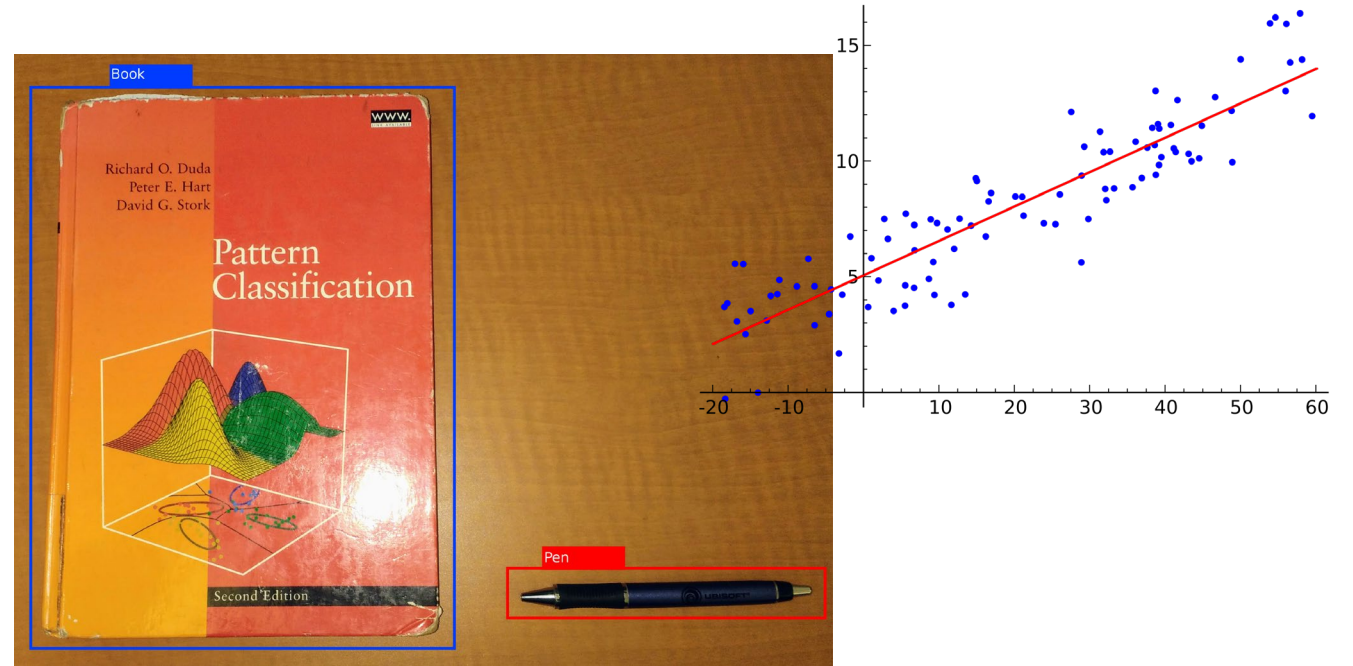


Optimal control policy to avoid danger

Reachable set (States leading to danger)

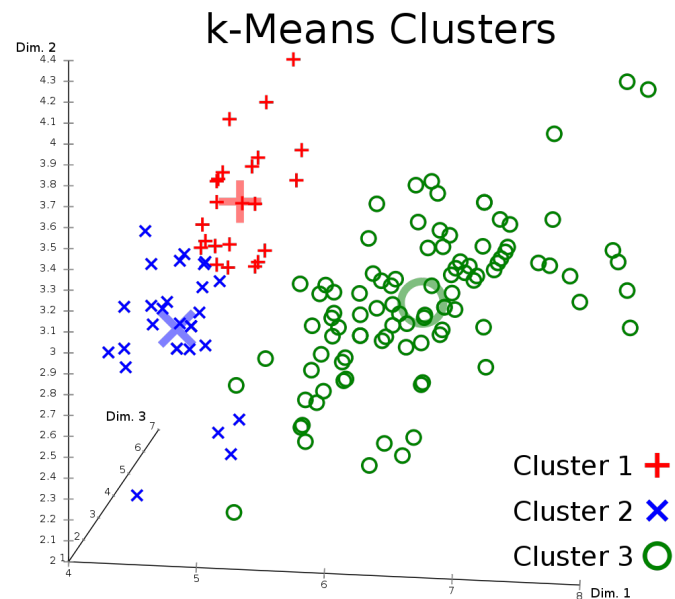
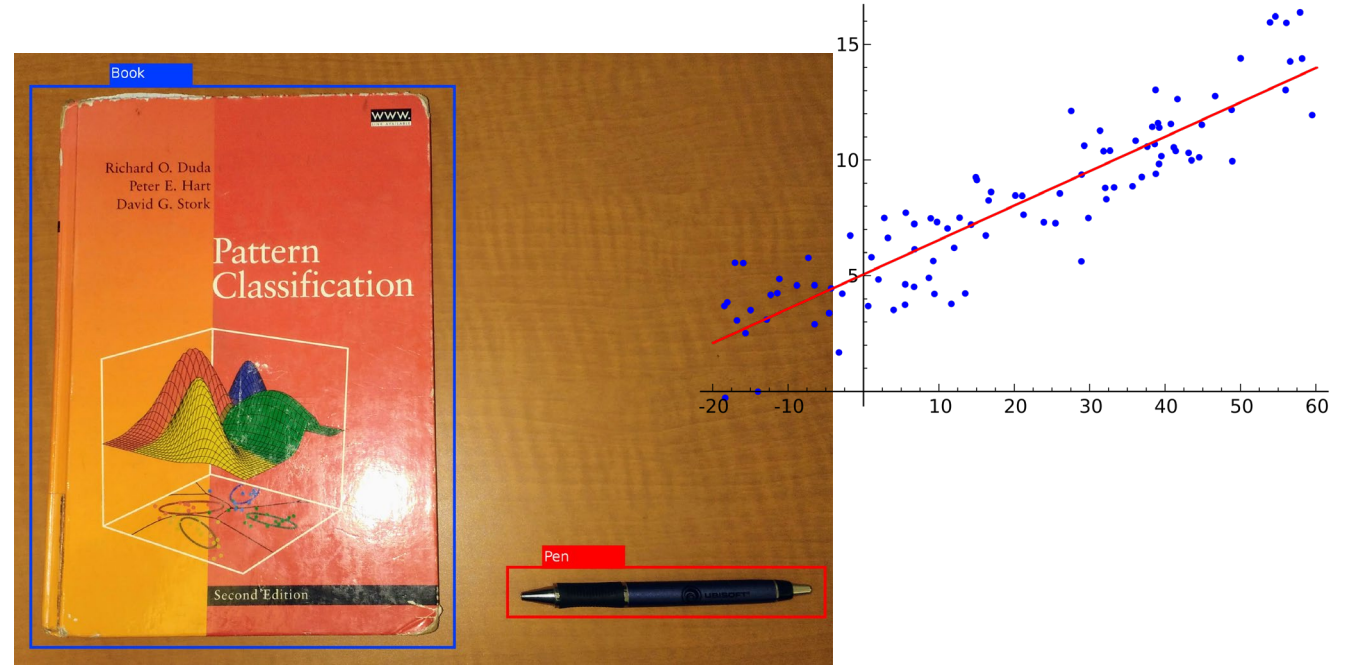
# Machine Learning

