A Puzzle For You: What's the bug?

```
int main () {
    float x = 0.0;
    while (x <= 1.0) {
        printf("x = %f\n", x);
        x = x + 0.1;
    }
    printf("the final value of x = %f\n", x);

    double y = 0.2;
    while (y < 2.0) {
        printf("y = %lf\n", y);
        y = y + 0.2;
    }
    printf("the final value of y = %lf\n", y);
}
```

OUTPUT

```
x = 0.000000
x = 0.100000
x = 0.200000
x = 0.300000
x = 0.400000
x = 0.500000
x = 0.600000
x = 0.700000
x = 0.800000
x = 0.900000
the final value of x = 1.000000
y = 0.200000
y = 0.400000
y = 0.600000
y = 0.800000
y = 1.000000
y = 1.200000
y = 1.400000
y = 1.600000
y = 1.800000
y = 2.000000
the final value of y = 2.200000
```

Announcements

- Assignment 9
  - Deadline extended to 23:59:59 Monday, Apr. 1
  - Do not submit it last minute!

- Make-up office hours
  - 9:30am on Monday, Apr. 1

Binary Encodings

CMPT 125
Mar. 29

Lecture 32

Today:

- Integer Encodings
- Floating Point Representation
- Endian-ness

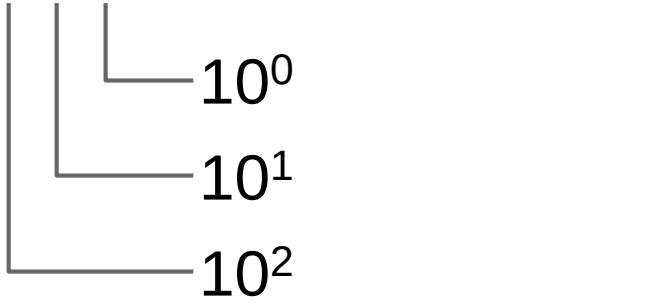
Positional Value (Review)

The value of binary digits are positional

- just like decimal, except 2-fold instead of 10-fold

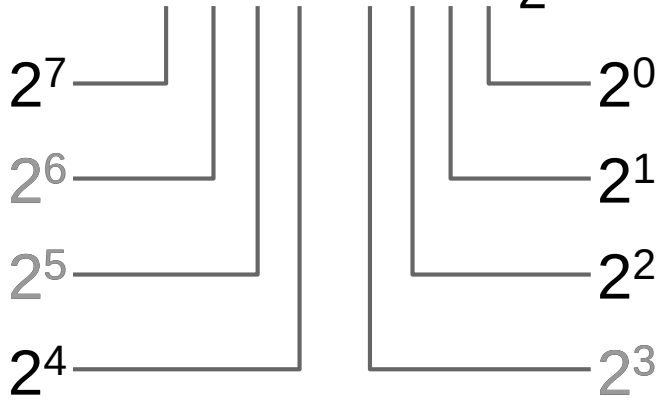
Decimal:

$739 = 700 + 30 + 9$



Binary:

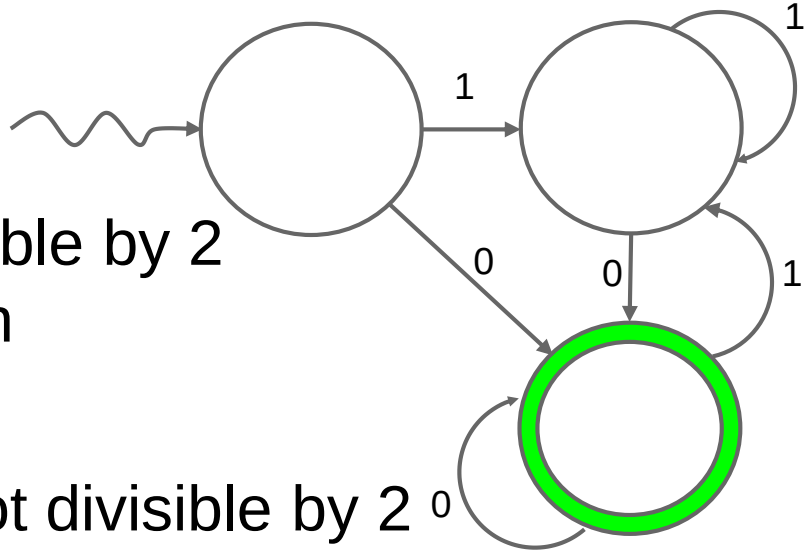
$1001\ 0111_2$



$= 2^7 + 2^4 + 2^2 + 2^1 + 2^0$

Divisibility By 2

- Construct an FSM that accepts all binary strings divisible by 2
- Intuition: everything that ends with 0 should be accepted
- Adding 0
  - Multiply by 2: result is divisible by 2
  - 2n is divisible by 2 for any n
- Adding 1
  - multiply by 2, and add 1: not divisible by 2
  - 2n+1 always has remainder 1 when divided by 2



Fixed Width Encodings

Simple data types are usually fixed in width

- usually multiples of 8 bits
- E.g., `char` (8 bits), `int` (often 32 bits), `long` (often 64 bits)

