Course Outline

• Overview of algorithms used for robotic decision making
  • Theory-focused
  • Fundamentals for doing many areas of robotics research

• Dynamical systems
• Optimization and optimal control
• Machine learning in robotics
• Localization and mapping
Logistics

• Academic Quadrangle 5601,
  • Mondays 10:30-12:20
  • Wednesdays 10:30-11:20

• Office hour: Wednesdays 13:00-14:30, TASC 1 8225

• Course website: https://coursys.sfu.ca/2019fa-cmpt-419-x1/pages/

• Contact:
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Caveats

• This class is in “experimental mode”

• Slight changes are expected

• Some things may not be super polished

• Please provide feedback and comments
Grading

- 40% Homework
  - 3 assignments

- 60% Project
Project suggestions

• Thoroughly understand and critically evaluate 3 to 5 papers in an area covered in this course

• Reproduce the results of 1 to 2 papers in an area covered in this course, and suggest or make improvements

• Mini Research project related to an area covered in this course

• Other: Please consult instructor
Project timeline

• Proposal (1-2 paragraphs)
  • Due Oct. 7

• Poster session
  • Last lecture of the term, Dec. 2

• Report (6 pages maximum)
  • Due Dec. 2
Recommended textbooks


Dynamical systems

- Mathematical models of robotic systems
  - Deterministic vs. stochastic
  - Continuous vs. discrete time

- Configuration of system described by the state, often denoted \( x \)
  - State changes, or evolves, according to the model

- Deterministic, continuous time
  - \( \frac{dx}{dt} = \dot{x}(t) = f(x, u) \)

- Stochastic, discrete time
  - \( x_{k+1} \) obtained from the probability distribution \( p(x_{k+1} | x_k, u_k) \)
System State

- Defined in terms of any variables of interest
  - Often denoted $x(t)$ or $x_k$

- Position
- Heading

- Velocity
- Angular velocity

- Voltages, concentrations of chemicals
- Human comfort, degree of trust
Control and disturbance

• Control/action: usually used to achieve a desired goal
  • Usually denoted $u(t)$ or $u_k$
  • Acceleration
  • Turn rate

• Disturbance
  • Usually denoted $d(t)$ or $d_k$
  • Bumps on the road
  • Input noise
Mathematical models of robotic systems

- Continuous state
  - Continuous time
    - Linear
      - Time-varying
      - Time-invariant
    - Nonlinear
      - Time-varying
      - Time-invariant
  - Discrete time
    - Discrete state
Mathematical models of robotic systems

- Continuous state
  - Continuous time
  - Linear
    - Time-varying
    - Time-invariant
  - Nonlinear
    - Time-varying
    - Time-invariant
- Discrete state
  - Discrete time
    - Linear
    - Time-varying
    - Time-invariant
    - Time-invariant
Examples of Robotic Systems
Car models

1. 
\[
\begin{align*}
\dot{x} &= v \cos \theta \\
\dot{y} &= v \sin \theta \\
\dot{\theta} &= \omega \\
\end{align*}
\]
States: \((x, y, \theta)\); position and heading
Control: \(\omega\); turn rate (angular speed)

2. 
\[
\begin{align*}
\dot{x} &= v \cos \theta \\
\dot{y} &= v \sin \theta \\
\dot{\theta} &= \omega \\
\dot{v} &= a \\
\dot{\omega} &= \alpha \\
\end{align*}
\]
States: \((x, y, \theta, v, \omega)\);
position, heading, speed, turn rate
Control: \((a, \alpha)\);
acceleration, angular acceleration

3. Bicycle model
\[
\begin{align*}
\dot{x} &= v_x \\
\dot{y} &= v_y \\
\dot{v}_x &= \omega v_y + a_{x2} \\
\dot{v}_y &= -\omega v_x + \frac{1}{m} (F_{c,f} \cos \delta_f + F_{c,r}) \\
\dot{\psi} &= \omega \\
\dot{\omega} &= \frac{2}{I_z} (l_f F_{c,f} - l_r F_{c,r}) \\
\dot{\chi} &= v_x \cos \psi - v_y \sin \psi \\
\dot{\chi} &= v_x \sin \psi + v_y \cos \psi
\end{align*}
\]
Models

• All models are wrong; some are useful

• Definition of “useful” depends on situation
  • Simulation
  • Analysis and control
  • Verification

• Considerations
  • Does the model capture the desired system behaviours
  • Is the model amenable to tractable computation
Nonlinear Optimization

• Choose $x$ to minimize some cost, subject to constraints

\[
\begin{align*}
\text{minimize} & \quad f(x) & \quad \text{Fuel cost, distance to obstacles, distance from goal, prediction error in machine learning} \\
\text{subject to} & \quad g_i(x) \leq 0, \ i = 1, \ldots, n \\
& \quad h_j(x) = 0, \ j = 1, \ldots, m
\end{align*}
\]

• Equivalently, maximize $-f(x)$: Maximize reward, maximize profit

• Robotics spans many fields
  • Many conventions
  • Many notations clashes
Nonlinear Optimization

• A very difficult problem in general for $x \in \mathbb{R}^n$ where $n$ is large
  • Calculus facts: necessary and sufficient conditions
  • Rely on gradients (if possible)

• Sometimes, some components of $x$ may be integers
  • Can we do better than brute force?

