CMPT 310, Spring 2019, Written Assignment 08

Due date: March 18, 2019

Problem 1. Consider the variable elimination algorithm (Use as reference the algorithm on the book "*Artificial Intelligence*", page 528).

(a) Section 14.4 applies variable elimination to the query $P(Burglary|JohnCalls = true, MaryCalls = true)$. Perform the calculations indicated and check that the answer is correct.

(b) Count the number of arithmetic operations performed, and compare it with the number performed by the enumeration algorithm.

Problem 2. In this exercise, we examine that happens to the probabilities in the umbrella world in the limit of long time sequences. Suppose we observe an unending sequence of days on which the umbrella appears. Show that, as the days go by, the probability of rain on the current day increases monotonically toward a fixed point. Calculate this fixed point.

Problem 3 (Optional). Suppose a network has the form of a *chain*: a sequence of Boolean variables $X_1, ..., X_n$ where $Parents(X_i) = \{X_{i-1}\}$ for $i = 2, ..., n$. What is the complexity of computing $P(X_1|X_n = true)$ using enumeration? Using variable elimination?

Problem 4 (Optional). A professor wants to know if students are getting enough sleep. Each day, the professor observes whether the students sleep in class, and whether they have red eyes. The professor has the following domain theory:

- The prior probability of getting enough sleep, with no observation, is 0.7.
- The probability of getting enough sheep on night t is 0.8 given that the student got enough sleep the previous night, and 0.3 if not.
- The probability of having red eyes is 0.2 if the student got enough sleep, and 0.7 if not.
- The probability of sleeping in class is 0.1 if the student got enough sleep, and 0.3 if not.

Formulate this information as a dynamic Bayesian network that the professor could use to filter or predict form a sequence of observations. Then reformulate it as a hidden Markov model that has only a single observation variable. Five the complete probability tables for the model.

Problem 5 (Optional). For the DBN specified in Question 4 and for the evidence values
e_1 = not red eyes, not sleeping in class
e_2 = red eyes, not sleeping in class
e_3 =red eyes, sleeping in class
perform the computation of the state estimation $P(EnoughSleep_t|e_{1:t})$ for each of $t = 1, 2, 3$.