- Application of nonlinear optimization
  - Takes advantage of available data
- Supervised learning
  - Regression
  - Classification



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- Application of nonlinear optimization
  - Takes advantage of available data
- Supervised learning
  - Regression
  - Classification
- Unsupervised learning
  - Clustering
  - Reinforcement learning



# Machine Learning

- Very scalable with additional data
- Requires a lot of data

# Machine Learning

- Very scalable with additional data
- Requires a lot of data
- Computer vision
- Natural language processing
- Game playing
- Simulated robotics
- Physical robotics?

# Localization and Mapping

- Localization
  - Given a map, figure out where the robot is (with respect to the map) using sensor information
  - Continuously do this while moving around in the environment



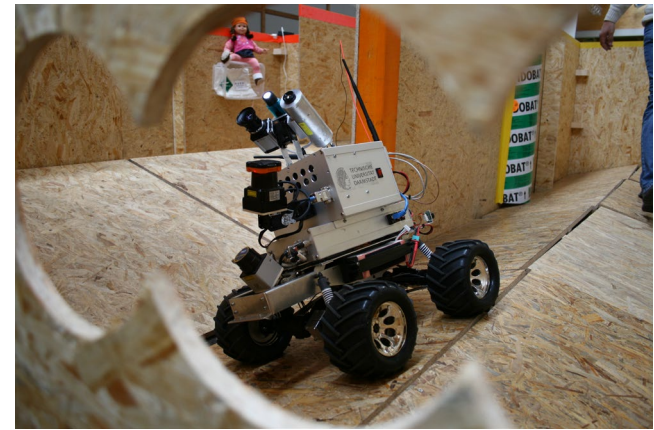


# Localization and Mapping

- Localization
  - Given a map, figure out where the robot is (with respect to the map) using sensor information
  - Continuously do this while moving around in the environment
- Simultaneous localization and mapping
  - Figure out the map and localize at the same time



# Localization and Mapping

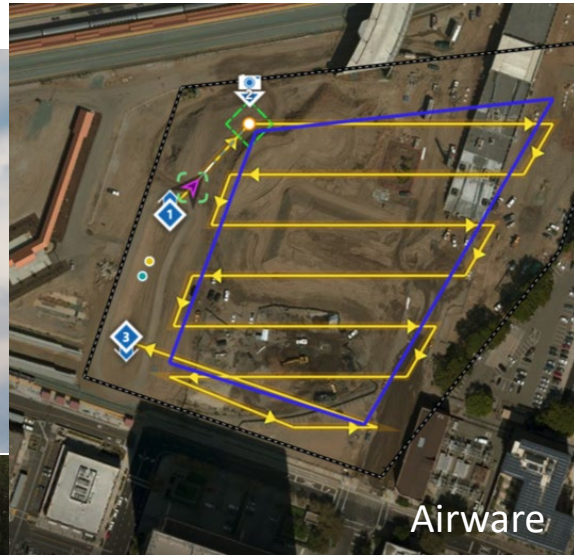


- Localization
  - Given a map, figure out where the robot is (with respect to the map) using sensor information
  - Continuously do this while moving around in the environment
- Simultaneous localization and mapping
  - Figure out the map and localize at the same time
- Probabilistic models
  - of how the robot moves
  - of how the robot senses the environment

