Problem 1. Consider the problem of placing $k$ knights on an $n \times n$ chessboard such that no two knights are attacking each other, where $k$ is given and $k \leq n^2$.

(a) Choose a CSP formulation. In your formulation, what are the variables?

Solution . .
Solution A: There is a variable corresponding to each of the $n^2$ positions on the board. Solution B: There is a variable corresponding to each knight.

(b) What are the possible values of each variable?

Solution . .
Solution A: Each variable can take one of two values, occupied, vacant Solution B: Each variables’s domain is the set of squares.

(c) What sets of variables are constrained, and how?

Solution . .
Solution A: every pair of squares separated by a knights move is constrained, such that both cannot be occupied. Furthermore, the entire set of squares is constrained, such that the total number of occupied squares should be $k$. Solution B:every pair of knights is constrained, such that no two knights can be on the same square or on squares separated by a knights move. Solution B may be preferable because there is no global constraint, although Solution A has the smaller state space when $k$ is large.

Problem 2. Consider a vocabulary with only four propositions, A, B, C and D. How many models are there for the following sentences?

Solution . These can be computed by counting the rows in a truth table that come out true, but each has some simple property that allows a short-cut.

(a) $B \lor C$.

Solution . Sentence is false only if B and C are false, which occurs in 4 cases for A and D, leaving 12.
(b) \( \neg A \lor \neg B \lor \neg C \lor \neg D \).

**Solution** . Sentence is false only if A, B, C, and D are false, which occurs in 1 case, leaving 15.

(c) \( (A \Rightarrow B) \land A \land \neg B \land C \land D \).

**Solution** . The last four conjuncts specify a model in which the first conjunct is false, so 0.

**Problem 3 (Optional).** Suppose the agent has progressed to the point shown in Figure 2, having perceived nothing in \([1,1]\), a breeze in \([2,1]\), and is now concerned with the contents of \([1,3]\), \([2,2]\), and \([3,1]\). Each of these can contain a pit, and at most one can contain a wumpus. Following the example of Figure 3, construct the set of possible worlds. (You should find 32 of them). Mark the worlds in which the KB is true and those in which each of the following sentences is true:

\( \alpha_2 = \text{“There is no pit in \([2,2]\)”} \)

\( \alpha_3 = \text{“There is a wumpus in \([1,3]\)”} \)

**Solution** . See list of models as a Table (Figure BLA) rather than a collection of diagrams. There are eight possible combinations of pits in the three squares, and four possibilities for the wumpus locations (including nowhere). We can see that \( KB \models \alpha_2 \) because every line where \( KB \) is true also has \( \alpha_2 \) true. Similarly for \( \alpha_3 \)

**Problem 4 (Optional).** Prove, or find a counterexample to, each of the following assertions:

(a) If \( \alpha \models \gamma \) or \( \beta \models \gamma \) (or both) then \( (\alpha \land \beta) \models \gamma \).

**Solution** . True. This follows from monotonicity.

(b) If \( \alpha \models (\beta \land \gamma) \) then \( \alpha \models \beta \) and \( \alpha \models \gamma \).

**Solution** . True. If \( \beta \lor \lambda \) is true in every model of \( \alpha \), then \( \beta \) and \( \gamma \) are true in every model of \( \alpha \), so \( \alpha \models \beta \) and \( \alpha \models \gamma \).

(c) If \( \alpha \models (\beta \lor \gamma) \) then \( \alpha \models \beta \) or \( \alpha \models \gamma \) (or both).

**Solution** . False. Consider \( \beta \equiv A \), \( \gamma \equiv \neg A \).

**Problem 5 (Optional).** We have defined four binary logical connectives.

(a) Are there any others that might be useful?

(b) How many binary connectives can there be?
Figure 1: A truth table constructed for Question 3. Propositions not listed as true on a given line are assumed false, and only true entries are shown in the table.

(c) Why are some of them not very useful?

Solution. A binary logical connective is defined by a truth table with 4 rows. Each of the four rows may be true or false, so there are $2^4 = 16$ possible truth tables, and thus 16 possible connectives. Six of these are trivial ones that ignore one or both inputs; they correspond to True, False, $P$, $Q$, $\neg P$ and $\neg Q$. Four of them we have already studied: $\lor$, $\land$, $\Rightarrow$, $\Leftarrow$, $\Leftrightarrow$. The remaining six are potentially useful. One of them is reverse implication ($\Leftarrow$ instead of $\Rightarrow$), and the other five are the negations of $\lor$, $\land$, $\Rightarrow$, $\Leftarrow$, $\Leftrightarrow$. The first three of these are sometimes called nand, nor and xor.
Figure 2: Question 3 - Stage in the progress of the agent.

<table>
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<th>3,4</th>
<th>4,4</th>
</tr>
</thead>
<tbody>
<tr>
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<td>W!</td>
<td>2,3</td>
<td>3,3</td>
<td>4,3</td>
</tr>
<tr>
<td>1,2</td>
<td>$\alpha$</td>
<td>2,2</td>
<td>3,2</td>
<td>4,2</td>
</tr>
<tr>
<td>1,1</td>
<td>V</td>
<td>2,1</td>
<td>$\beta$</td>
<td>3,1</td>
</tr>
<tr>
<td>V</td>
<td>OK</td>
<td>B</td>
<td>OK</td>
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</tr>
</tbody>
</table>

\(\alpha_1\) = Agent
\(\beta\) = Breeze
\(G\) = Glitter, Gold
\(OK\) = Safe square
\(P\) = Pit
\(S\) = Stench
\(V\) = Visited
\(W\) = Wumpus

Figure 3: Question 3 - Possible models for the presence of pits in squares [1, 2], [2, 2], and [3, 1]. The KB corresponding to the observations of nothing in [1, 1] and a breeze in [2, 1] is shown by the solid line. (a) Dotted line shows models of $\alpha_1$ (no pit in [2, 1]). (b) Dotted line shows models of $\alpha_2$ (no pit in [2, 2])