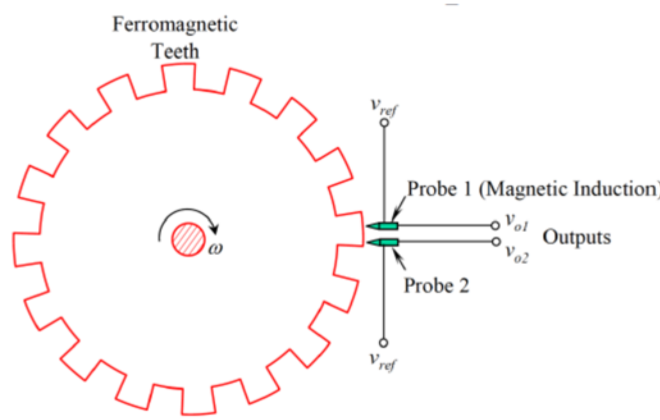
Digital Tachometers:



- 2 proximity sensors located at ¼ pitch from each other.
- This is a *magnetic induction, pulse tachometer of the variable-reluctance type.*
- The teeth on the wheel are made of a ferromagnetic material.
- The two magnetic-induction (and variable-reluctance) proximity probes *are placed radially facing the teeth*, at quarter-pitch apart (*pitch = tooth-to-tooth spacing*).
- When the toothed wheel rotates, the two probes generate output signals that are 90° out of phase (i.e., quadrature signals).
- One signal leads the other in one direction of rotation and lags the other in the opposite direction. *In this manner, a directional reading (i.e., velocity rather than speed) is obtained.*
- The speed is computed either by counting the pulses over a sampling period or by timing the pulse width, as in the case of an incremental encoder.
- Advantages: Simplicity, robustness, and low-cost.
- Disadvantages: Poor resolution, mechanical loading.

Alternative types of *digital tachometers use eddy current proximity probes* or capacitive proximity probes discussed in chapter-5.

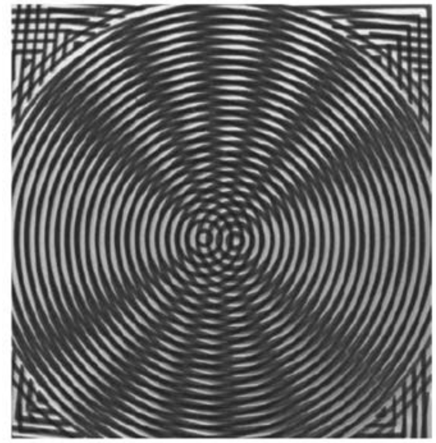
- In the case of an eddy current tachometer, *the teeth of the pulsing wheel are made of (or plated with) electricity- conducting material*.
- The probe consists of an active coil connected to an ac bridge circuit excited by a radio-frequency (i.e., in the range 1–100 MHz) signal.
- The resulting magnetic field at radio frequency is modulated by the tooth-passing action. The bridge output may be demodulated and shaped to generate the pulse signal.
- In the case of a capacitive tachometer, *the toothed wheel forms one plate of the capacitor*; *the other plate is the probe and is kept stationary*.
- As the wheel turns, the gap width of the capacitor fluctuates. If the capacitor is excited by an ac voltage of high frequency (typically 1 MHz), a nearly pulse-modulated signal at that carrier frequency is obtained.
- This can be detected through a bridge circuit as before but using a capacitance bridge rather than an inductance bridge.
- Demodulated pulse signal generated in this manner is used in the angular velocity computation.

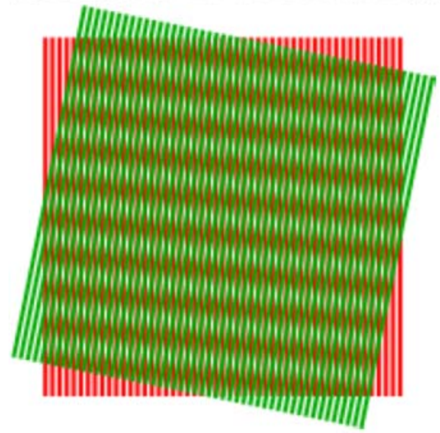
Moiré Fringe Displacement Sensors:

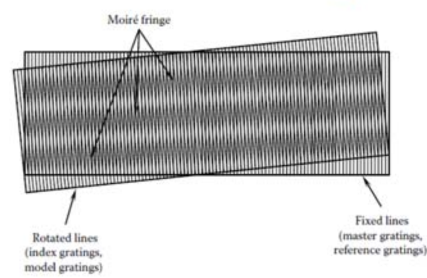
Suppose that a piece of transparent fabric is placed on another similar fabric. If one piece is moved or deformed with respect to the other, we will notice various designs of light and dark patterns (lines) in motion. Dark lines of this type are called moire fringes.

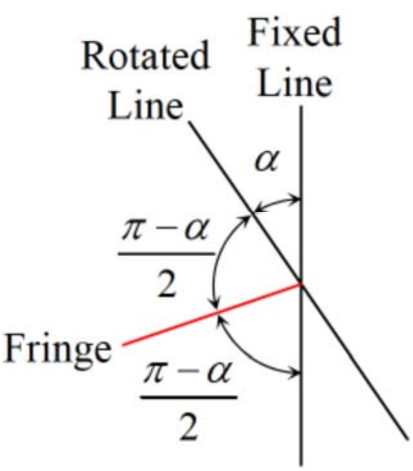
The rotation of the index plate with respect to the reference plate can be measured by sensing the orientation of the fringe lines with respect to the fixed (master or reference) gratings.