• Simpler cases
  • Differentiable functions
  • Linear, convex, quasiconvex
  • Unconstrained problems
Nonlinear Optimization

minimize $f(x)$
subject to $g_i(x) \leq 0, i = 1, \ldots, n$
$h_j(x) = 0, j = 1, \ldots, m$

• Nonlinear optimization:
  • Decision variable is $x \in \mathbb{R}^n$
Optimal Control

\[
\begin{align*}
\text{minimize} & \quad \ell(x(t_f), t_f) + \int_0^{t_f} c(x(t), u(t), t) \, dt \\
\text{subject to} & \quad \dot{x}(t) = f(x(t), u(t)) \\
& \quad g(x(t), u(t)) \geq 0 \\
& \quad x(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^m, x(0) = x_0
\end{align*}
\]

• Nonlinear optimization:
  • Decision variable is \( x \in \mathbb{R}^n \)

• Optimal control:
  • Decision variable is a function \( u(\cdot) \)

Cost functional, \( J(x(\cdot), u(\cdot)) \)
Dynamic model
Additional constraints
  • Eg. actuation limits
Robotic Safety

• Verification methods

  Assumptions → Control policy → Prove safety

• Considers all possible system behaviours, given assumptions

• Can be written as an optimal control problem
Reachability Analysis

- Model of robot
- Unsafe region

Reachable set

Unsafe region

Optimal control policy to avoid danger

Reachable set (States leading to danger)
Machine Learning

• Application of nonlinear optimization
  • Takes advantage of available data

• Supervised learning
  • Regression
  • Classification

• Unsupervised learning
  • Clustering
  • Reinforcement learning
Machine Learning

• Very scalable with additional data
• Requires a lot of data

• Computer vision
• Natural language processing
• Game playing
• Simulated robotics

• Physical robotics?
Localization and Mapping

• Localization
  • Given a map, figure out where the robot is (with respect to the map) using sensor information
  • Continuously do this while moving around in the environment

• Simultaneous localization and mapping
  • Figure out the map and localize at the same time

• Probabilistic models
  • of how the robot moves
  • of how the robot senses the environment
Sample of MARS Research

- https://sfumars.com
- Control algorithms
- Computational complexity
- Reinforcement learning
- Human intent inference
- Theory
- Computation
- Experiments
Safety: A Crucial Perspective in Automation
Challenges in Safety-Critical Systems

• Account for all possible system behaviours
• Complex systems
• Complex environment
  • Weather conditions
  • Other robots
Reachability Analysis

• Model of robot
• Unsafe region

Reachable set (States leading to danger)
Optimal control policy to avoid danger
Autonomous quadrotors

Human-controlled intruder
Main Challenge:
Exponential Computational Complexity with DP

\[ O(N^d) \] time and space complexity!

Number of system dimensions

1D: < 0.1s
negligible RAM

2D: minutes
negligible RAM

3D: tens of megabytes

4D: hours
hundreds of megabytes

5D: intractable!
gigabytes

6D: days
Research Directions

Dimensionality reduction
Self-Contained Subsystems

• Motivating example: Dubins Car

\[
\begin{align*}
\dot{x} &= v \cos \theta \\
\dot{y} &= v \sin \theta \\
\dot{\theta} &= \omega
\end{align*}
\]

• Subsystems are coupled through state and control

• Many systems have states that are not directly coupled to each other
  • Most common in vehicle dynamics
Research Directions

Dimensionality reduction

Parallel computing

Perception systems
Research Directions

Human intent understanding

Campbell et al.
Proactive Human Intent Understanding

Is the red car
• A pursuer,
• Or a benign vehicle?

Robot car (green) *proactively* changes lanes to determine intent
Multi-Modal Human Intent Understanding

Motion

Emotion

Engagement

Audio
Research Directions

Human intent understanding

Robotic learning

Campbell et al.

Global Robots Ltd.
Curriculum Reinforcement Learning

Without curriculum | Distance-based reward shaping | Reachability-based curriculum | Random curriculum
Curriculum Reinforcement Learning

Task performance

Curriculum performance

- Reachability-based curriculum
- Random curriculum
- Without curriculum
- Distance-based reward shaping