

Forward Checking (4-queens):

Step 0:


Q1={1,2,3,4}, Q2={1,2,3,4}, Q3={1,2,3,4}, Q4={1,2,3,4}

Step 1:

Q1	x	x	x
	x		
		x	
			x

Q1={1,2,3,4}, Q2={3,4}, Q3={2,4}, Q4={2,3}

Step 2:

Q1	x	x	x
	x	x	
	Q2	x	x
		x	x

Q1={1,2,3,4}, Q2={3,4}, Q3={}, Q4={2}

Forward Checking has detected that partial assignment is inconsistent with the constraints and backtracking can occur.

Step 3:

	x		
Q1	x	x	x
	x		
		x	

Q1={1,2,3,4}, Q2={4}, Q3={1,3}, Q4={1,3,4}

Step 4:

	x		
Q1	x	x	x
	x	x	
	Q2	x	x

Q1={1,2,3,4}, Q2={4}, Q3={1}, Q4={1,3}

Step 6:

	x	Q3	x
Q1	x	x	x
	x	x	
	Q2	x	x

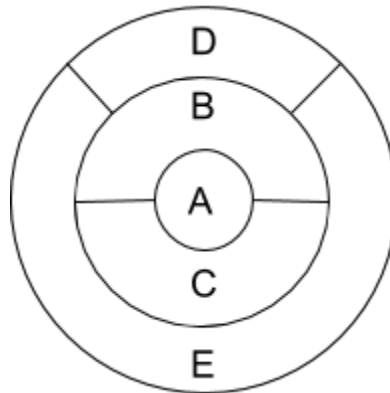
Q1={1,2,3,4}, Q2={4}, Q3={1}, Q4={3}

Step 7:

	x	Q3	x
Q1	x	x	x
	x	x	Q4
	Q2	x	x

Q1={1,2,3,4}, Q2={4}, Q3={1}, Q4={3}. Now 4 ladies peacefully live together on the board!

Forward Checking (3-colors Mahmoud's map): keep track of remaining legal values for unassigned variables.

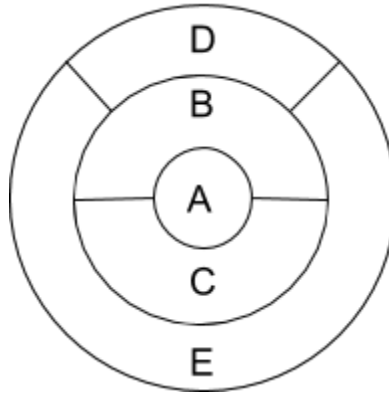


	A	B	C	D	E
	1, 2, 3	1, 2, 3	1, 2, 3	1, 2, 3	1, 2, 3
After A = 1	<u>1</u>	2, 3	2, 3	1, 2, 3	1, 2, 3
After D = 2	<u>1</u>	3	2, 3	<u>2</u>	1, 3
After C = 3	<u>1</u>		<u>3</u>	<u>2</u>	1

Forward checking has detected that the partial assignment  $\{A=1, D=2, C=3\}$  is inconsistent with the constraints of the problem (it allows 0 values for B), and the algorithm will therefore backtrack immediately.

Minimum Remaining Values (MRV):

Choose the variable with the fewest legal values (most constrained variable) or fail first heuristic.



	A	B	C	D	E
	1, 2, 3	1, 2, 3	1, 2, 3	1, 2, 3	1, 2, 3
After A = 1	<u>1</u>	2, 3	2, 3	1, 2, 3	1, 2, 3
After B = 2	<u>1</u>	<u>2</u>	3	1, 3	1, 3
After C = 3	<u>1</u>	<u>2</u>	<u>3</u>	1, 3	1
After E = 1	<u>1</u>	<u>2</u>	<u>3</u>	3	<u>1</u>
After D = 3	<u>1</u>	<u>2</u>	<u>3</u>	<u>3</u>	<u>1</u>

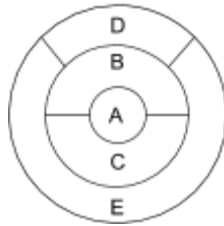
Degree heuristic:

Select the variable that is involved in the largest number of constraints on other unassigned variables. Degree heuristic often used as a tie breaker, e.g., in conjunction with MRV.