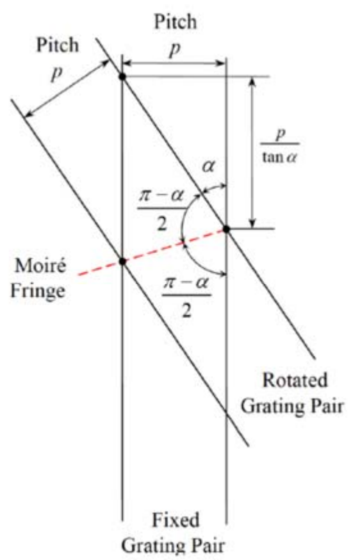
- Measurement is done by a CCD camera.
- Very high resolutions: 0.002 mm











Optical Sensors, Lasers, and Cameras:

There are many sensors that use light or laser as the basis of measurement. Also, camera images are widely used for sensing purposes.

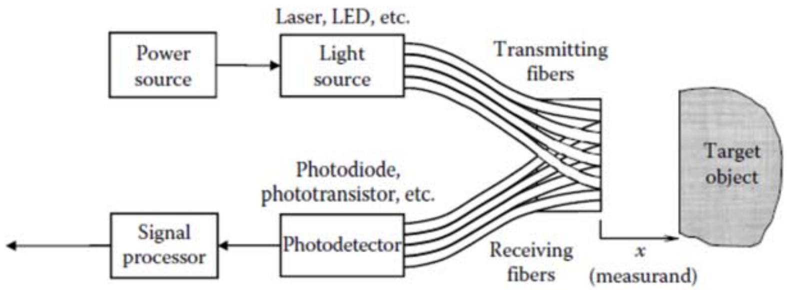
***Laser*:**

- ***L**ight **a**mplification by **s**timulated **e**mission of **r**adiation:*
- Produces electromagnetic radiation in the ultraviolet, visible, or infrared bands of the spectrum.
- Can provide a single-frequency (*monochromatic*) light source.
- Electromagnetic radiation in a laser is *coherent* in the sense that all waves generated have constant phase angles.
- Uses oscillations of atoms or molecules of various elements.
- Useful in fiber optics, but it can also be used directly in sensing and gauging applications.
- Helium-neon (HeNe) laser and the semiconductor laser are commonly used in optical sensor applications.

***Fiber-optic sensors*:**

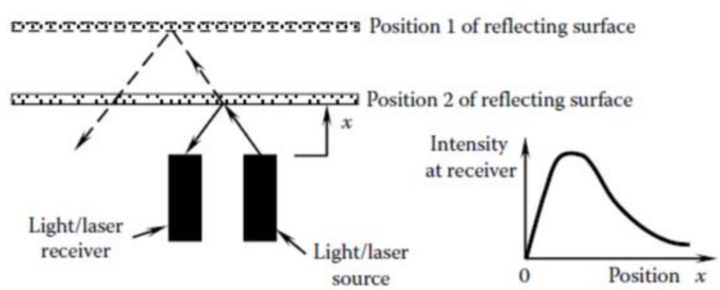
- The characteristic component in a fiber-optic sensor is a bundle of glass fibers (typically a few hundred) that can carry light.
- Each optical fiber may have a diameter on the order of a few μm to about 0.01 mm.
- There are two basic types of fiber-optic sensors:
    - In one type—the *indirect* or the *extrinsic* type—the optical fiber acts only as the medium in which the sensor light is transmitted. In this type, the sensing element itself does not consist of optical fibers.
    - In the second type—the *direct* or the *intrinsic* type—the optical fiber itself acts as the sensing element. When the conditions of the sensed medium change, the light-propagation properties of the optical fibers change as well (e.g., due to micro-bending of a straight fiber as a result of an applied force), providing a measurement of the change in conditions.
    - Examples of the first (extrinsic) type of sensor include fiber-optic position sensors, proximity sensors, and tactile sensors. The second (intrinsic) type of sensor is found, for example, in fiber-optic gyroscopes, fiber- optic hydrophones, and some types of micro-scale displacement or force sensors.

Fiber-Optic Position Sensor:



A schematic representation of a fiber-optic position sensor (or proximity sensor or displacement sensor) is shown and the optical fiber bundle is divided into *two* groups: *transmitting fibers and receiving fibers.*

- Light from the light source is *transmitted along the first bundle* of fibers to the target object whose position is being measured.
- Light reflected (or, diffused) *onto the receiving fibers* by the surface of the target object is carried to a photodetector.
- The intensity of the *light received by the photodetector* will depend on position x of the target object.
- In particular, if x = 0:
  - The *transmitting bundle will be completely blocked off*
  - And *the light intensity at the receiver will be zero*.
- As distance x is increased, the intensity of the received light will increase, because more and more light will be reflected onto the tip of the receiving bundle. This will reach a peak at some value of x.

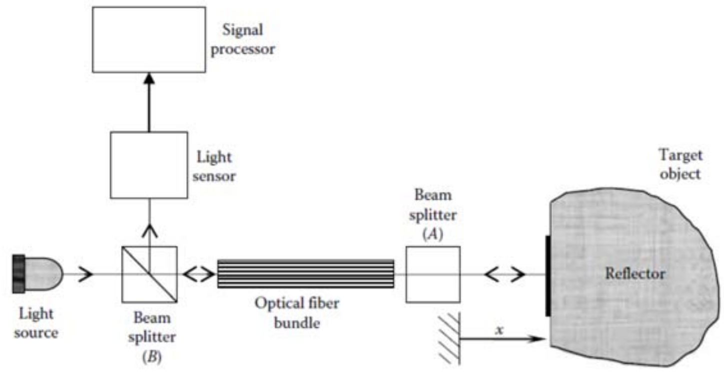


- When x is increased beyond that value:
  - more and more light will be reflected outside the receiving bundle;
  - Hence, the intensity of the received light will drop.

- The proximity–intensity curve for an optical proximity sensor will be nonlinear and will have the shape shown in diagram.
- Using this (calibration) curve, we can determine the position (x) once the intensity of the light received at the photosensor is known.
- This type of fiber-optic sensors can be used, with a suitable front-end device (such as bellows, springs, etc.) to measure pressure, force, etc. as well.

Laser Interferometer:

- Useful in the accurate measurement of small displacements.
- This is an *extrinsic application of fiber optics* where optical *fiber is used for light transmission* rather than light sensing.