Puts a limit on the range of possible numbers

- for k bits, gives a max of 2^k possibilities
- E.g., `int`: $[-2^{31}, 2^{31}-1]$

Step outside the range and you lose precision

- E.g., `x = 2147483647; x++;`

  results in an *overflow*
- You can also lose precision due to round-off errors

From *Stack Overflow*:

Q. What's the maximum value for an int32? I can never remember that number. I need a memory rule.

A. It's 2,147,483,647. Easiest way to memorize it is via a tattoo.

(4923 upvotes)

Quick Estimates

- 2^10 = 1024, approximately 1000 (10^3)
- 2^20 is about 1000 * 1000, or 1 million (10^6)
- 2^30 is about 1 billion (10^9)
- 2^31 is about 2 billion (2 * 10^9)
- The last bit is used for the sign
  - So largest positive number is about 2 billion

- Another couple of facts:
  - 32-bit operating systems can access 4GB of RAM
  - 64-bit operating systems can access 4 * 2^32 GB of RAM

Non-Integer Arithmetic

Two common decimal numbers:

⅓ = 0.3333333333… ⅔ = 0.6666667 (+1) 6666…

It's easy if you have an infinite amount of paper!

But what if you have a fixed width of digits?

- you have to *truncate* and *round*.

These are the *significant digits* of the number

- also known as the *significand*

Scientific Notation (Review)

A convention to express numbers by their significand and their magnitude (exponent)

- E.g., 6.022 x 10^{23} atoms/mol = N_A (Avogadro's Const.)
- E.g., 2.99792458 x 10^{8} m/s = c (speed of light)
- E.g., 1.073741824 x 10^{9} bytes = 1 gigabyte

Common usage is to place one significant digit before the radix (decimal point)

- E.g., ⅓ = 3.333333 x 10^{-1}

The same conventions are used for binary.

Floating Point Encoding

A `float` is composed of 32 bits:

- 1 bit for the sign — `0` → positive, `1` → negative
- 23 bits for the significand — `1.`$b_{22}b_{21}\ldots b_1b_0$
  - approximately 7 decimal digits of precision
- 8 bits for the exponent — ranges from [-126,127]

Range of representations:

- +/- `1.`$b_{22}b_{21}\ldots b_1b_0$ x 2^{exp}, where:
  - the largest number ≈ 2^{128} ≈ 3.40 x 10^{38}
  - the smallest number ≈ 2^{-126} ≈ 1.17 x 10^{-38}
- There is a “special” representation for 0
- . . . and a handful of other special cases

Sign-magnitude representation of negatives

Example: -0.625

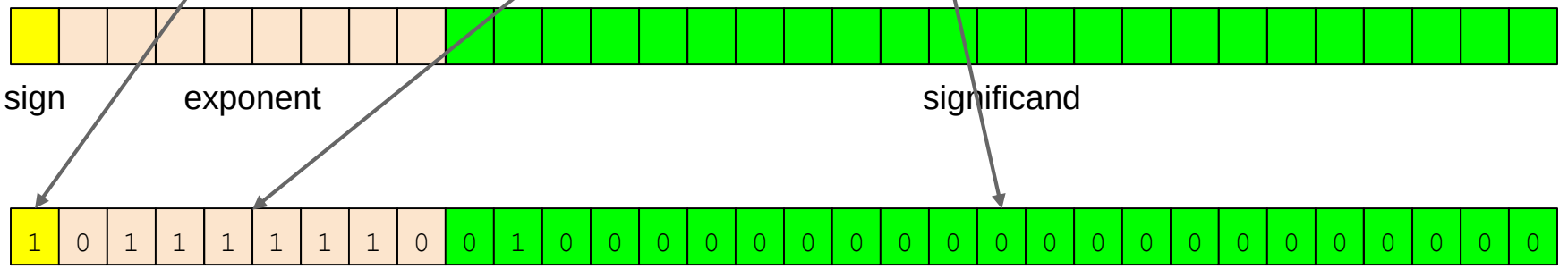
Decimal fraction: -⅝

- sign bit? = 1 (negative)
- significand and exponent? ⅛ = 0.001, so ⅝ = 0.101

An exact number! No rounding!

- -0.625 = −1.01000000000000000000000 x 2^{-1}

Format:



Example: 0.1

Decimal fraction: 1/10

- sign bit? = 0
- significand and exponent? — try long division

```
                                                                +1
          0.0001100110011001100110011001100110...
      ---------------------------------------------
1010 )  1.0000000000000000000000000000000000...
```

- repeating decimal — truncate and round
- exponent = −4

0	0	1	1	1	1	0	1	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	**1**

Special Exponents

- Exponent has 8 bits
  - but ranges from [-126,127]?
  - 8 bits has 2^8 = 256 possible values
  - -126 to 127 has 127 – (-126) + 1 = 254 values
- How to represent 0?
  - 1.(anything) times 2^(anything) is never going to be zero
- Two exponents reserved for special cases
  - All zero exponent bits → sign*0.(significand)*2^-126
  - All one exponent bits → infinity, NaN
  - NaN: “not a number”, for e.g. when dividing by 0

A Note about Endian-ness

A multiple-byte quantity, like `int` or `float`, is stored across a contiguous sequence of addresses in memory or in a file.

- two possible memory / file layouts