# Sample of MARS Research

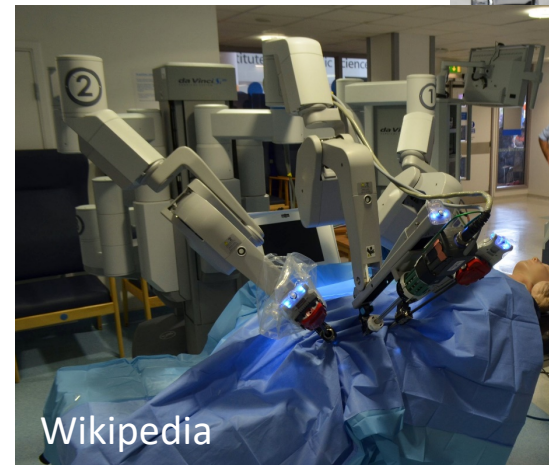
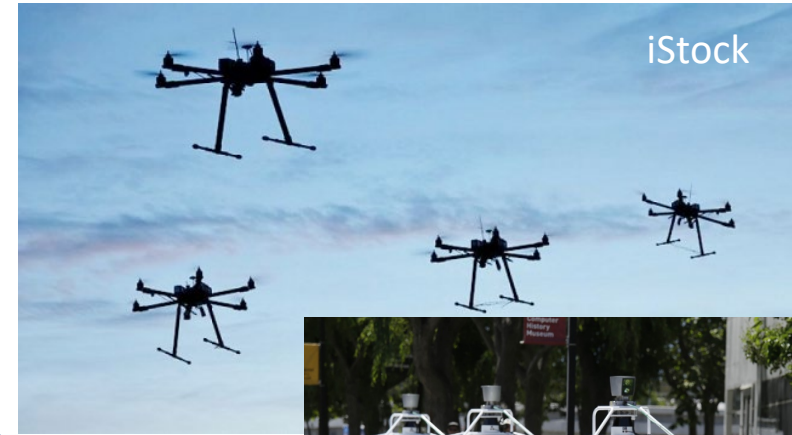
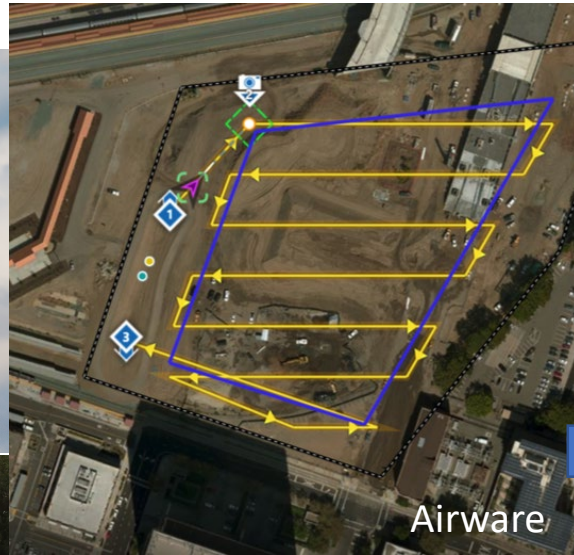
- <https://sfumars.com>
- Control algorithms
- Computational complexity
- Reinforcement learning
- Human intent inference
- Theory
- Computation
- Experiments

# Safety: A Crucial Perspective in Automation





# Safety: A Crucial Perspective in Automation

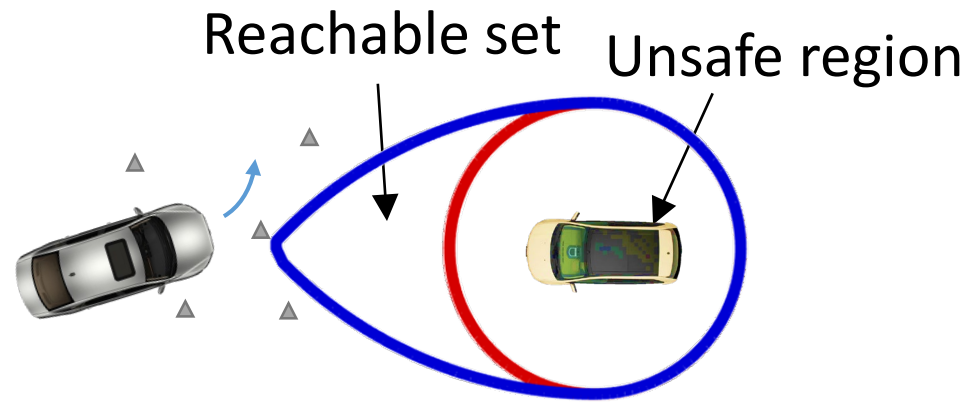


# Challenges in Safety-Critical Systems

- Account for all possible system behaviours
- Complex systems
- Complex environment
  - Weather conditions
  - Other robots