- In this fiber-optic position sensor, the *same bundle of fibers is used for sending and receiving* a monochromatic beam of light (typically, laser).
- *Alternatively, monomode fibers, which transmit only monochromatic light (of a specific wavelength) may be used for this purpose*.

- In either case, as shown above, a beam splitter (A) is used so that:
  - Part of the light is directly reflected back to the bundle tip and
  - The other part reaches the target object (as in Figure last page) and reflected back from it (*using a reflector mounted on the object*) on to the bundle tip.
  - In this manner, part of the light returning through the bundle had not traveled beyond the beam splitter while the other part had traveled between the beam splitter (A) and the object (through an extra distance equal to twice the separation between the beam splitter and the object).
  - As a result, the two components of light will have a phase difference ϕ, which is given by: $\phi = \frac{2x}{\lambda} 2\pi$
    - *x is the distance of the target object from the beam splitter*
    - *λ is the wavelength of monochromatic light*
  - The returning light is directed to a light sensor using a beam splitter B.
  - The sensed signal is processed using principles of interferometry to determine ϕ, and from Equation above, the distance x.
  - Very fine resolutions better than a fraction of a micrometer (μm) can be obtained using this type of fiber-optic position sensors.

<u>Advantages:</u>

The advantages of fiber optics include:

- Insensitivity to electrical and magnetic noise (due to optical coupling);
- Safe operation in explosive, High- temperature, corrosive, and hazardous environments; and high sensitivity.
- Mechanical loading and wear problems do not exist because fiber-optic position sensors are *noncontact* devices with no moving parts.

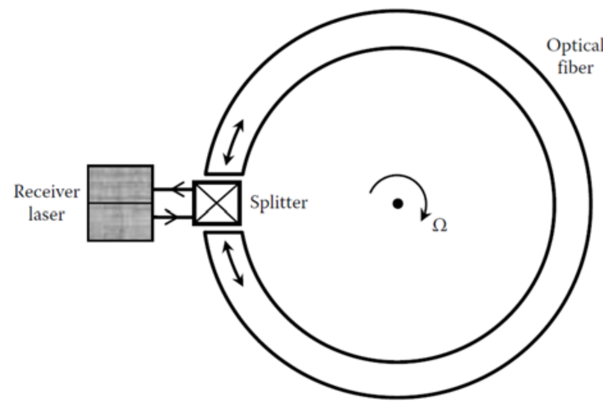
<u>Disadvantages:</u>

The disadvantages of fiber optics include:

- Direct sensitivity to variations in the intensity of the light source and
- Dependence on ambient conditions (temperature, dirt, moisture, smoke, etc.). Compensation can be made, however, with respect to temperature.

Intrinsic application - Fiber-optic Gyroscope

This is an angular speed sensor that uses fiber optics. Contrary to its name, however, it is not a gyroscope in the conventional sense.



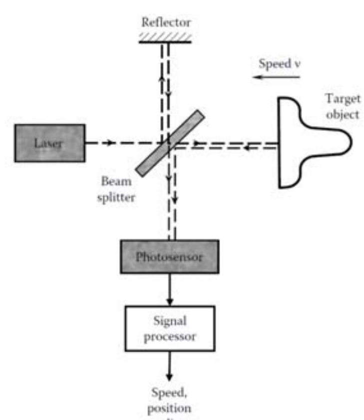
- *Two loops of optical fiber wrapped around a cylinder are used in this sensor, and they rotate with the cylinder, at the same angular speed that needs to be sensed.*
- *One loop carries a monochromatic light (or laser) beam in the clockwise direction, and the other loop carries a beam from the same light (laser) source in the counterclockwise direction.*
- *Since the laser beam traveling in the direction of rotation of the cylinder attains a higher frequency than that of the other beam. The difference in speed or frequencies (known as the Sagnac effect) of the two laser beams received at a common location will measure the angular speed of the cylinder.*
- *This may be accomplished through interferometry, because the combined signal is a sine beat. As a result, light and dark patterns (fringes) will be present in the detected light, and they will measure the frequency difference and hence the rotating speed of the optical fibers.*
- *Note that in a laser (ring) gyroscope, it is not necessary to have a circular path for the laser. Triangular and square paths are used as well. In general the beat frequency* **Δω** *of the combined light from two laser beams traveling in opposite directions is given by:*

$$\Delta\omega = \frac{4A}{p\lambda}\Omega$$

A is the area enclosed by the travel path (πr^2 for a cylinder of radius r)
p is the length (perimeter) of the traveled path ($2\pi r$ for a cylinder)
λ is the wavelength of the laser
Ω is the angular speed of the object (or, optical fiber)

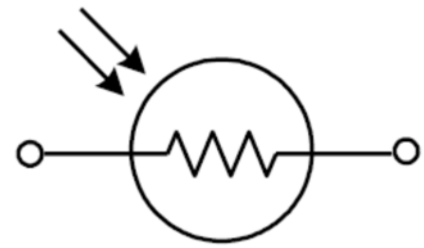
Laser Doppler Interferometer:

- Used for accurate measurement of speed.
- Based on two phenomena: Doppler Effect (DE) and light wave interference: Constructive and Destructive
- DE -- Consider a wave source (e.g., a light source or sound source) that is moving w.r.t. a receiver (observer).
  - If source moves toward the receiver, the Frequency of received wave appears ↑ or $f_2 = f + \Delta f$
  - If the source moves away from the receiver, the frequency of received wave ↓ or $f_2 = f - \Delta f$
  - $\Delta f \propto$ Velocity of the source relative to the receiver. This phenomenon is known as the *Doppler Effect*. $\Delta f = \frac{2f}{c} v = k\, v$



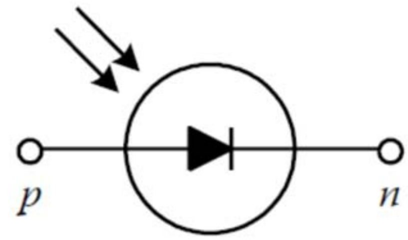
Light Sensors

Semiconductor-based light sensors as well as light sources are needed in optoelectronics. A light sensor is also known as a *photodetector* or *photosensor*.



