Assignment 2: Corrections

Q1c:

```
void strcopy2(char *s1, char *s2) {
}
int main () {
  char s2[10] = "copy this";
  char s1[10];
  strcopy2(s1, s2);
}
```

Q2, first expression: $0.001 \log_4 n + \log_2(\log_2 n)$

- Course website has corrected version
- Due next Wednesday (Jan. 23) in **CSIL assignment boxes** before class

Algorithm Performance

(the Big-*O*)

Lecture 7

Today:

- Barometer instructions
- Manipulating Big-O expressions
- Growth rates of common functions

The Story So Far . . .

- Often consider the *worst-case* behaviour as a benchmark
- Derive total steps (T) as a function of input size (N)
    - use `time` command to measure for various N
    - OR . . . count the elementary operations
- Use Big-*O* to express the growth rate
    - compares algorithms' behaviour as N gets large
    - leading constants are removed
    - a hardware-independent analysis

Leading Constants (Review)

Leading constants are affected by:

- CPU speed
- other tasks in the system
- characteristics of memory
- program optimization

Regardless of leading constants, a $O(N \log N)$ algorithm will outperform a $O(N^2)$ algorithm as N gets large

As N Gets Large, The Algorithm is Most Important

A carefully crafted algorithm can make the difference between software that is usable and useless

- e.g., if it costs a $O(N)$ algorithm 0.5s to search 1 billion bank records, but a $O(\log N)$ algorithm 0.005s
- e.g., real-time computing - where a nearly instant response is required

Optimizing Algorithms

If you find yourself trying all sorts of clever implementation tricks to speed up an algorithm, you should:

- Step back and ask if you're trying to improve a fundamentally inefficient algorithm
- Consider if there might be a better one

. . . but also realize that there might not be!

It's more important to reduce your running time by a factor of N, than by a factor of 10

- both are important, but not equally important

Big-*O* and Barometer Instructions

Problem: Given an algorithm, how do you determine its Big-*O* growth rate?

- Rule of Thumb: the frequency of the algorithm's *barometer instructions* will be proportional to its Big-O running time

So, find the most frequent operation(s) and count them!

As N Gets Large, The Algorithm is Most Important

“You can’t make a racehorse of a pig, but you can make a very fast pig.”

“When you find yourself trying all sorts of clever implementation tricks to speed up an algorithm, you should step back for a moment and ask if you’re trying to make a racehorse out of a pig. It might be more productive to use your time to look for a racehorse.”

“It’s important to know whether an efficient algorithm is possible. There’s no sense spending time looking for a unicorn. In this situation, the best you can hope for is a fast pig and a short racecourse.”

- Lou Hafer, SFU CS

Loops ⇨ Multiply

```
int max10by10(int a[N][N]) {
    int best = 0;
    for (int u_row = 0; u_row < N-10; u_row++) {
        for (int u_col = 0; u_col < N-10; u_col++) {
            int total = 0;
            for (int row = u_row; row < u_row+10; row++) {
                for (int col = u_col; col < u_col+10; col++) {
                    total += a[row][col];
                }
            }
            best = max(best, total);
        }
    }
    return best;
}
```

x(*N*-10)

x(*N*-10)

x10

x10

barometer instructions

$f(N) = 3 \times 10 \times 10 \times (N-10) \times (N-10) = O(N^2)$

Loops ⇨ Multiply

Q. What is N?
- The number of elements in the array

```
        int dup_chk(int a[], int length) {
1           int i = length;
N + 1       while (i > 0) {
N               i--;
N               int j = i - 1;
i + 1           while (j >= 0) {
i                   if (a[i] == a[j]) {
                        return 1;
                    }
i                   j--;
                }
            }
1           return 0;
        }
```

Outside of loop: 2 (steps)

Outer loop: $3N + 1$

Inner loop: $3i + 1$ for all possible i from 0 to $N - 1$.

$= 3/2\, N^2 - 1/2\, N$

Grand total $= 3/2\, N^2 + 5/2\, N + 3$

A *quadratic* function!

