

Sampling-Based Motion Planning

CMPT 419/983

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SFU Computing Science

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Outline

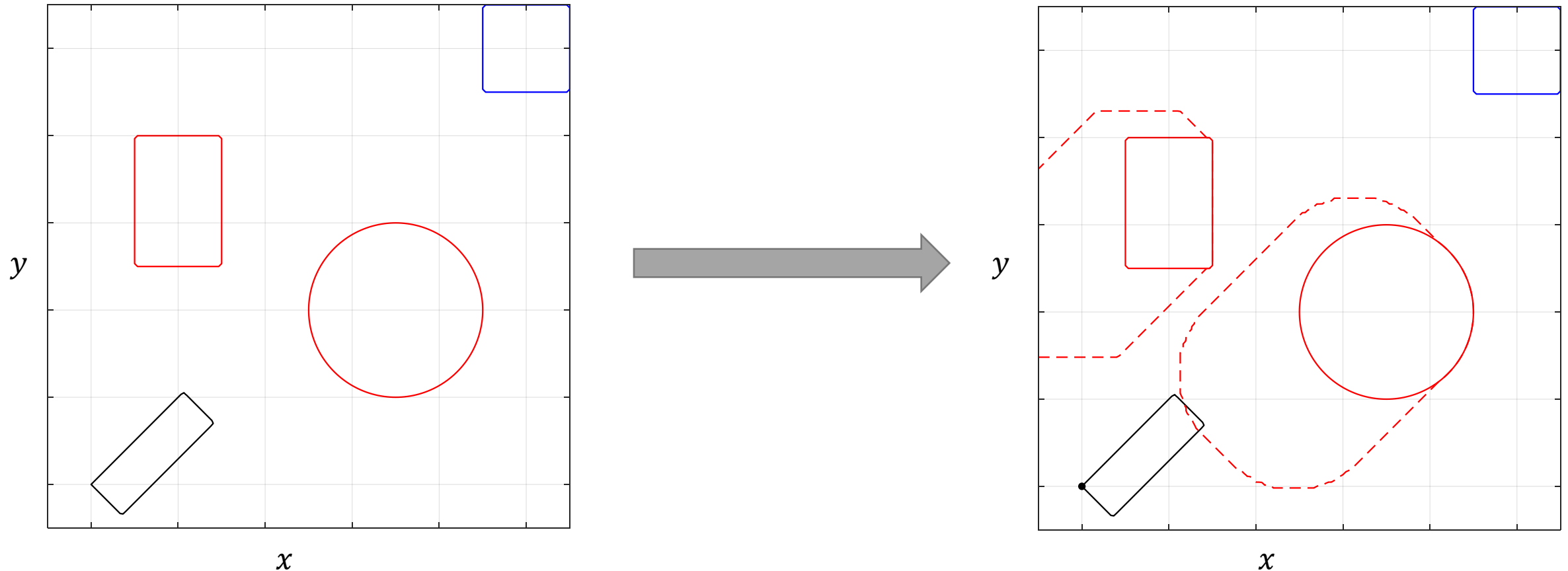
- Configuration space
- Probabilistic road maps (PRM)
 - PRM*
- Rapidly-exploring random trees (RRT)
 - RRT*
- Robust real-time planning (FaSTrack)

Configuration Space (C-Space)

- Similar to state space, but considers reachability
 - Usually state space does not consider the set of states that a system can reach
 - Configuration space is the subset of the state space reachable by the system
 - Example: mechanical joints
- Rigid bodies in 2D:
 - 2D position and one rotation angle
- Rigid bodies in 3D:
 - 3D position and three rotation angles
- Connected rigid bodies
 - Concatenate positions and angles (but not every position and angle is reachable)

