Bayesian network example:

Suppose you are using a Bayesian network to infer the relationship in between raining, traffic and being late (to office). The probability of raining and the conditional probability of traffic jam, given raining, and being late, given traffic jam are all depicted in this graph.

R	P(R)
t	0.1
f	0.9

R	T	P(T|R)
t	t	0.8
t	f	0.2
f	t	0.1
f	f	0.9

T	L	P(L|T)
t	t	0.3
t	f	0.7
f	t	0.1
f	f	0.9

Enumeration example:

What's the probability of being late, that is $P(L = true)$?

The joint distribution defined by the above Bayesian network is:

$P(R,\ T,\ L) \ = \ P(L|T)\,P(T|R)\,P(R)$

$\Rightarrow P(R,\ T,\ L = true) \ = \ P(L = true|T)\,P(T|R)\,P(R)$

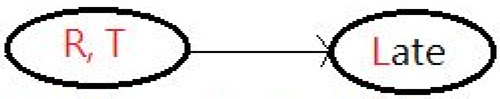
$\Rightarrow P(L = true) \ = \sum_T \sum_R P(L = true|T)\,P(T|R)\,P(R)$

$$\Rightarrow P(L = true) \ = P(L = true|T = true)\,P(T = true|R = true)\,P(R = true) \ + \\ P(L = true|T = true)\,P(T = true|R = false)\,P(R = false) \ + \\ P(L = true|T = false)\,P(T = false|R = true)\,P(R = true) \ + \\ P(L = true|T = false)\,P(T = false|R = false)\,P(R = false)$$

$\Rightarrow P(L = true) = (0.3 \times 0.8 \ \times 0.1) \ + \ \ (0.3 \times 0.1 \ \times 0.9) \ + \ (0.1 \times 0.2 \ \times 0.1) \ + ($
$0.1 \times 0.9 \ \times 0.9) \ = \ 0.134$

Computing $P(L = true)$ using Variable Elimination:

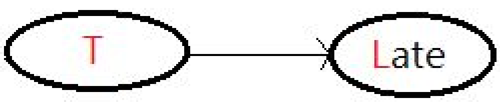
Step 1: choose 2 or more of the factors. In this case, we choose $P(R)$ and $P(T|R)$, to combine them together to form a new factor which represents the joint probability of all variables R and T in that new factor $P(R,T)$. We perform the operation of joining factors on these 2 factors, $P(R)$ and $P(T|R)$, getting a new factor which is part of the existing network. Below exhibits what we have now.



R	T	P(R,T)
t	t	0.08
t	f	0.02
f	t	0.09
f	f	0.81

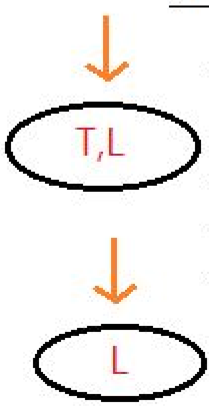
T	L	P(L|T)
t	t	0.3
t	f	0.7
f	t	0.1
f	f	0.9

Step 2: The second is the operation of elimination, also called summing out or marginalization, to take the table $P(R,T)$, reduce it to $P(T)$, finally combine it with $P(L|T)$ to get $P(L)$:



T	P(T)
t	0.17
f	0.83

T	L	P(L|T)
t	t	0.3
t	f	0.7
f	t	0.1
f	f	0.9



T	L	P(T,L)
t	t	0.051
t	f	0.119
f	t	0.083
f	f	0.747

L	P(L)
t	0.134
f	0.886

$P(L = true)$=0.134