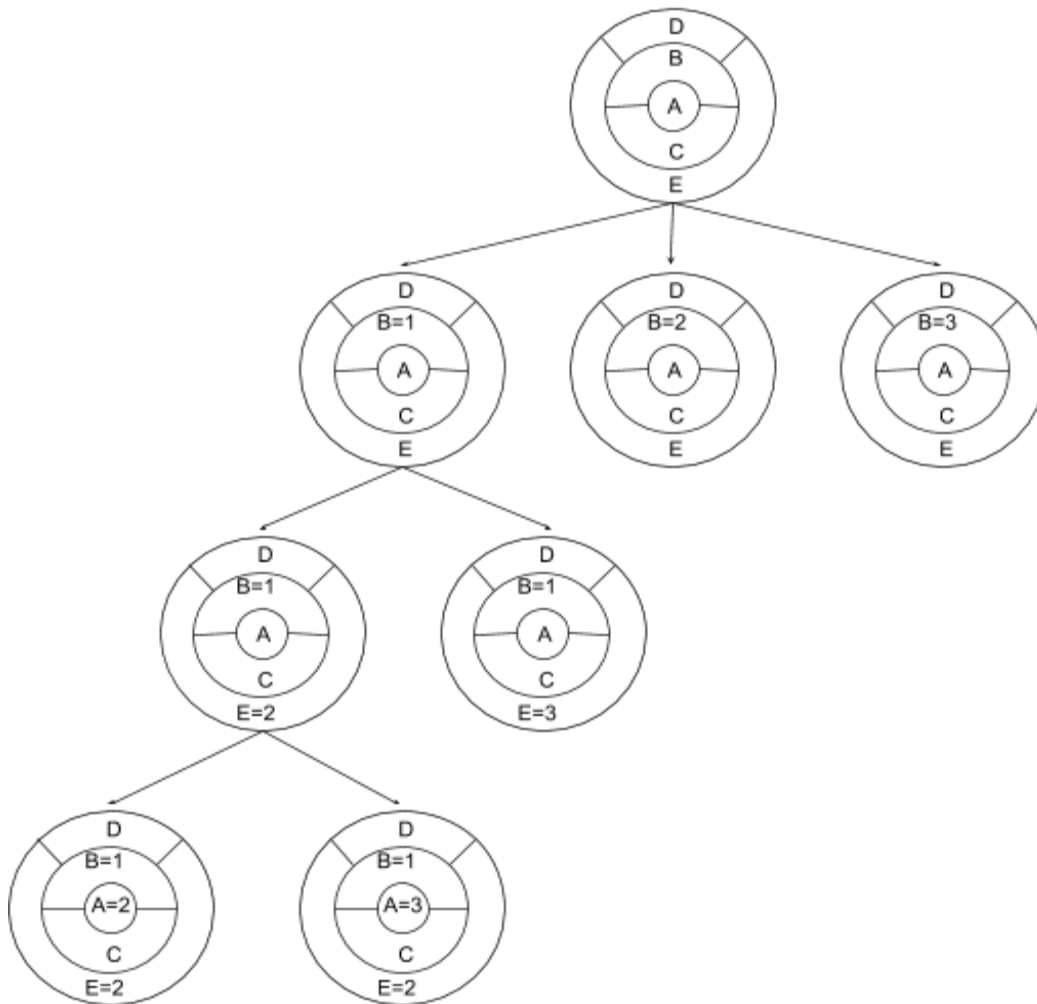
	A	B	C	D	E
	1, 2, 3	1, 2, 3	1, 2, 3	1, 2, 3	1, 2, 3
After B = 1	2, 3	<u>1</u>	2, 3	2, 3	2, 3
After E = 2	2, 3	<u>1</u>	3	3	<u>2</u>
After C = 3	2	<u>1</u>	<u>3</u>	3	<u>2</u>
After D = 3	2	<u>1</u>	<u>3</u>	<u>3</u>	<u>2</u>
After A = 2	<u>2</u>	<u>1</u>	<u>3</u>	<u>3</u>	<u>2</u>

Backtracking:

Problem: The 3-colors Mahmoud's map!