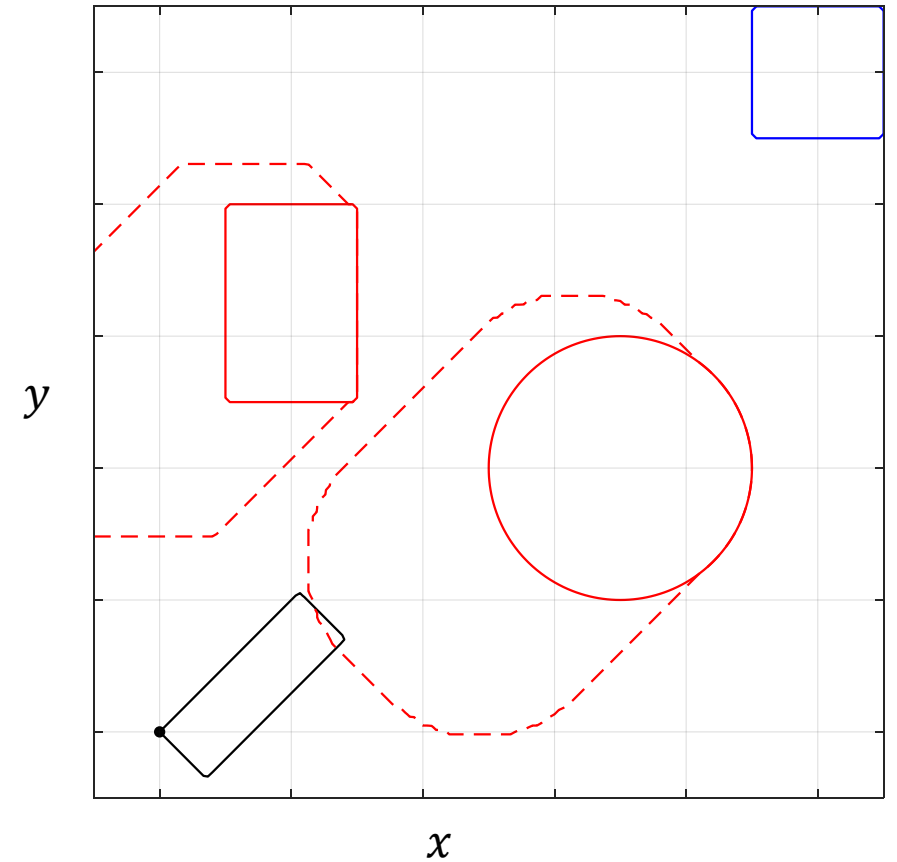
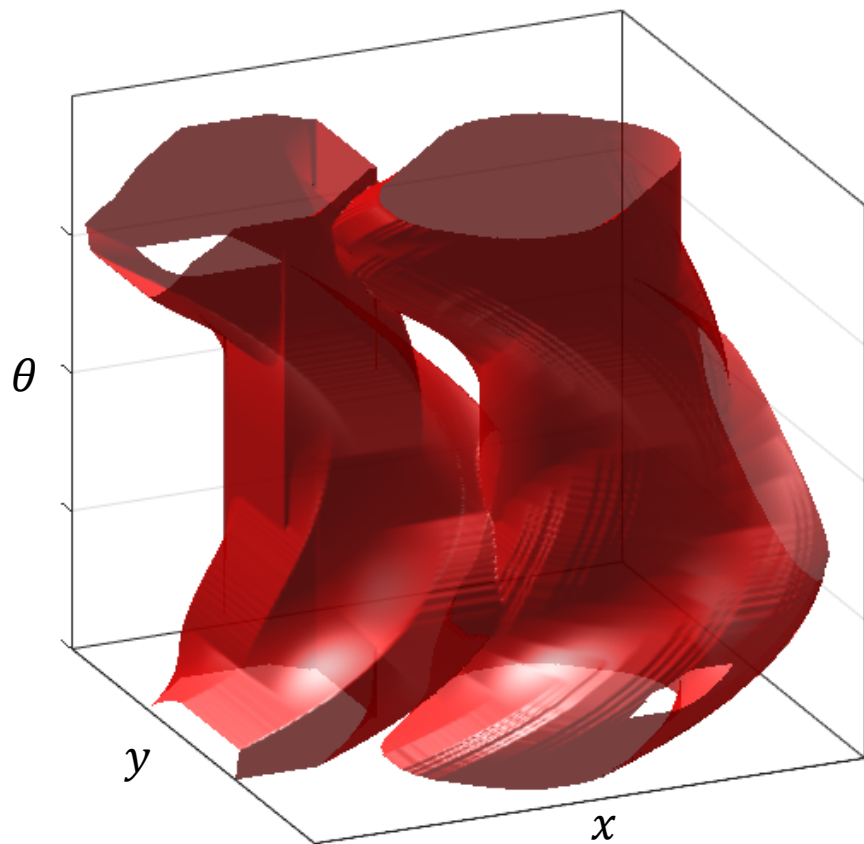
Planning in C-Space

- Reduce objects to a point, and augment obstacles
- Example: Sliding rectangular block to goal