# Reachability Analysis



- Model of robot
- Unsafe region



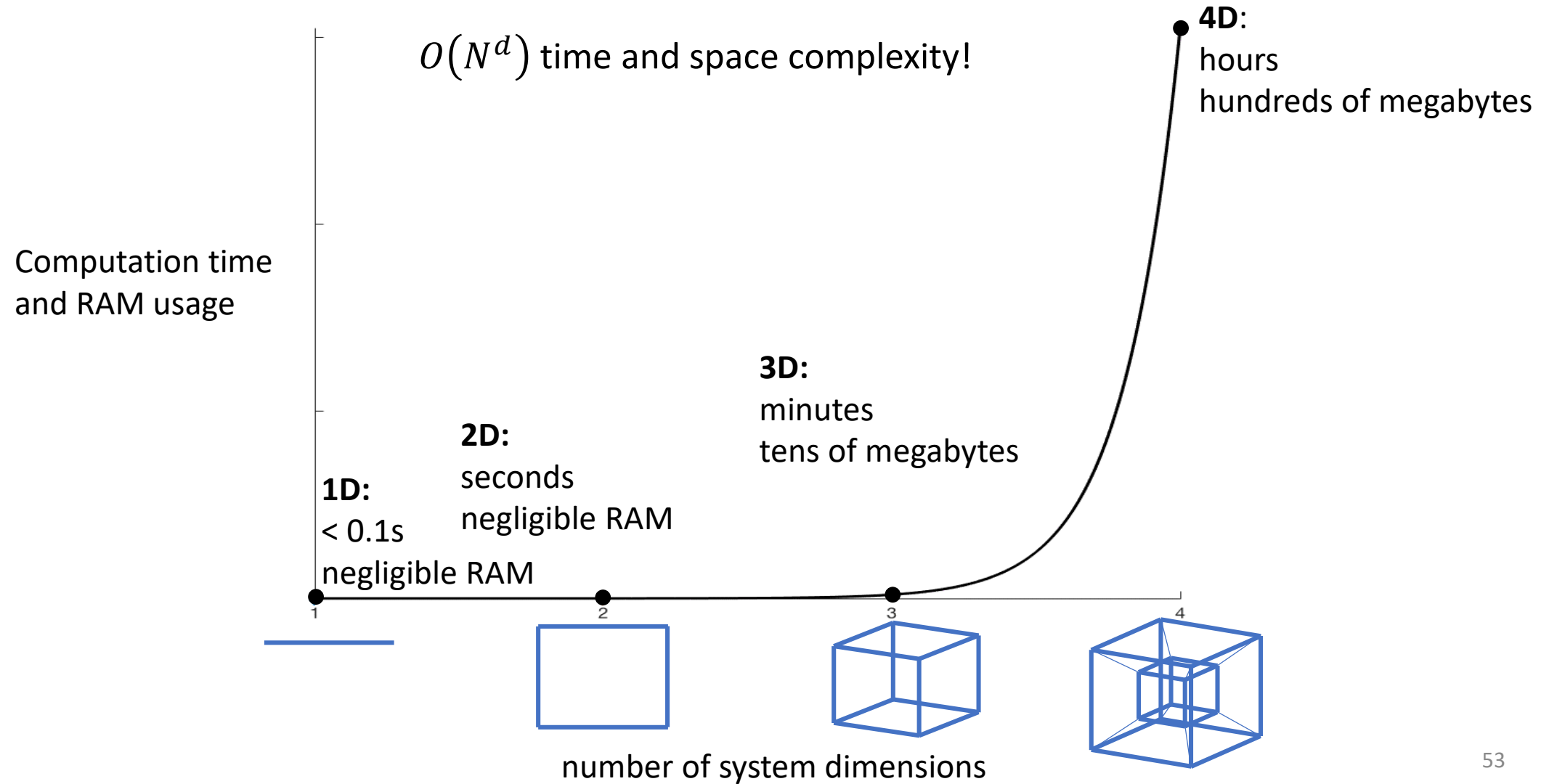
Reachable set (States leading to danger)