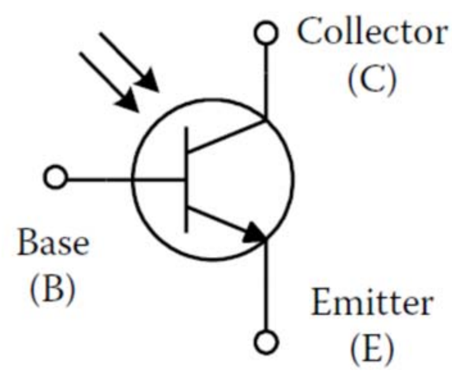
Photoresistor (or *photoconductor*):

- R $\downarrow$ (or $\uparrow \sigma$) as the intensity of light falling on it $\uparrow$
- Typically, the resistance of a photoresistor could change from very high values (megohms) in the dark to reasonably low values (less than 100 Ω) in bright light. As a result, *very high sensitivity to light is possible*.
- e.g: *cadmium sulfide* (CdS) or *cadmium selenide* (CdSe) between two electrodes. *Lead sulfide* (PbS) or *lead selenide* (PbSe) may be used in infrared photoresistor.



Photodiode:

- *pn* junction of semiconductor material that produces electron–hole pairs in response to light. Two types of photodiodes are available.
- A *photovoltaic* diode generates a sufficient potential at its junction in response to light (photons) falling on it. Hence an external bias source is not necessary for a photovoltaic diode.
- A *photoconductive* diode undergoes a resistance change at its junction in response to photons.



Phototransistor:

- Semiconductor photosensor with amplification circuitry built into the same package (chip) is popularly called a phototransistor. Hence a photodiode with an amplifier circuit in a single unit might be called a phototransistor and this is an *npn* transistor.
- *i_c is nearly proportional to the intensity of the light falling, hence, i_c can be used as a measure of the light intensity. Germanium or silicon is the semiconductor material that is commonly used in phototransistors.*

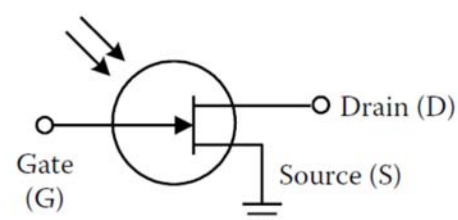
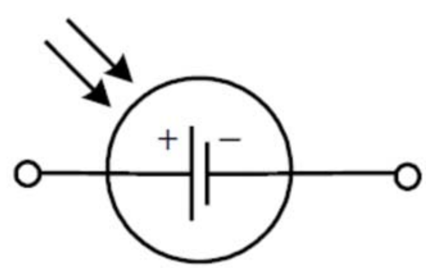


Photo-FET:

- A photo-field effect transistor is similar to a conventional FET. The symbol shown is for an n-channel photo-FET.
- This consists of an n-type semiconductor element (e.g., silicon doped with boron), called channel.
- When *light is projected at the gate, the drain current i_d will increase*.
- Hence, drain current (current at the D lead) can be used as a measure of light intensity.



Photocell:

- Photocells are similar to photosensors except that a photocell is used as an electricity source rather than a sensor of radiation.
- Solar cells, which are more effective in sunlight, are commonly available.
- A *typical photocell is a semiconductor junction element made of a material such as single-crystal silicon, polycrystalline silicon, and cadmium sulfide*.
- Cell arrays are used in moderate-power applications and typical power output is 10 mW/cm2 of surface area, with a potential of about 1.0 V.

Charge-Coupled Device

- A charge-coupled device (CCD) is an integrated circuit element (a *monolithic device*) of semiconductor material.
- A silicon wafer (*p* type or *n* type) is oxidized to generate a layer of SiO2 on its surface.
- A *matrix of metal electrodes is deposited on the oxide layer and is linked to the CCD output leads*.
- When light falls onto the CCD element (from an object), a *charge packets* are generated within the substrate *silicon wafer*. Now if an external potential is applied to a particular electrode of the CCD, a *potential well* is formed under the electrode and a charge packet is deposited here. This charge packet can be moved across the CCD to an output circuit

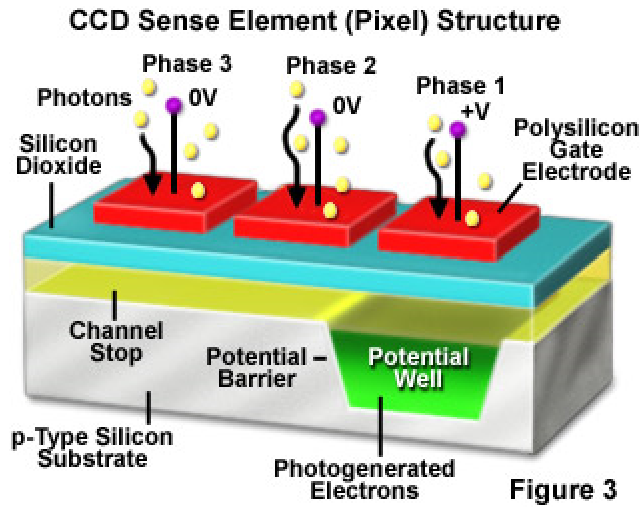


Figure 3

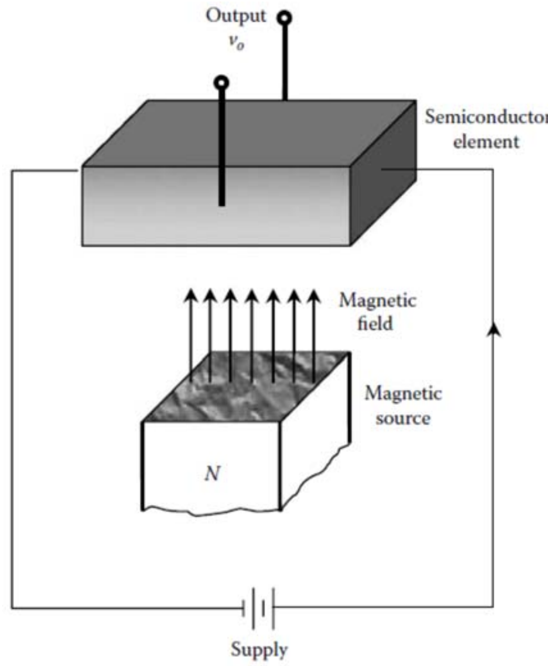
Image Sensors:

- Imaging device is a sensor, and an image is the sensed data.
- Depending on the imaging device, an image can be of many varieties such as optical, thermal or infrared, x-ray, ultraviolet, acoustic, ultrasound, and so on.