Planning in C-Space

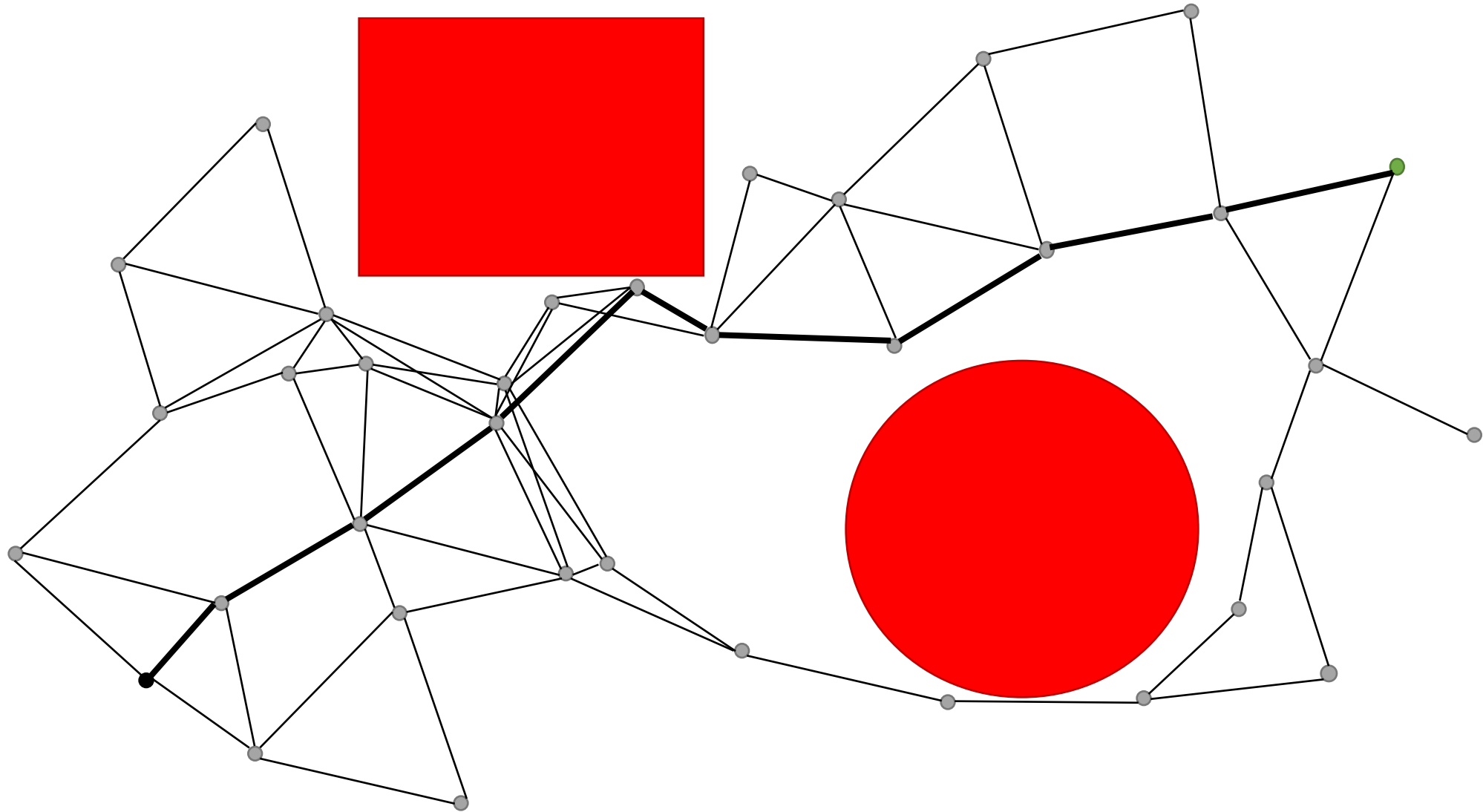
- Reduce objects to a point, and augment obstacles
 - Example: Sliding **and rotating** rectangular block to goal



Probabilistic Road Map

- Draw N samples
 - Keep points outside of obstacles
- Choose a disk radius
- For each kept point, draw edge between it and all other points within the disk
 - Keep edges that are collision free (expensive)
- Use graph search algorithm (e.g. A*, Dijkstras) on the resulting graph to find a path