Optimal control policy to avoid danger



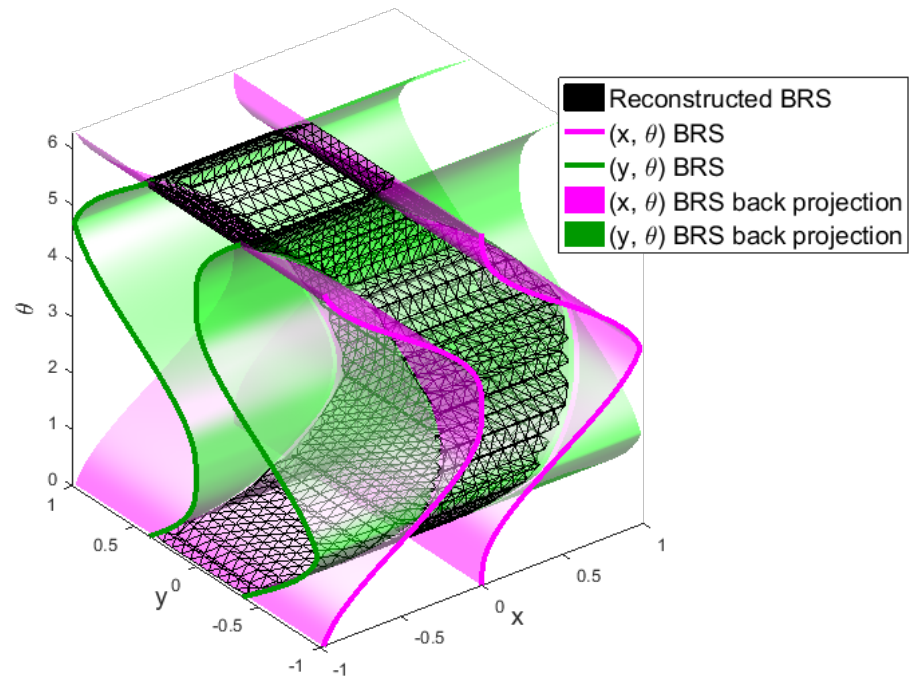


# Main Challenge: Exponential Computational Complexity with DP



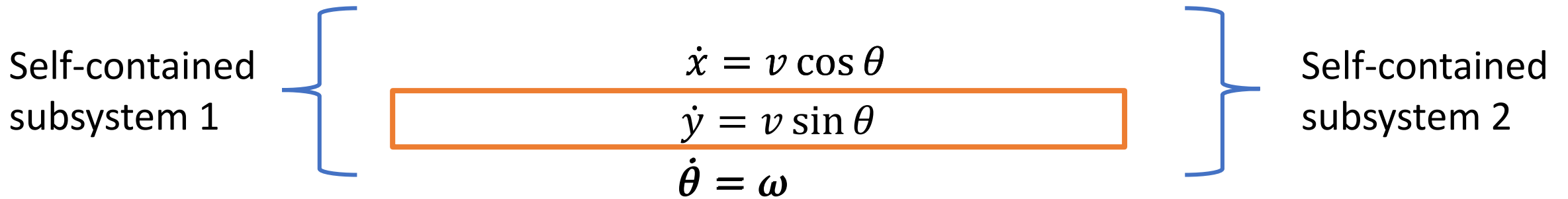
# Research Directions

## Dimensionality reduction



# Self-Contained Subsystems

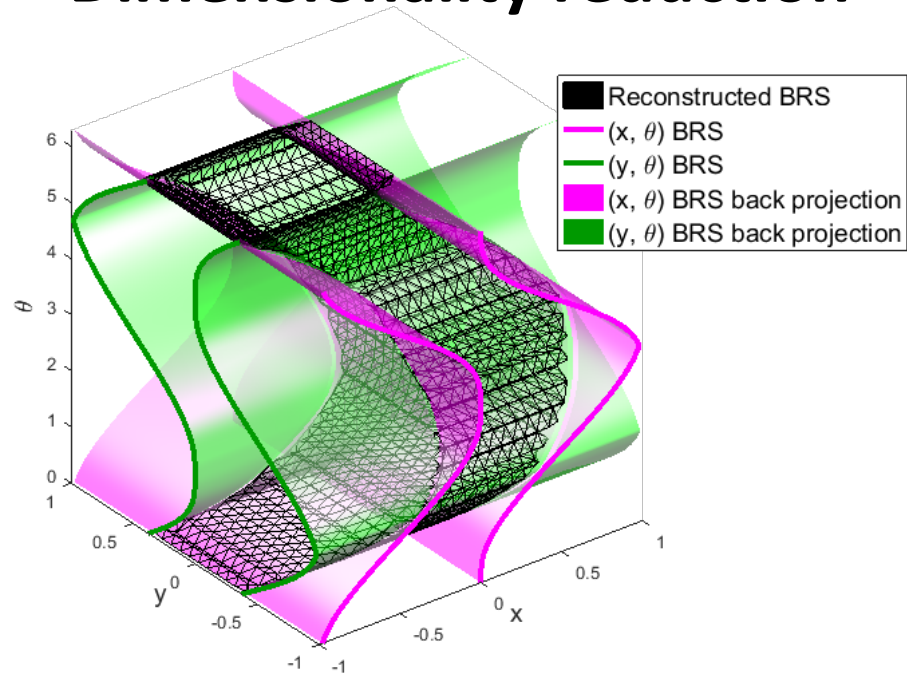
- Motivating example: Dubins Car



- Subsystems are coupled through state and control
- Many systems have states that are not directly coupled to each other
  - Most common in vehicle dynamics

