# CMPT 310, Spring 2019, Written Assignment Replacement 

Due date: No submission required

Problem 1. In this question we consider decision trees with continuous input attributes $A_{1}, \ldots, A_{n}$ and a Boolean output attribute $Y$. In such trees, the test at each internal node is an inequality of the form $A_{k}>c$, where $c$, the split point may be any real number (to be chosen by the learning algorithm. The value at each leaf is true or false. In a test-once tree, each attribute may be tested at most once on any path in the tree; in a test-many tree, each attribute may be tested more than once on a path.

Suppose we are given the four examples (also shown in the Figure 1 ).

| $A_{1}$ | $A_{2}$ | Y |
| :---: | :---: | :---: |
| 3 | 3 | false |
| 6 | 13 | true |
| 15 | 14 | true |
| 14 | 22 | false |



Figure 1: Examples used in Problems 1 and 2
(a) Draw a test-once decision tree that classifies the examples correctly.
(b) Write down the information gain of your root test and the child test (your answer may contain logs; numerical evaluation not required).

Problem 2. Considering the Figure 1 and the Problem 1 description:
(a) Can every non-noisy training set can be correctly classified by a test-once decision tree? Why or why not?
(b) Can every non-noisy training set can be correctly classified by a test-many decision tree?Why or why not?
(c) Now consider the four training sets below (Figure 2). Which are correctly classifiable by test-many decision trees with a reasonably few nodes?


Figure 2: Training sets for Problem 2.d and 2.e.
(d) Which are correctly classifiable by a single-layer perceptron (Figure 2)?

Problem 3. Consider the class of neural networks with inputs are either 0 or 1 and where $g$ is a step function.
(a) Describe how to specify a network that computes the majority function on $n$ inputs. That is, it should output 1 if at least half the inputs are 1.
(b) Draw a decision tree that represents the disjunction of five inputs.
(c) Suppose you're training a neural network in a genuinely nondeterminstic domain. The training set consists of $N$ copies of the same example, a fraction $p>0.5$ of which are positive and a fraction $1-p$ of which are negative. Suppose we use the absolute error function $E=\sum_{i=1}^{N}\left|T_{i}-O\right|$ where $T_{i}$ is the correct value for example $i$ and $O$ is the network's output for this example. Suppose that $O$ must also be in the range [0,1]. By writing out an expression for the error in terms of $O$, find the value of $O$ that minimizes the error.

Problem 4. The Surprise Candy company makes candy in two flavors: $70 \%$ are strawberry flavor and $30 \%$ are anchovy flavor. Each new piece of candy starts out with a round shape; as it moves down the production line, a machine randomly selects a certain percentage to be trimmed into a square; then, each piece is wrapped in a wrapper whose color is chosen randomly to be red or brown. $80 \%$ of strawberry candies are round and $80 \%$ have a red wrapper, while $90 \%$ of the anchovy candies are square and $90 \%$ have a brown wrapper. All candies are sold individually in sealed, identical black boxes.

Now you, the customer, have just bought a Surprise candy at the store but have not yet opened the box. Consider these three Bayes nets shown in Figure 3.
(a) Which network(s) can correctly represent $P$ (Flavor, Wrapper, Shape)? Why?
(b) Which network is the best representation for this problem? Why?
(c) True/False: Network (i) asserts that $P$ (Wrapper|Shape) $=P$ (Wrapper).


(iii)

Figure 3: Bayes Nets used in Problems 4 and 5

Problem 5. Considering the Bayes Nets in Figure 3 and the situation described in Problem 4, answer the following items.
(a) What is the probability that your candy has a red wrapper?
(b) In the box is a round candy with a red wrapper. The probability that its flavor is strawberry is ?
(c) An unwrapped strawberry candy is worth $s$ on the open market and an unwrapped anchovy candy is worth $a$. Write an expression for the value of an unopened candy box.