Probabilistic Road Map



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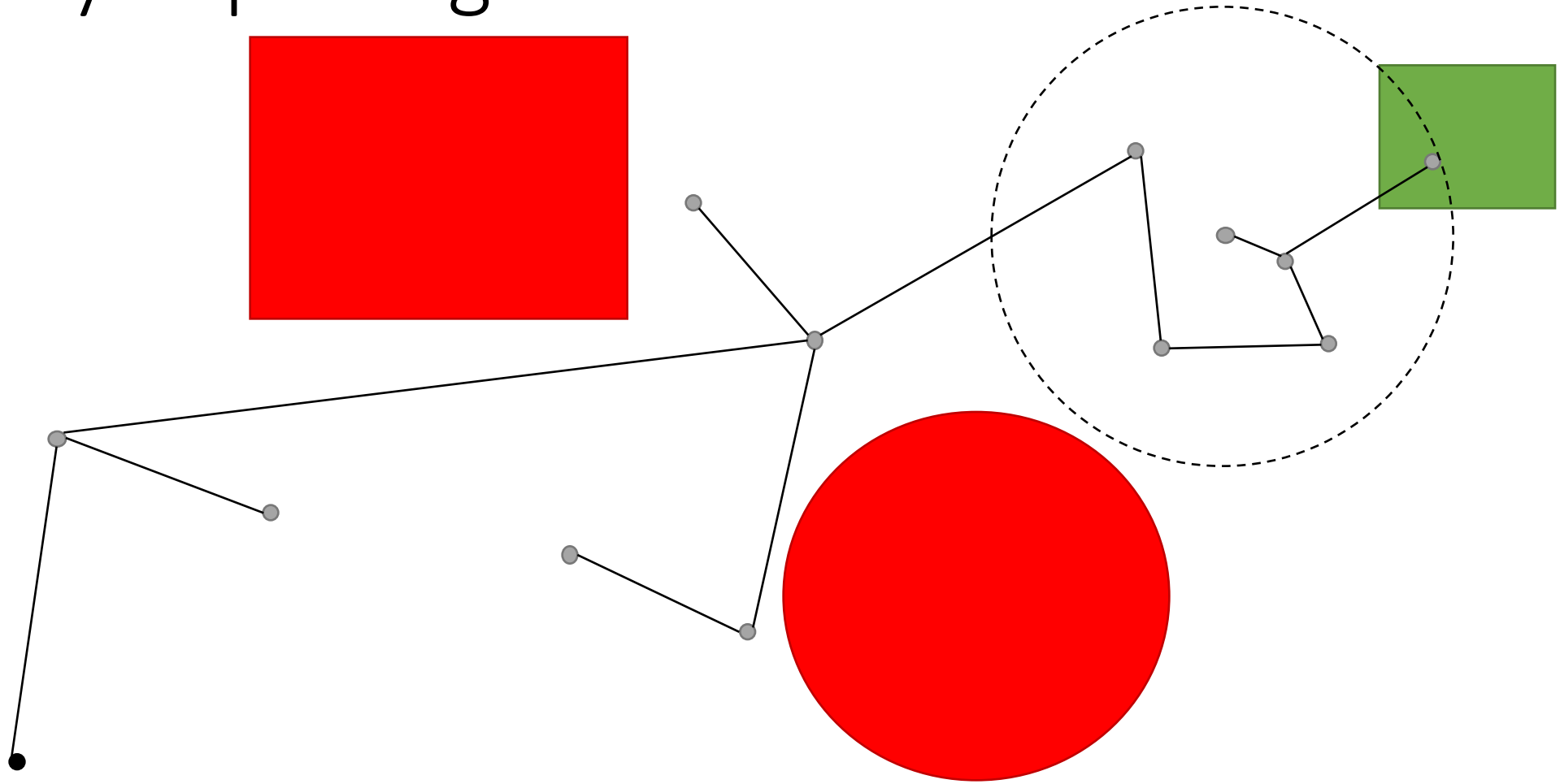
Tuning Parameters

- Sampling distribution
- Deterministic samples are possible
- Collision checker – determines type of obstacles that can be considered