# Research Directions

## Dimensionality reduction

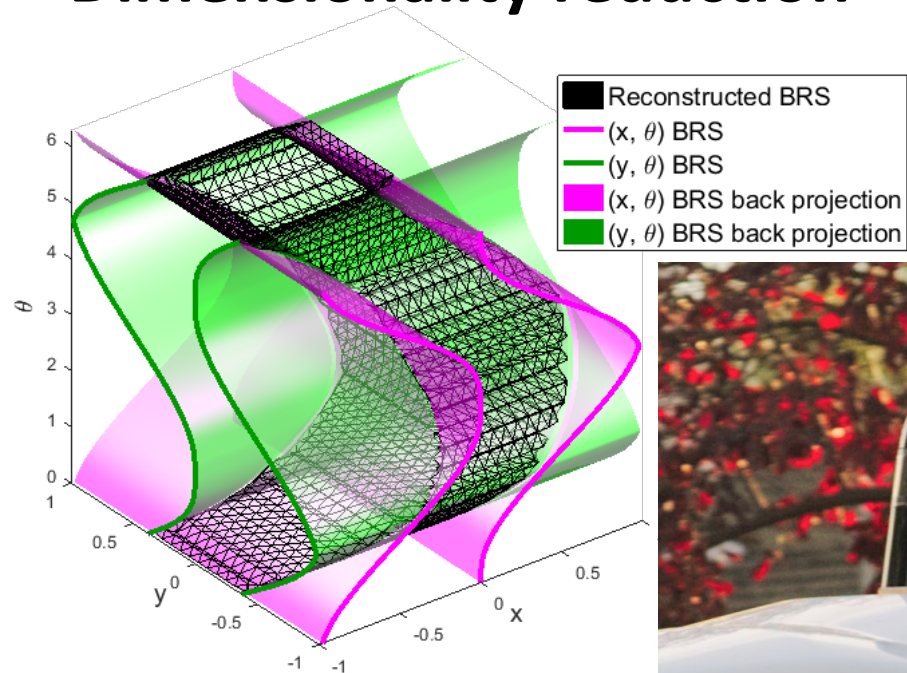


## Parallel computing



# Research Directions

## Dimensionality reduction



## Parallel computing



## Perception systems

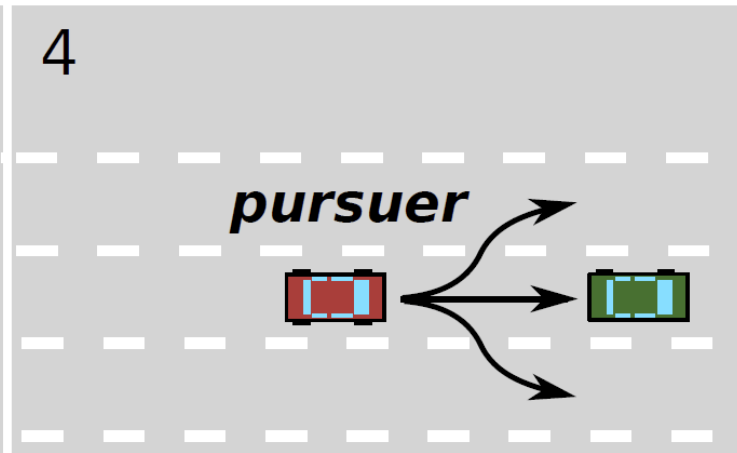
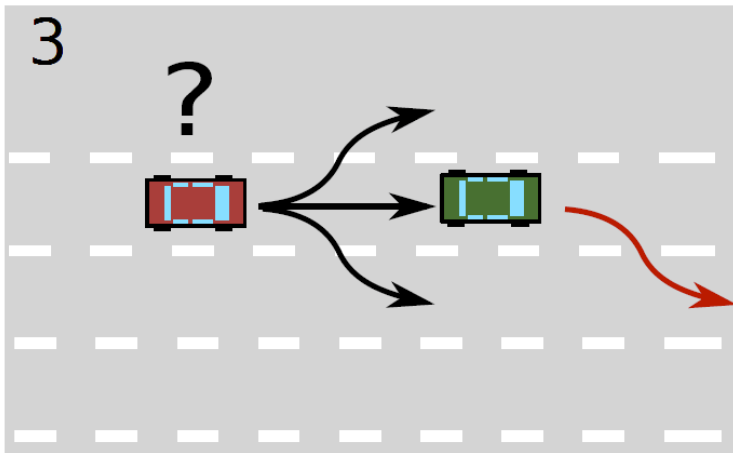
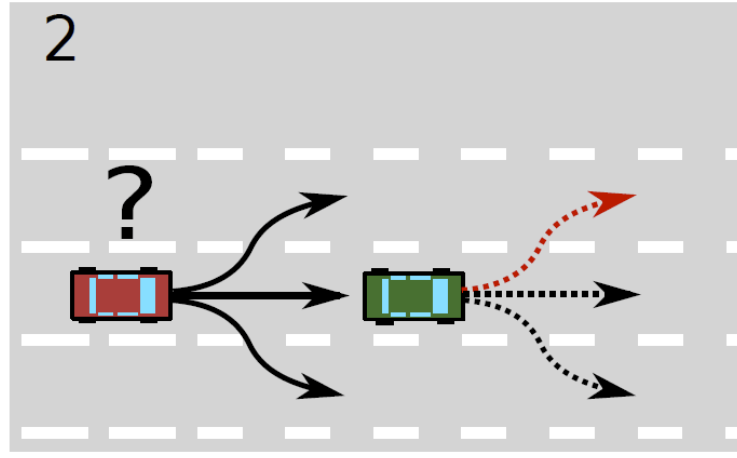
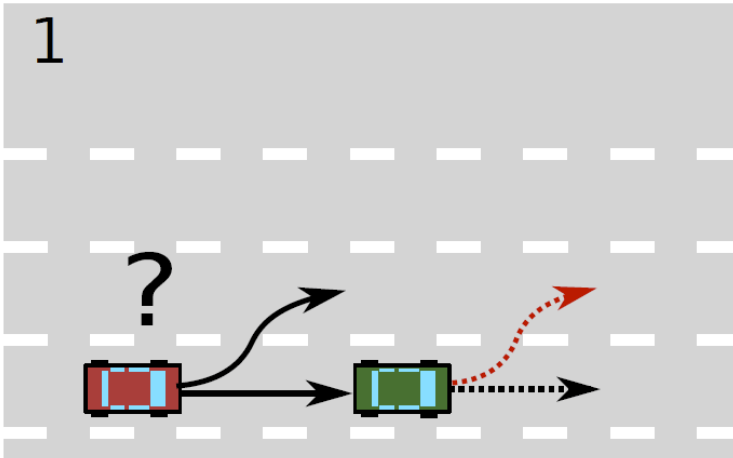
# Research Directions

## Human intent understanding



Campbell et al.