Image Processing and Computer Vision:

- An image may be processed (analyzed) to obtain a more refined image from which useful information such as edges, contours, areas, and other geometrical information can be determined.
- Computer Vision involves higher level operations than image processing and is akin to what humans infer based on what they see.
  1. ***Filtering:*** *(to remove noise and enhance the image) including directional filtering (to enhance edges, for edge detection)*
  2. ***Thresholding:*** *(to generate a two-level black-and-white image where the gray levels above a set threshold are assigned white and those below the threshold are assigned black)*
  3. ***Segmentation:*** *(to subdivide an enhanced image, identify geometric shapes/objects, and capture properties such as area and dimensions of the identified geometric entities)*
  4. ***Morphological processing:*** *(sequential shrinking, filtering, stretching, etc. to prune out unwanted image components and extract those that are important)*
  5. ***Subtraction:*** *(e.g., subtract the background form the image)*
  6. ***Template matching:*** *(to match a processed image to a template—useful in object detection)*
  7. ***Compression:*** *(to reduce the quantity of data that is needed to represent the useful information of an image)*

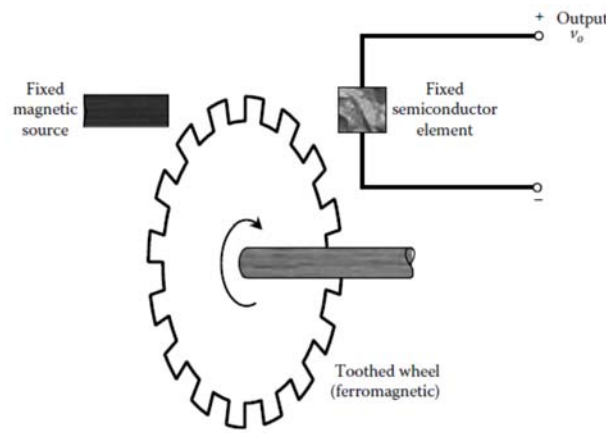
Hall-Effect Sensor:



- Consider a semiconductor element subject to a dc voltage v_{ref}.

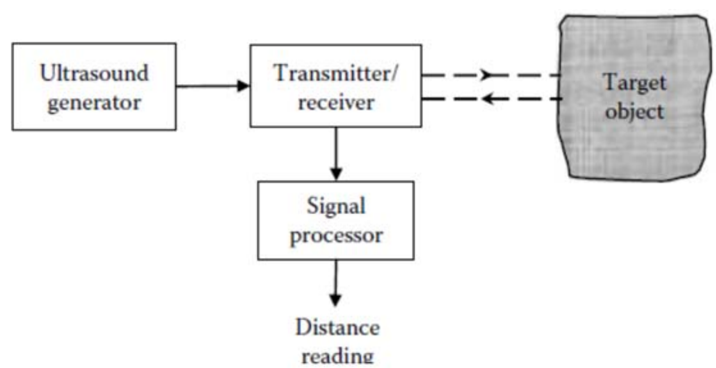
- If a magnetic field is applied perpendicular to the direction of this voltage, a voltage v_o will be generated in the third orthogonal direction within the semiconductor element. This is known as the Hall Effect (observed by E.H. Hall in 1879).

Hall-Effect Motion Sensors:



- A Hall-effect sensor may be used for motion sensing in many ways; for example, as an analog proximity sensor, a limit switch (digital), or a shaft encoder.
- Output voltage v_o increases as the distance from the magnetic source to the semiconductor element decreases, the *output signal v_o can be used as a measure of proximity*.
- Digitally speaking, certain threshold level of the output voltage v_o can be used to generate a binary output, which represents the presence/absence of an object.
- The use of a toothed ferromagnetic wheel (as for a digital tachometer) to alter the magnetic flux will result in a shaft encoder.

Ultrasonic Sensors:

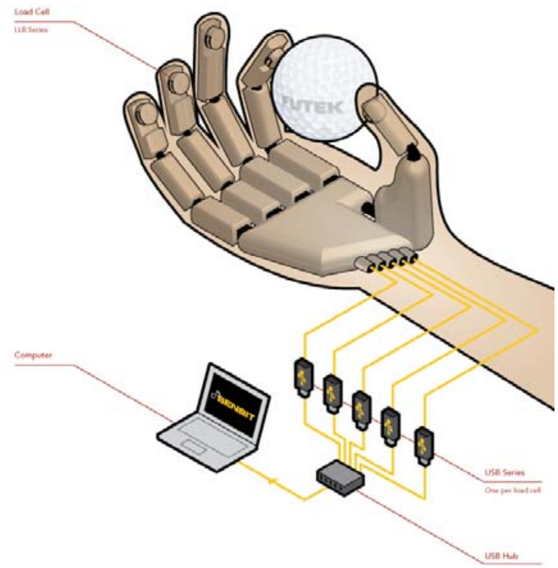


- Ultrasound waves are pressure waves, just like sound waves, but their frequencies are higher (ultra) than the audible frequencies (range of 20 Hz to 20 kHz).
- Ultrasonic sensors are used in many applications, including medical imaging, ranging systems for cameras with autofocusing capability, level sensing, and speed sensing.
- Velocity - Using Doppler Effect.