Rapidly-Exploring Random Tree

- Draw a sample
 - Connect to nearest neighbour
- Continue until path is found

Rapidly-Exploring Random Tree



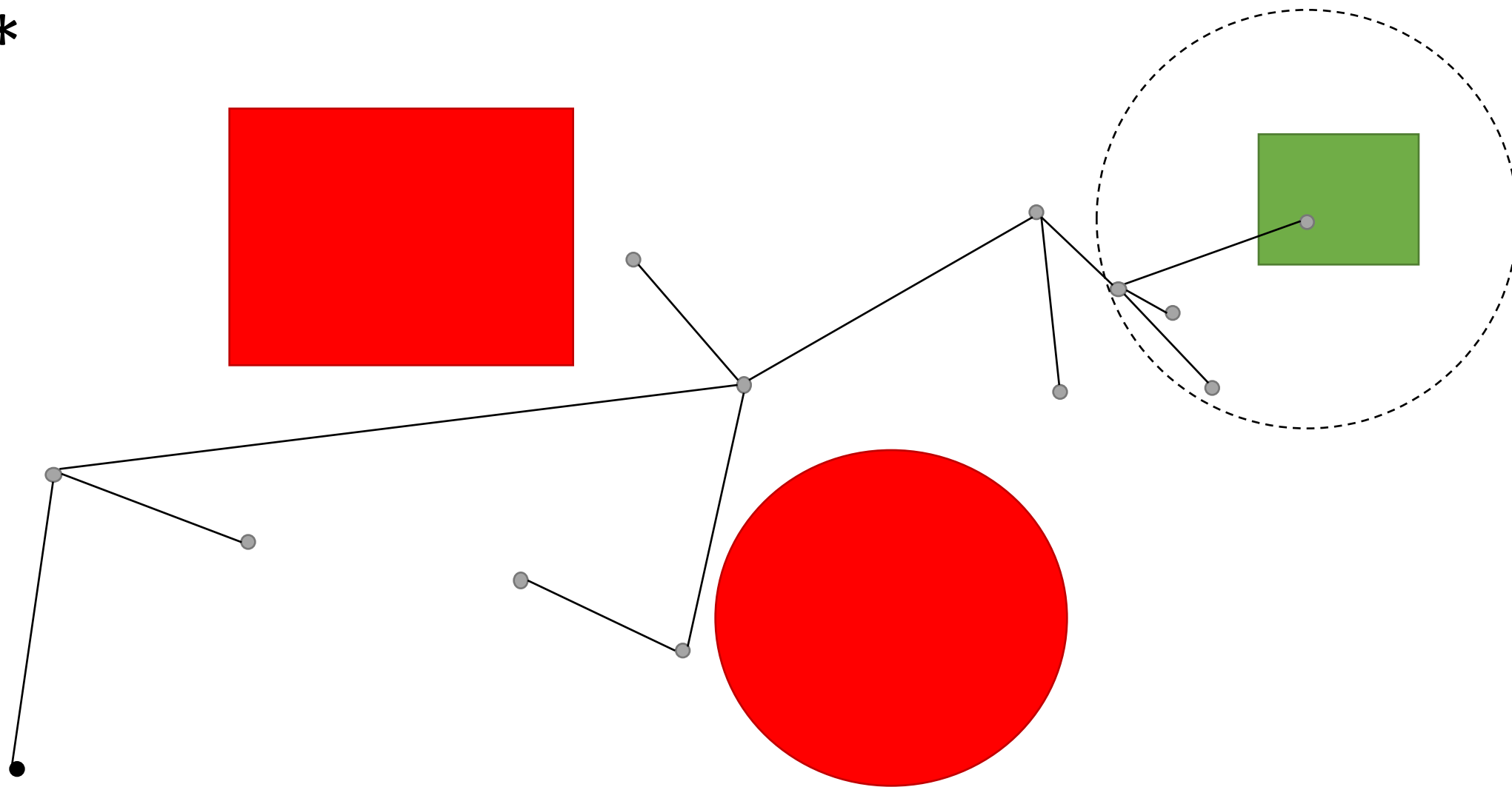
Rapidly-Exploring Random Tree

- Draw a sample
 - Connect to nearest neighbour
- Continue until path is found

RRT*

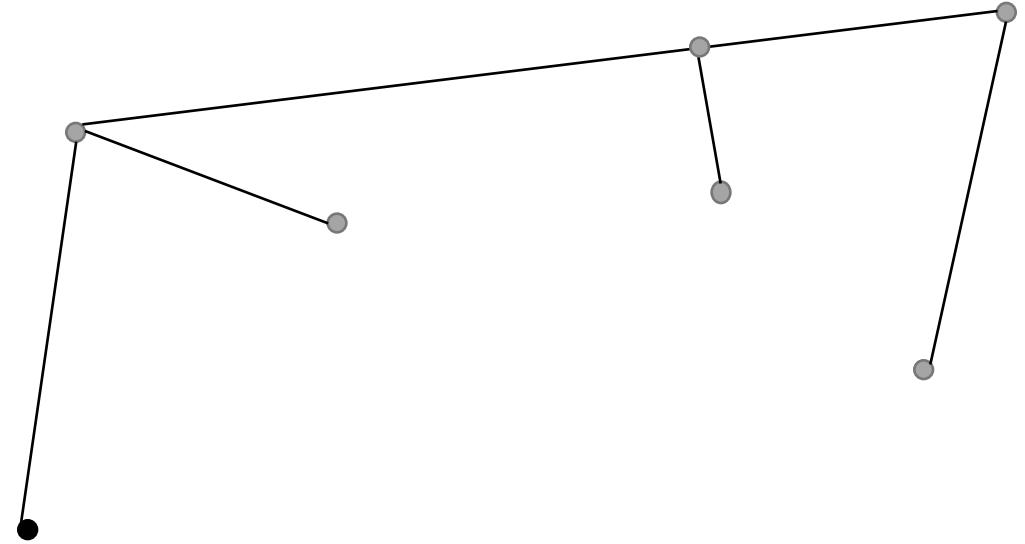
- Draw a sample
 - Connect to nearest neighbour
 - Rewire paths in a cheaper way
 - Look within some radius
 - Can we get to the sample in a cheaper way?
 - Is there a cheaper way to get to the samples within the radius?
- Continue until path is found

RRT*



Tuning Parameters

- Sampling distribution
 - Deterministic samples are possible
- Nearest sample vs. nearest point covered by the tree
- Collision checker – determines type of obstacles that can be considered
- Number of trees
 - Having more than one tree may decrease time needed to find a feasible path