# Proactive Human Intent Understanding



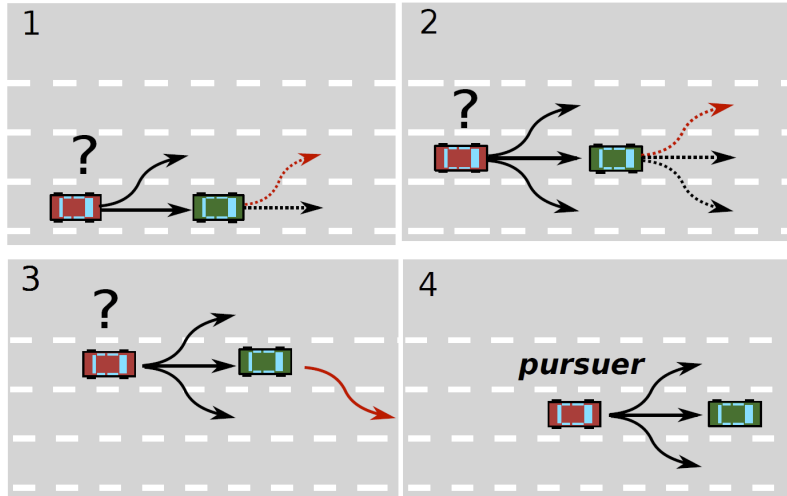
Is the red car

- A pursuer,
- Or a benign vehicle?

Robot car (green)  
*proactively* changes lanes  
to determine intent



# Multi-Modal Human Intent Understanding

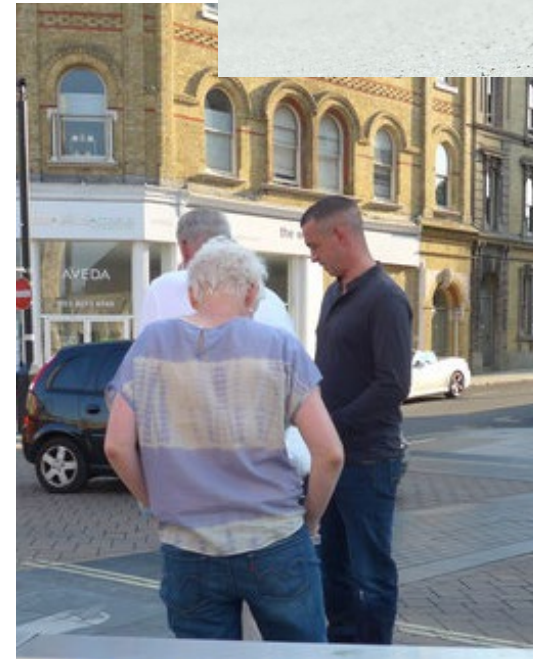


Motion

Emotion

Engagement

Audio





# Research Directions

## Human intent understanding



Campbell et al.

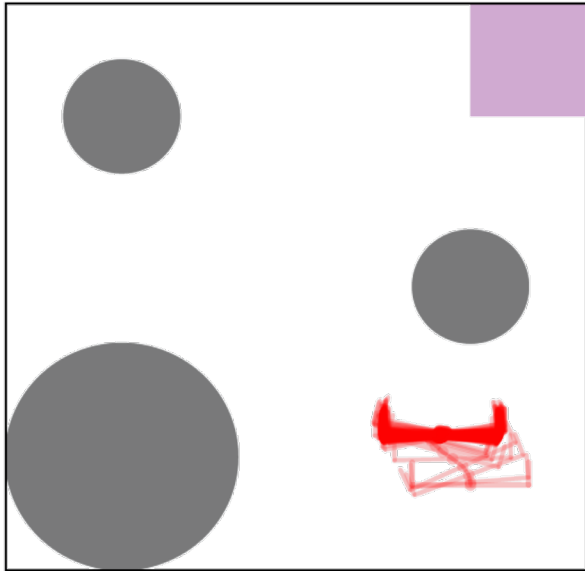
## Robotic learning



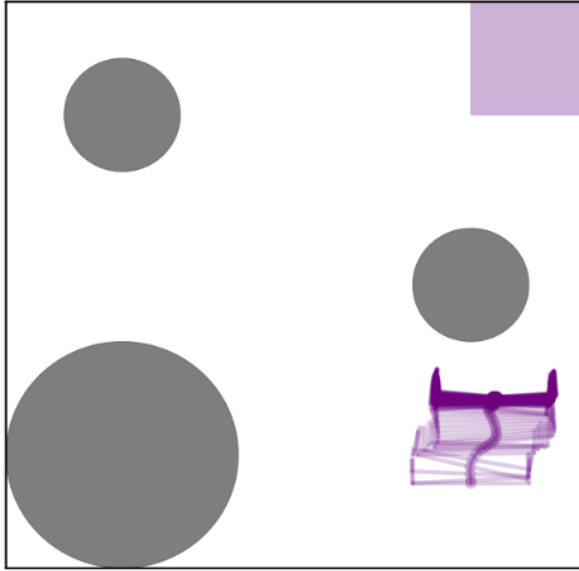
Global Robots Ltd.