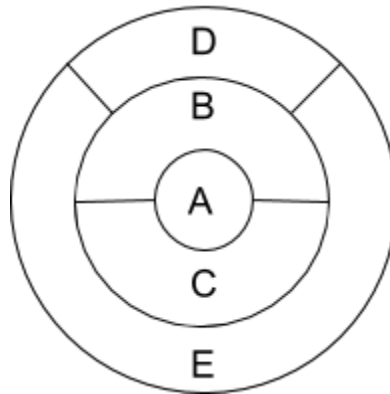


	A	B	C	D	E
	1, 2, 3	1, 2, 3	1, 2, 3	1, 2, 3	1, 2, 3
After B = 1	2, 3	<u>1</u>	2, 3	2, 3	2, 3
After E = 2	2, 3	<u>1</u>	3	3	<u>2</u>
After A = 2	<u>2</u>	<u>1</u>	3	3	<u>2</u>



### Least Constraining Value heuristic

Given a variable, choose the least constraining value: the one that rules out the fewest values in the remaining variables.



	A	B	C	D	E
	1, 2, 3	1, 2, 3	1, 2, 3	1, 2, 3	1, 2, 3
After B = 1	2, 3	<u>1</u>	2, 3	2, 3	2, 3
After E = 2	2, 3	<u>1</u>	3	3	<u>2</u>
After A = 2	<u>2</u>	<u>1</u>	3	3	<u>2</u>

	A	B	C	D	E
	1, 2, 3	1, 2, 3	1, 2, 3	1, 2, 3	1, 2, 3
After B = 1	2, 3	<u>1</u>	2, 3	2, 3	2, 3
After E = 2	2, 3	<u>1</u>	3	3	<u>2</u>
After A = 3	<u>3</u>	<u>1</u>		3	<u>2</u>

Least Constraining Value heuristic prefers A = 2, since A = 3 allows 0 values for C.

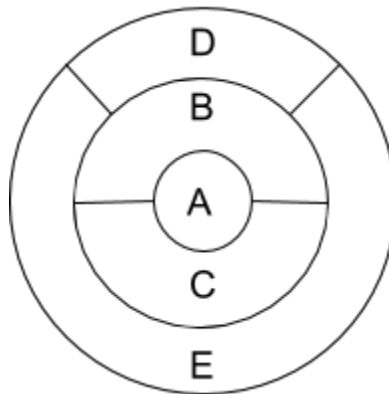
Hill-climbing search (4-queens):

The heuristic cost function  $h$  is the number of pairs of queens that are attacking each other, either directly or indirectly.

Q	3	5	4
4	Q	Q	4
3	<u>2</u>	6	3
4	<u>2</u>	4	Q

A 4-queens state with heuristic cost estimate  $h = 4$ , showing the value of  $h$  for each possible successor obtained by moving a queen within its column. The best moves are marked. The hill-climbing algorithm choose to move the queen in the second column to the third or fourth row.

Hill-climbing search (3-colors Mahmoud's map!):



The heuristic cost function  $h$  is the number of pairs of adjacent regions which have the same color.

	A	B	C	D	E
Red	x	x	3	3	2
Bule	<u>1</u>	2	<u>1</u>	x	2
Green	2	3	x	3	x

A state (A = Red, B = Red, C = Green, D = Blue, E = Green) with heuristic cost estimate  $h = 2$ , showing the value of  $h$  for each possible successor obtained by changing the color each region.

The best moves are marked. Thus, the hill-climbing algorithm choose to change the color of either A or C to Blue.