$= O(N^2)$

Inner Loops that Depend on Outer Loop

```
int count = 0;
int N = 1000000;


for (int i = 0; i < N; i++) {
  for (int j = 0; j < i; j++) {
    count++;
  }
}
```

Thought process

i goes from 0 to N-1, so N iterations

j goes from 0 to i-1, so i iterations

- But i keeps changing!
- 1 iteration, 2 iterations, 3 iterations, …, N iterations
- Average: N/2 iterations

Overall:

- Loops → multiply!
- O(N^2)

Inner Loops that Depend on Outer Loop

```
int count = 0;
int N = 1000000;


for (int i = 0; i < N; i++) {
  for (int j = 5; j < i; j = j + 3) {
    count++;
  }
}
```

Thought process

`i` goes from 0 to N-1, so N iterations

`j` goes from 5 to i-1

- But `i` keeps changing!
- 5 iteration, 8 iterations, 11 iterations, …, approx. N iterations
- Average: N/2 iterations

Overall:

- Loops → multiply!
- O(N^2)

Inner Loops that Depend on Outer Loop

```
int count = 0;
int N = 1000000;


for (int i = 1; i < N; i=i*2) {
  for (int j = 1; j < i; j++) {
    count++;
  }
}
```

Thought process

i goes like 1, 2, 4, … approx. N

- So, log N

j goes from 0 to i-1, so i iterations

- But i keeps changing!
- 1 iteration, 2 iterations, 4 iterations, …, approx. N iterations
- Average is not N/2!

Sometimes you need to do the math, at least partially

- Arithmetic series
- Geometric series
- Logarithms

Function Calls ⇨ Substitute

```
int range(int A[], int n) {
   int lo = min(A, n);
   int hi = max(A, n);
   return hi-lo;
}
```

$O(N)$

$O(N)$

$O(1)$

$T(N) = O(N) + O(N) + O(1)$

$= O(N)$

Function calls are not elementary operations

- substitute their Big-*O* running times

If / Else ⇒ Max

```
int search(int A[], int n, int key) {
   if (!sorted(A, n)) {
      return lsearch(A, n, key);
   } else {
      return bsearch(A, n, key);
   }
}
```

$O(N)$

$O(N)$

$O(\log N)$

$T(N) = O(N) + \max(O(N), O(\log N))$

$= O(N) + O(N)$
$= O(N)$

`if` / `else` is not an elementary operation

- pick the largest of the two running times
  - remember this is worst case analysis

Rules of the Big-*O* (Review)

Usually, take the dominant term, remove the leading constant, and put *O*(. . .) around it

- Properties:
  - constant factors don't matter
  - low-order terms don't matter

Rules about Polynomials

1. The powers of N are ordered according to their exponents
   - i.e., $N^a = O(N^b)$ if and only if $a \leqslant b$
   - e.g., $N^2 = O(N^3)$, but N^3 is not $O(N^2)$
2. A logarithm grows more slowly than any positive power of N
   - e.g., $\log_2 N = O(N^{1/2})$

For most functions, can apply L'Hôpital's Rule:

- Theorem: If $\lim_{N\to\infty} \frac{f(N)}{g(N)}$ exists then $f(N) = O(g(N))$

More Rules

3. Transitivity: if $f(N) = O(g(N))$ and $g(N) = O(h(N))$ then $f(N) = O(h(N))$
4. Addition: $f(N) + g(N) = O(\max(f(N), g(N)))$
5. Multiplication: if $f_1(N) = O(g_1(N))$ and $f_2(N) = O(g_2(N))$ then $f_1(N) * f_2(N) = O(g_1(N) * g_2(N))$

