CMPT 310, Spring 2019, Written Assignment 07

Due date: March 04, 2019

Problem 1. We have a bag of three biased coins a,b and c with probabilities of coming up heads of 20%, 60%, and 80%, respectively. One coin is drawn randomly from the bag (with equal likelihood of drawing each of the three coins), and then the coin is flipped three times to generate the outcomes X_1, X_2, and X_3.

(a) Draw the Bayesian network corresponding to this setup and define the necessary CPTs.

(b) Calculate which coin was most likely to have been drawn from the bag if the observed flips come out heads twice and tails once.

Problem 2. Consider the Bayesian network in Figure 1.

(a) If no evidence is observed, are *Burglary* and *Earthquake* independent? Prove this from the numerical semantics and from the topological semantics.

(b) If we observe $Alarm = true$, are *Burglary* and *Earthquake* independent? Justify your answer by calculating whether the probabilities involved satisfy the definition of conditional independence.

Problem 3 (Optional). Investigate the complexity of exact inference in general Bayesian networks. Prove that any 3-SAT problem can be reduced to exact inference in a Bayesian network constructed to represent the particular problem and hence that exact inference is NP-hard. (Hint: consider a network with one variable for each proposition symbol, one for each clause, and one for the conjunction of clauses.)

Problem 4 (Optional). In your local nuclear power station, there is an alarm that senses when a temperature gauge exceeds a given threshold. The gauge measures the temperature of the core. Consider the Boolean variables A (alarm sounds), F_A(alarm is faulty), and F_G(gauge is faulty) and the multivalued nodes G (gauge reading) and T (actual core temperature).

(a) Draw a Bayesian network for this domain, given that the gauge is more likely to fail when the core temperature gets too high.

(b) Suppose there are just two possible actual and measured temperatures, normal and high; the probability that the gauge gives the correct temperature is x when it is working, but y when it is faulty. Give the conditional probability table associated with G.

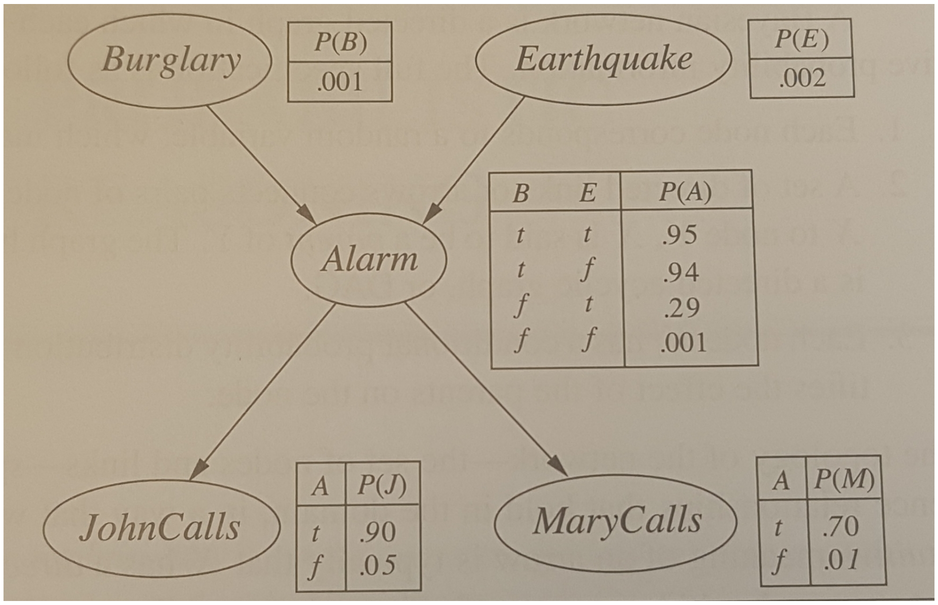

Figure 1: A typical Bayesian network, showing both the topology and the conditional probability tables (CPTs). In the CPTs, the letters B, E, A, J, and M stand for *Burglary, Earthquake, Alarm, John calls, and Mary calls*, respectively.