$$\boldsymbol{x = \frac{ct}{2}};$$

- t is the time of flight of the ultrasound pulse (from generator to receiver)
- x is the distance between the ultrasound generator/receiver and the target object
- c is the speed of sound in the medium (typically, air)

Tactile Sensing:



- Tactile sensing is usually interpreted as touch sensing, but tactile sensing is different from a simple clamping where very few discrete force measurements are made.
- In tactile sensing, a force distribution is measured, using a closely spaced array of force sensors and usually exploiting the skin-like properties of the sensor array.
- Tactile sensing is particularly important in two types of operations:
  - o Grasping and fine manipulation, and
  - o Object identification.
- In *grasping and fine manipulation*, the object has to be held in a stable manner without being allowed to slip and without being damaged.
- *Object identification* includes recognizing or determining the shape, location, and orientation of an object as well as detecting or identifying surface properties (e.g., density, hardness, texture, flexibility), and defects.

Ideally, these tasks would require two types of sensing:

- o Continuous spatial sensing of time-variable contact force
- o Sensing of surface deformation profiles (time-variable)

- Note that learning also can be an important part of tactile sensing.
- Typical specifications for an industrial tactile sensor are as follows:
  - Spatial resolution of about 1 mm (about 100 sensor elements)
  - Force resolution of about 2 g
  - Dynamic range of 60 dB
  - Force capacity (maximum touch force) of about 1 kg
  - Response time of 5 ms or less (a bandwidth of over 200 Hz)
  - Low hysteresis (low energy dissipation)
  - Durability under harsh working conditions
  - Robustness and insensitivity to change in environmental conditions (temperature, dust, humidity, vibration, etc.)
  - Capability to detect and even predict slip

Dexterity:

Dexterity is an important consideration in sophisticated manipulators and robotic hands that employ tactile sensing:

$$Motion\ Dexterity = \frac{\text{Number of degrees of freedom in the device}}{\text{Motion resolution of the device}}$$

$$Force\ Dexterity = \frac{\text{Number of degrees of freedom}}{\text{Force resolution}}$$

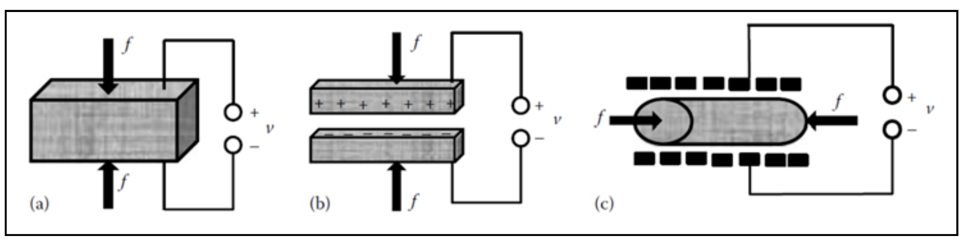
Read Example 6.10 and Strain Gauge example 6.11

MEMS Sensors:

- **M**icro**e**lectro**m**echanical **s**ystems (MEMS) are microminiature devices consisting of microminiature components such as sensors, actuators, and signal processing integrated and embedded into a single chip while exploiting both electrical/electronic and mechanical features of them.
- The device size can be in the sub-millimeter scale (0.01–1.0 mm) and
- The component size can be as small as a micrometer (micron), in the range 0.001–0.1 mm. Since MEMS exploits the integrated-circuit (IC) technologies in their fabrication, many components can be integrated into a single device (e.g., a few to a million).

The advantages of MEMS are primarily the advantages of IC devices which include:

- Microminiature size and weight
- Large surface area to volume ratio (when compared in the same measurement units)
- Large-scale integration (LSI) of components/circuits
- High performance
- High speed (20 ns switching speeds)
- Low power consumption
- Easy mass-production
- Low cost (in mass production)
- In particular, the microminiature size also means negligible mechanical loading, fast response, and negligible power consumption (and related electrical loading).



Energy Conversion Mechanism:
Piezoelectric: Mechanical strain in a piezoelectric material causes a charge separation across the material producing a voltage. Strain energy produced by the mechanical work that is needed to deform the material, is converted into electrostatic energy. This is a passive device.
Electrostatic: A voltage causes + and − charges to separate into the capacitor plates. The attraction force between the plates is supported by an external mechanical force. If *plates move apart, mechanical work is done, capacitance is reduced, and the voltage is increased*. Hence, mechanical energy is converted into electrical energy. This is a passive device.
Electromagnetic: As a coil moves in a magnetic field, a current is induced in the coil. In this process, *mechanical energy is converted into electrical energy*. This is a passive device.

Magnetic Circuits:

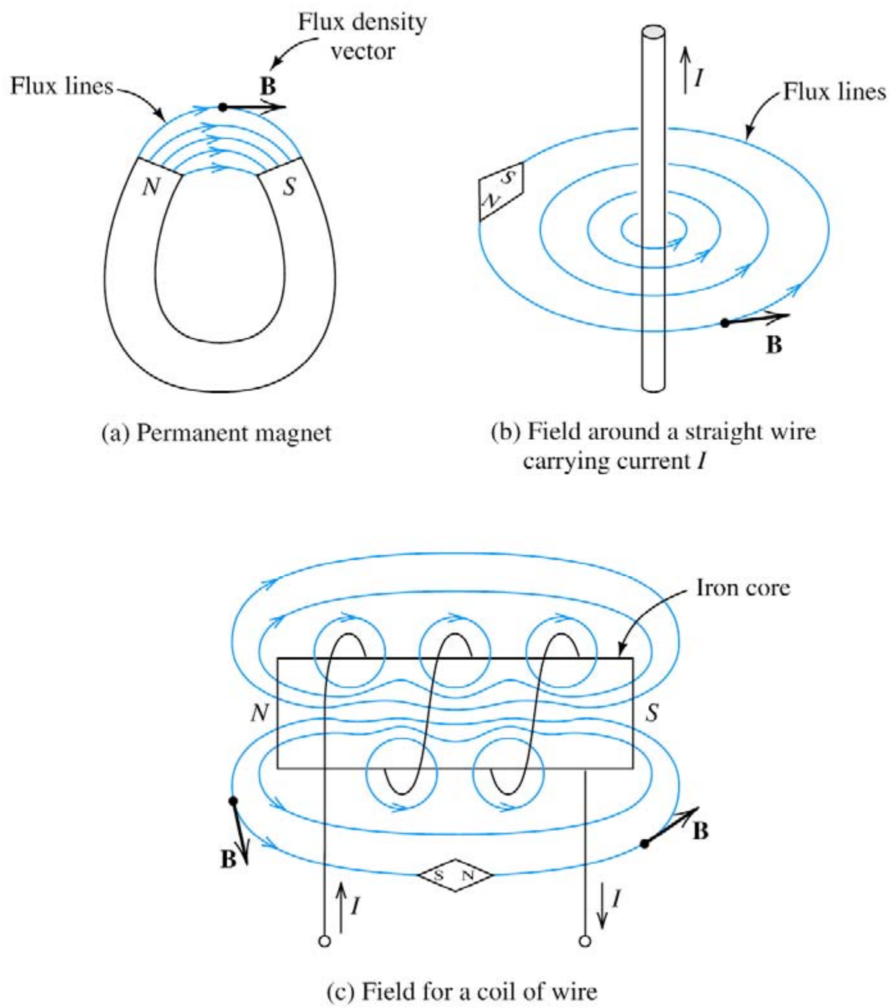


Figure 15.1 Magnetic fields can be visualized as lines of flux that form closed paths. Using a compass, we can determine the direction of the flux lines at any point. Note that the flux density vector **B** is tangent to the lines of flux.

See - Magnetic Circuits pdf copy on line

Stepper Motors:

The terms *stepper motor*, *stepping motor*, and *step motor* are synonymous and are often used interchangeably.

There are three basic types of stepper motors:

1. *Variable-reluctance (VR) stepper motors, which have soft-iron (ferromagnetic) rotors*
2. *Permanent-magnet (PM) stepper motors, which have magnetized rotors*
3. *Hybrid (HB) stepper motors, which have two stacks of rotor teeth forming the two poles of a permanent magnet located along the rotor axis*

The VR and PM steppers operate in a somewhat similar manner:

- Specifically, the *stator magnetic field (polarity) is stepped* so as to change the minimum reluctance (or detent) position of the rotor in increments.
- Hence, both types of motors undergo similar changes in reluctance (magnetic resistance) during operation.
- A disadvantage of VR steppers is that as the rotor is not magnetized, the holding torque is practically zero when the stator windings are not energized (i.e., power-off conditions). Hence, it is not capable to hold the mechanical load at a given position under power-off conditions, unless mechanical brakes are employed.
- An HB stepper motor possesses characteristics of both VR steppers and PM steppers.
  - The rotor of an HB stepper motor consists of two rotor segments connected by a shaft.
  - Each rotor segment is a toothed wheel and is called a stack.
  - The two rotor stacks form the two poles of a permanent magnet located along the rotor axis.
  - Hence, an entire stack of rotor teeth is magnetized to be a single pole (which is different from the case of a PM stepper where the rotor has multiple poles).

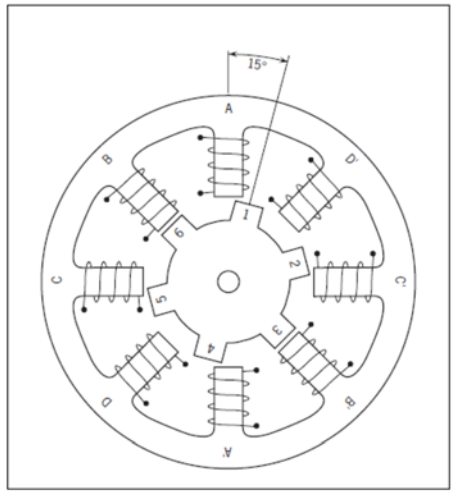


Figure 1. Cross-section of a variable-reluctance (VR) motor.

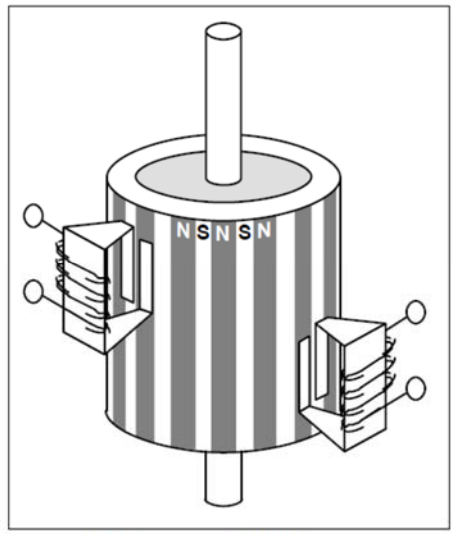


Figure 2. Principle of a PM or tin-can stepper motor.

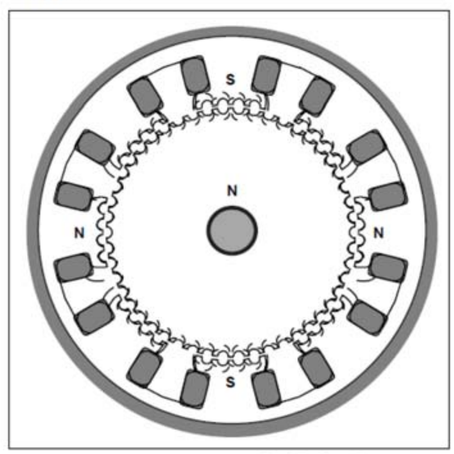
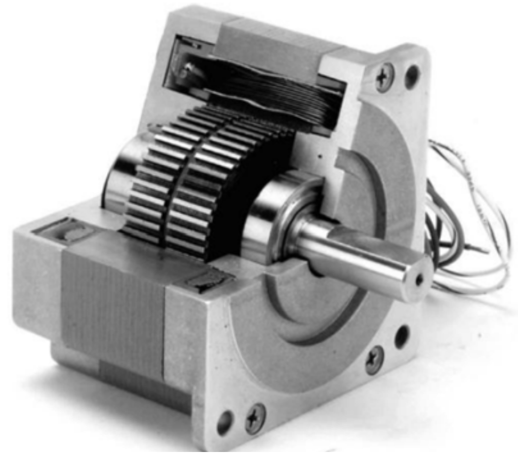