# Curriculum Reinforcement Learning

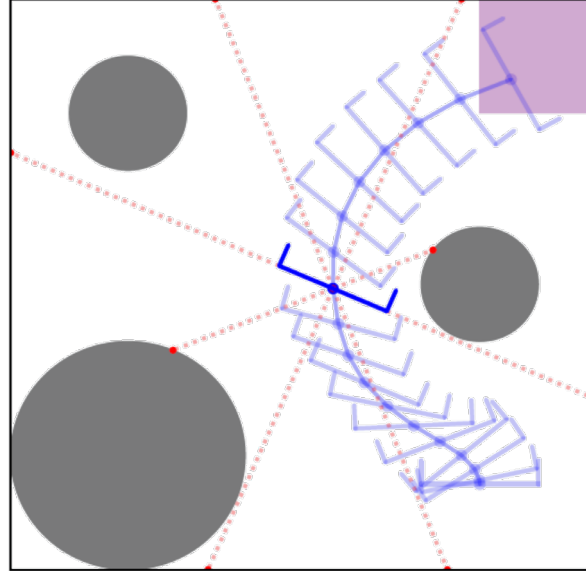
Without curriculum



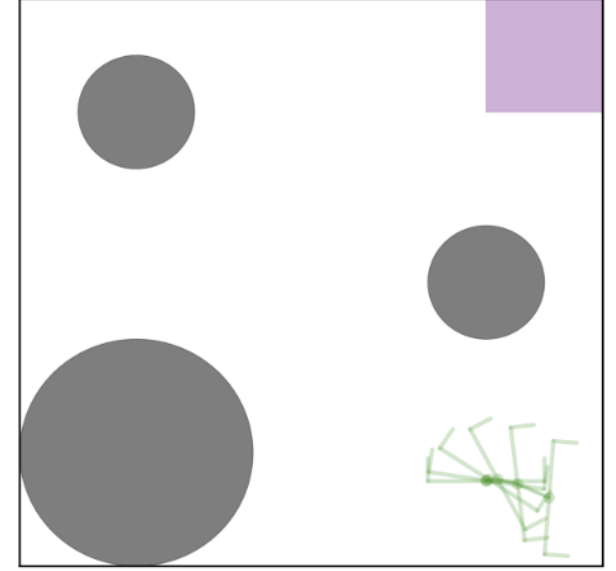
Distance-based reward shaping



Reachability-based curriculum

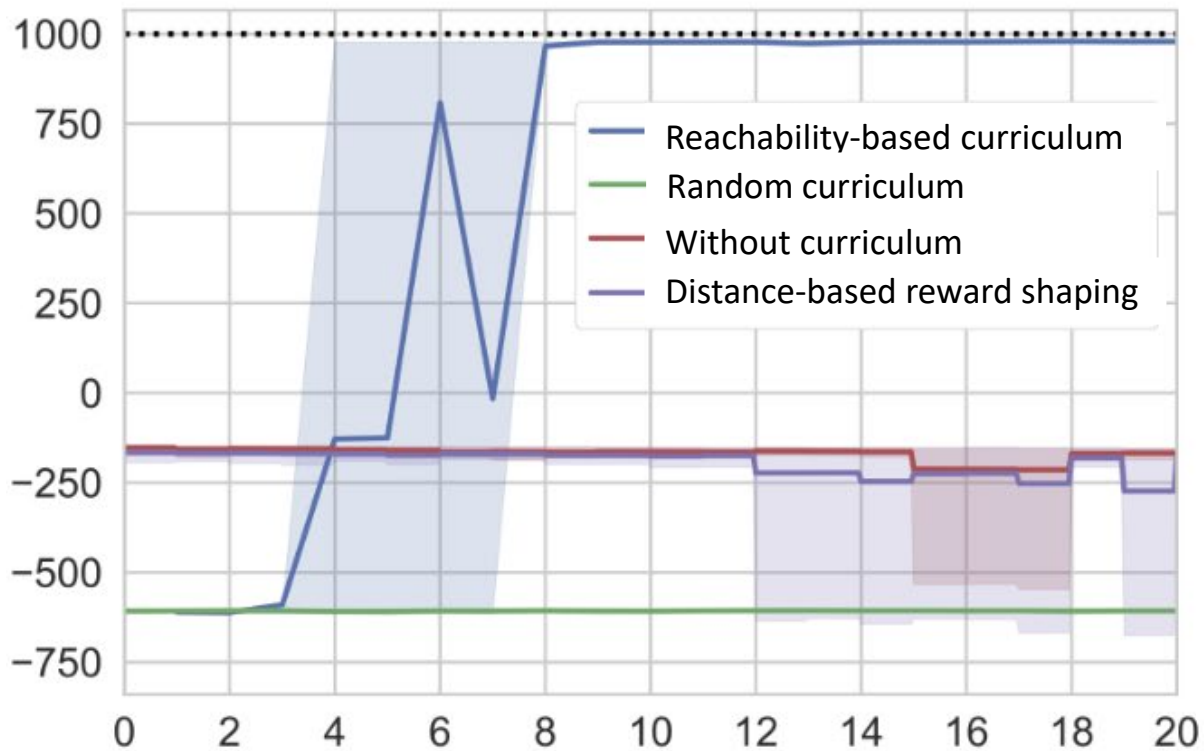


Random curriculum



# Curriculum Reinforcement Learning

Task performance



Curriculum performance