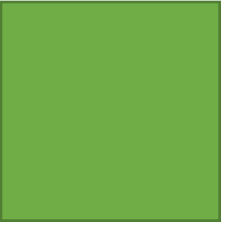
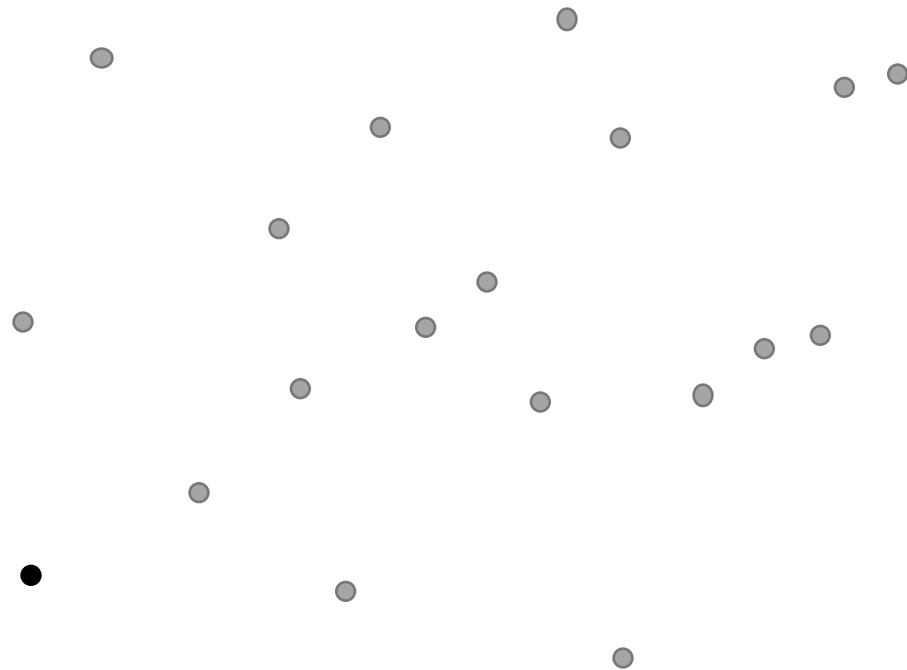


Comments

- PRM
 - Could be slow
 - May need many samples
- RRT
 - Incrementally draw samples → has potentially fewer samples
 - Quality of solution may be very poor → RRT*
- Inherently, complexity is still exponential
 - Hope: obtain feasible (potentially bad) solution quickly

A Difficult Case

- Narrow gaps



Analysis

- Potential computational bottlenecks
 - Collision checking
 - Nearest-neighbour finding
- Optimality
 - Asymptotic, as number of samples goes to infinity
 - Convergence rate: suboptimality bound is $O\left(n^{-\frac{1}{d}}\right)$
 - n – number of samples
 - d – number of dimensions

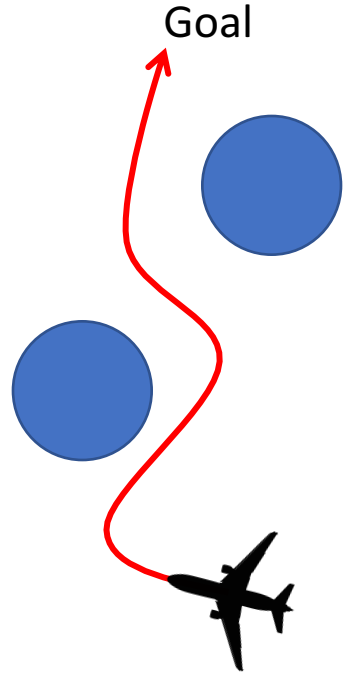
System Dynamics

- RRT and PRM, as presented, is a geometric planner
 - Cannot account for system dynamics
- Incorporating system dynamics
 - Make sure edges between nodes are dynamically feasible
 - Difficult in general, but has been done for special cases
 - Backward and forward reachability concepts are useful
 - Some references
 - LaValle, Kuffner. “Randomized Kinodynamic Planning,” 2001.
 - Webb, Van Den Berg. “Kinodynamic RRT*: Asymptotically optimal motion planning for robots with linear dynamics,” 2013.
 - Schmerling, Janson, Pavone. “Optimal Sampling-Based Motion Planning under Differential Constraints: the Drift Case with Linear Affine Dynamics,” 2015.

Robustness

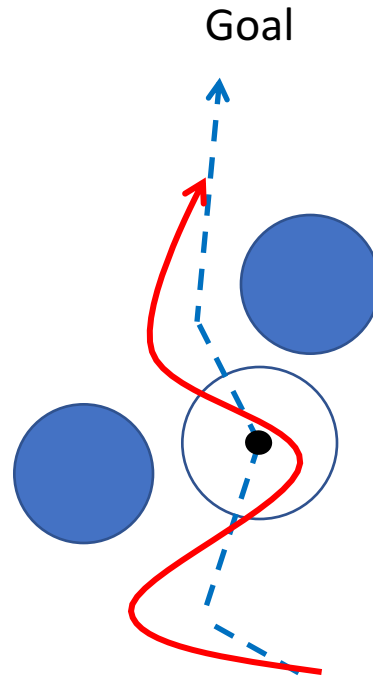
Slide courtesy of Sylvia Herbert

Slow and Accurate Planning:



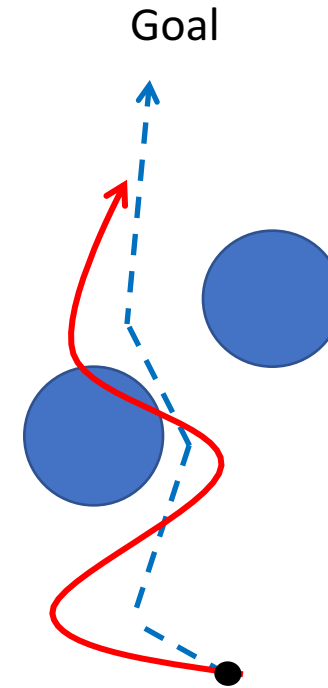
- Optimal control
- Guarantees on safety and goals
- Handles external disturbances (e.g. wind)
- Slow to compute

FaSTrack:



- Precompute a tracking error bound based on relative state between the true system and the planned path
- Make it modular & easy to incorporate in all sorts of real-time path/trajectory planners

Fast (but less accurate) Planning:



- Very fast with simple dynamics
- May not capture all system behavior
- Not necessarily robust to disturbances

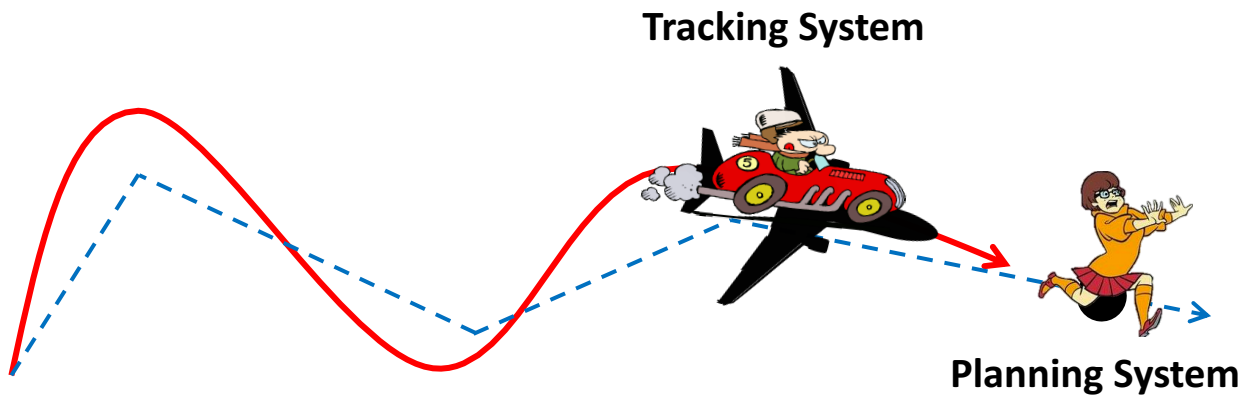
Herbert, et al. "FaSTrack: a modular framework for fast and guaranteed safe motion planning," 2017.

Precomputed Tracking Bound

Rufus Isaacs

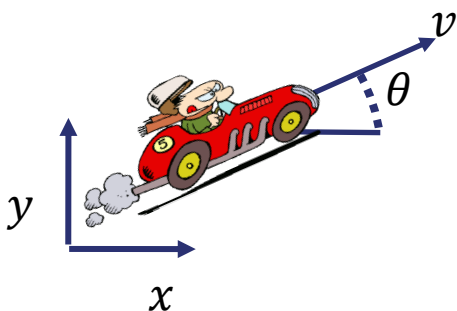


Homicidal Chauffeur Problem (1951)



Tracking System Model

Planning System Model

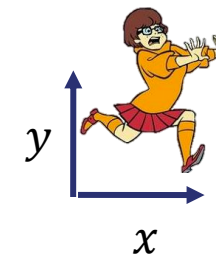


$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} v \cos \theta \\ v \sin \theta \\ \omega \end{bmatrix}$$

$$u_s = \omega$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} u_{px} \\ u_{py} \end{bmatrix}$$