(c) Suppose the alarm works correctly unless it is faulty, in which case it never sounds. Give the conditional probability table associated with A.

(d) Suppose the alarm and gauge are working and the alarm sounds. Calculate an expression for the probability that the temperature of the core is too high, in terms of the various conditional probabilities in the network.

Problem 5 (Optional). Let H_x be a random variable denoting the handedness of an individual x, with possible values l or r. A common hypothesis is that left- or right-handedness is inherited by a simple mechanism; that is, perhaps, there is a gene G_x also with values l or r, and perhaps actual handedness turns out mostly the same (with some probability s) as the gene an individual possesses. Furthermore, perhaps the gene itself is equally likely to be inherited from either of an individual's parents, with a small nonzero probability m of a random mutation flipping the handedness.

(a) Which of the three networks in Figure 2 claim that
$P(G_{father}, G_{mother}, G_{child}) = P(G_{father})P(G_{mother})P(G_{child})$.

(b) Which of the three networks make independence claims that are consistent with the hypothesis about the inheritance of handedness?

(c) Which of the three networks is the best description of the hypothesis?

(d) Write down the CPT for the G_{child} node in network (a), in terms of s and m.

(e) Suppose that $P(G_{father} = l) = P(G_{mother} = l) = q$. In network (a), derive an expression for $P(G_{child} = l)$ in terms of m and q only, by conditioning on its parent nodes.

(f) Under conditions of genetic equilibrium, we expect the distribution of genes to be the same across generations. Use this to calculate the value of q, and, given what you know about handedness in humans, explain why the hypothesis described at the beginning of this question must be wrong.

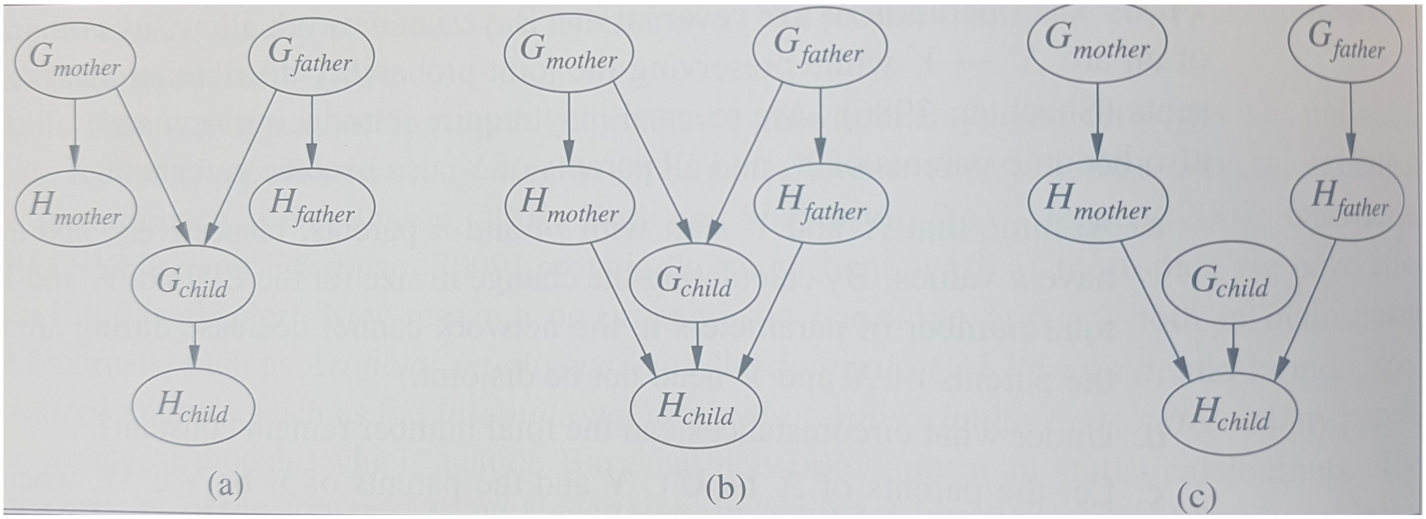

Figure 2: Three possible structures for a Bayesian network describing genetic inheritance of handedness.