Problem 1. Design a neural network that implements the XOR function. That is, define neurons, their connectivity and weights. Use threshold as your activation function (that is, the function that returns 0 or 1 if the input is negative or positive respectively). Extra challenge: do so using the fewest number of hidden neurons.

Problem 2. Imagine you have defined a neural network using the identity activation function; that is, \( g(x) = x \). Suppose the network has one hidden layer with 3 neurons, which is fully connected to the input. Write the expression for \( a_{out} \) as a function of the input. Show that you can express the same network as a single-layer perceptron. What does this tell you about the expressive power of a multi-layer neural network with the identity activation function?

Problem 3 (Optional). Consider the problem of learning to play tennis (or some other sport with which you are familiar). Explain how this process fits into the general learning model. Describe the percepts and actions, and the types of learning the player must do. Describe the subfunctions the player is trying to learn in terms of inputs and outputs, and available example data. Is this supervised or reinforcement learning?

Problem 4 (Optional). Suppose we generate a training set from a decision tree and then apply decision-tree learning to that training set. Is it the case that the learning algorithm will eventually return the correct tree as the training-set size goes to infinity? Why or why not?

Problem 5 (Optional). A decision graph is a generalization of a decision tree that allows nodes (i.e., attributes used for splits) to have multiple parents, rather than just a single parent. The resulting graph must still be acyclic. Now, consider the XOR function of three binary input attributes, which produces the value 1 if and only if an odd number of the three input attributes has value 1.

(a) Draw a minimal-sized decision tree for the three-input XOR function.

(b) Draw a minimal-sized decision graph for the three-input XOR function.