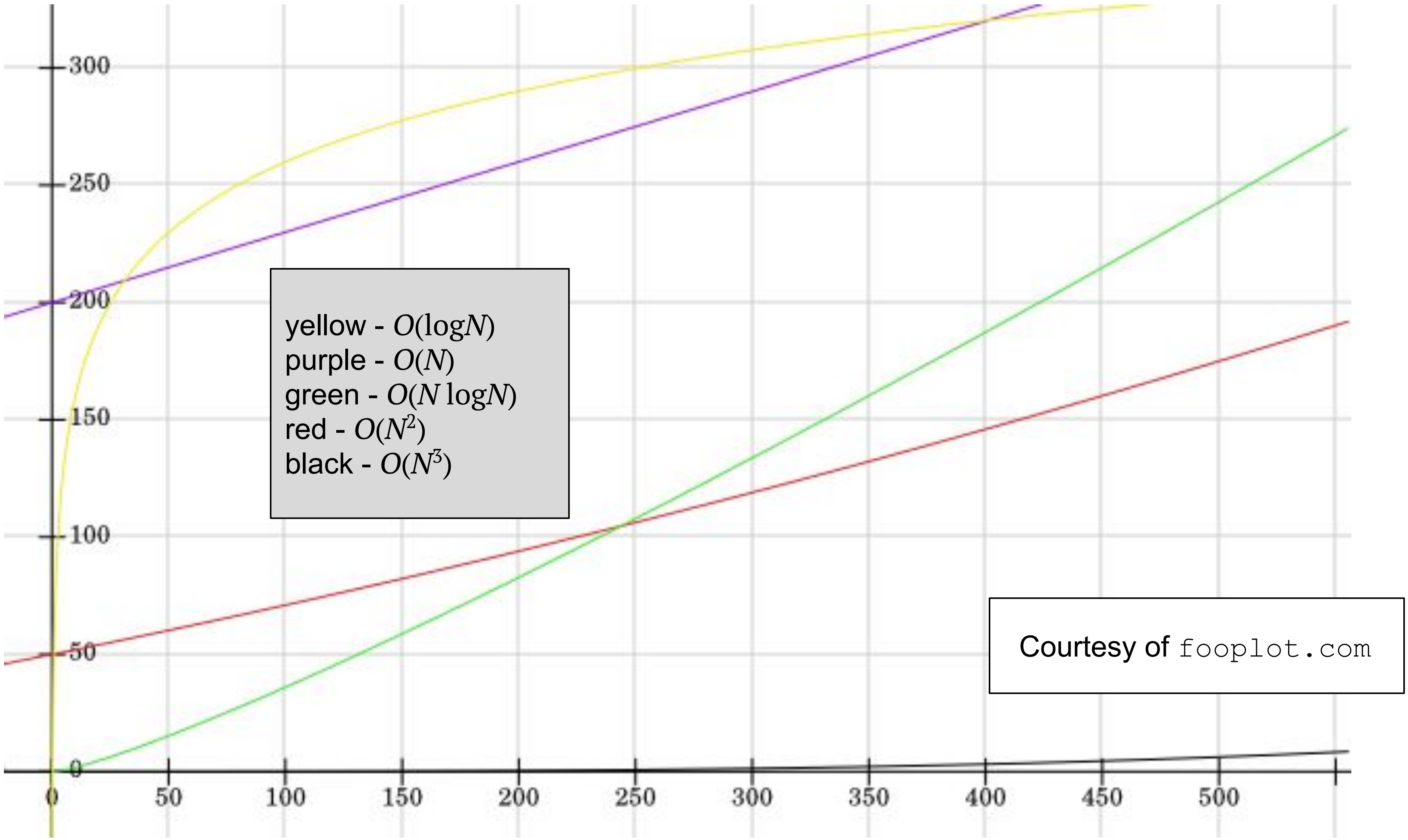
e.g., $\underbrace{(\cancel{10} + 5N^2)(10\log_2 N \cancel{+ 1})}_{\text{Ignore lower order terms}} + \underbrace{(5N + \cancel{\log_2 N})(\cancel{10N} + 2N\log_2 N)}_{\text{Ignore lower order terms}}$