$$u_p = [u_{px}, u_{py}]$$



Precomputed Tracking Bound



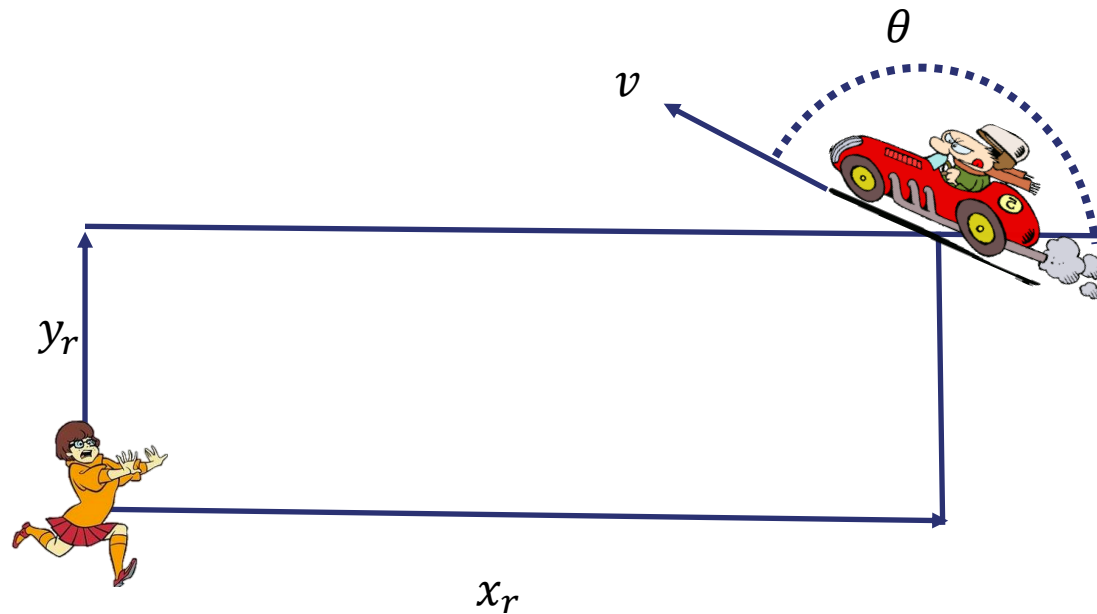
- Tracking system (car) pursues planning system (runner)
- Planning system tries to evade tracking system
- What will be the maximum relative distance over time?

Note:

Maximum relative distance over time
 $\equiv$ Worst possible tracking error over time
 $\equiv$ Tracking error bound

Precomputed Tracking Bound

Goal: Map initial relative state to worst possible tracking error over time

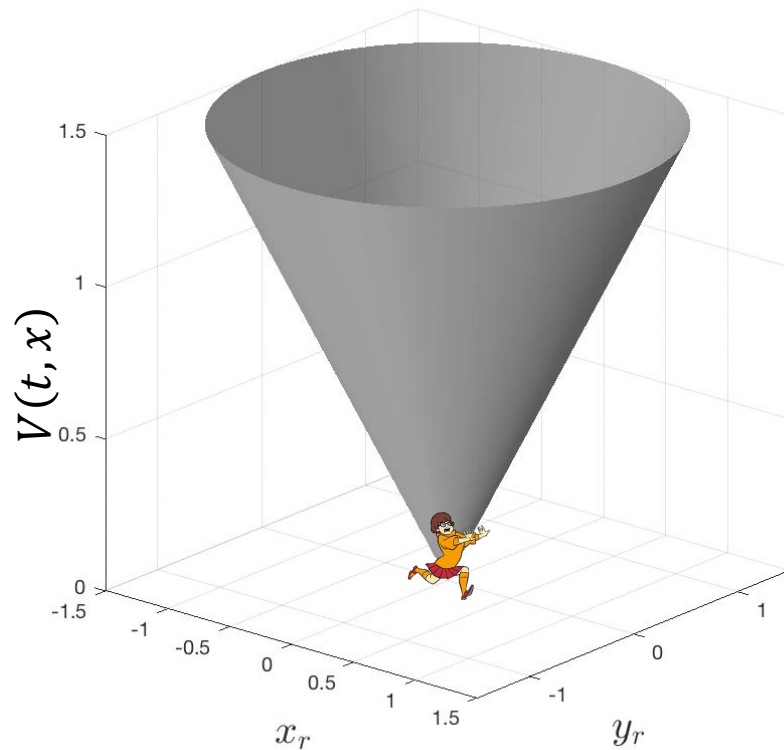


Relative System

$$\dot{r} = \begin{bmatrix} \dot{x}_r \\ \dot{y}_r \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} v \cos \theta - u_{px} \\ v \sin \theta - u_{py} \\ \omega \end{bmatrix}$$

Precomputed Tracking Bound

Goal: Map initial relative state to worst possible tracking error over time



Planning system tries to **maximize** error

Tracking system tries to **minimize** error

Keep track of **maximum** cost over time

$$V(t, x(t)) = \max_{\Gamma[u](\cdot)} \min_{u(\cdot)} \max_{s \in [t, 0]} l(x(s))$$

- Take $t \rightarrow -\infty$ for infinite time horizon case

Example: 10D Tracking 3D Single Integrator using RRT

10D near-hover quadrotor model

$$\begin{bmatrix} \dot{x} \\ \dot{v}_x \\ \dot{\theta}_x \\ \dot{\omega}_x \\ \dot{y} \\ \dot{v}_y \\ \dot{\theta}_y \\ \dot{\omega}_y \\ \dot{z} \\ \dot{v}_z \end{bmatrix} = \begin{bmatrix} v_x \\ g \tan \theta_x \\ -d_1 \theta_x + \omega_x \\ -d_0 \theta_x + n_0 u_x \\ v_y \\ g \tan \theta_y \\ -d_1 \theta_y + \omega_y \\ -d_0 \theta_y + n_0 u_y \\ v_z \\ k_T u_z - g \end{bmatrix}$$

3D single integrator

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$$