$$\underbrace{\text{O(N^2 log N)} + \text{O(N^2 log N)}}_{\text{O(N^2 log N)}}$$

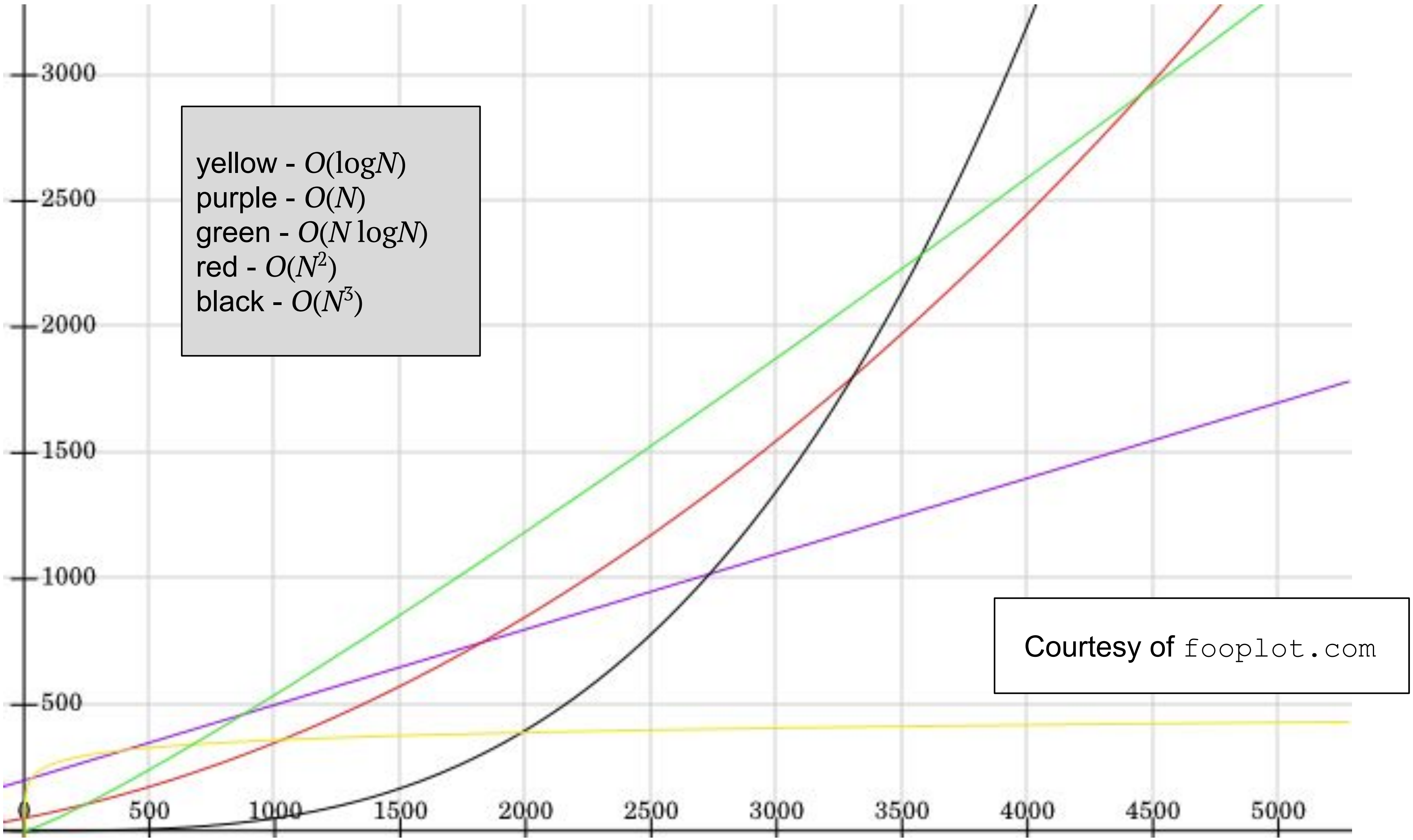
Typical Growth Rates

- $O(1)$ – *constant* time
  - The time is independent of N, e.g., array look-up
- $O(\log N)$ – *logarithmic* time
  - Usually the log is to the base 2, e.g., binary search
- $O(N)$ – *linear* time, e.g., linear search
- $O(N \log N)$ – e.g., quicksort, mergesort
- $O(N^2)$ – *quadratic* time, e.g., selection sort
- $O(N^k)$ – *polynomial* (where k is a constant)
- $O(2^N)$ – *exponential* time, very slow!

Some Plots to Convince You



Some Plots to Convince You



Acknowledgement

These slides are the work of Brad Bart (with minor modifications)