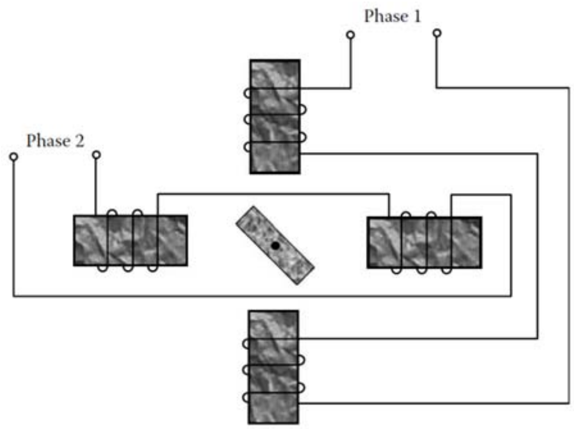
Figure 3. Cross-section of a hybrid stepper motor.

- The *rotor polarity of an HB stepper can be provided either by a permanent magnet, or by an electromagnet using a coil activated by a unidirectional dc source and placed on the stator to generate a magnetic field along the rotor axis.*
- A photograph of the internal components of a *two-stack stepping motor* is given in figure.



Permanent-Magnet Stepper Motor

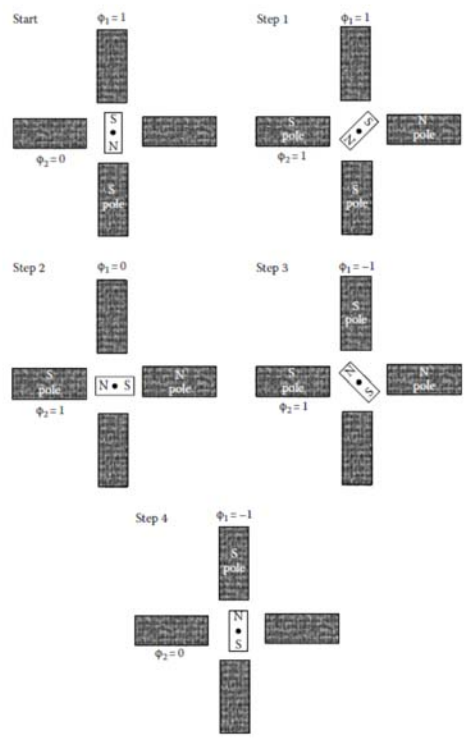
To explain the operation of a PM stepper motor, consider the simple schematic diagram shown in figure.



- The stator has two sets of windings (i.e., two *phases*) placed at 90°.
- This arrangement has four *salient poles* in the stator, each pole geometrically separated by a 90° angle from the adjacent one. The rotor is a two-pole permanent magnet. Each phase can take one of the three states 1, 0, and −1, which are defined as follows:
  1. State 1: current in a specified direction
  2. State −1: current in the opposite direction
  3. State 0: no current

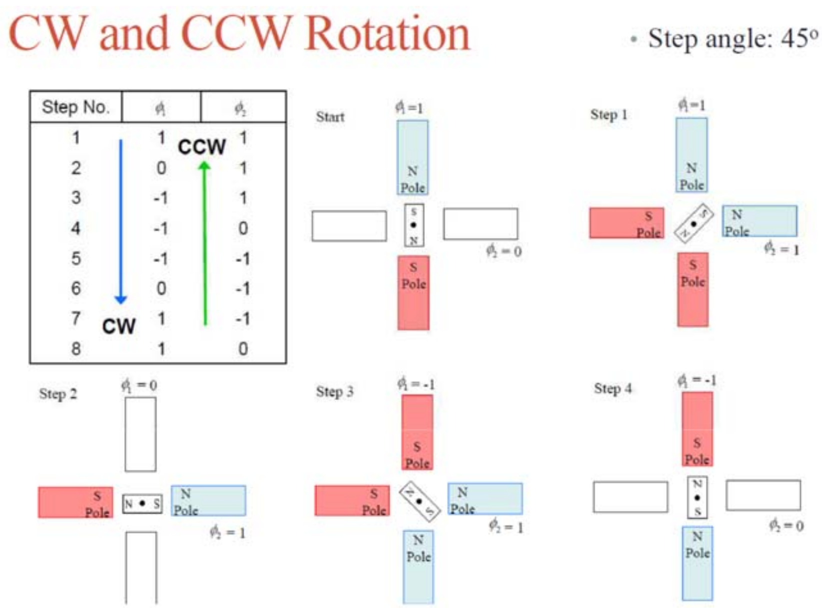
Note that:

- As −1 is the complement state of 1, in some literature the notation 1′ is used to denote the state −1.
- By switching the currents in the two phases in an appropriate sequence, either a clockwise (CW) rotation or a counterclockwise (CCW) rotation can be generated. The CW rotation sequence is shown.



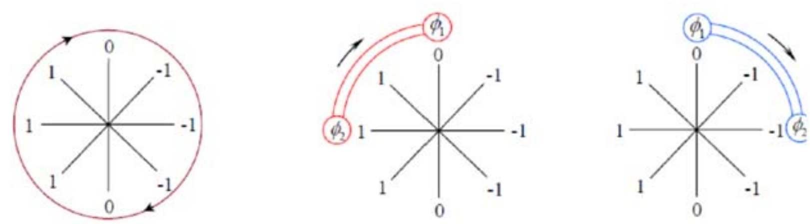
- ϕ_i *denotes* the state of the i_{th} phase.
- The step angle for this motor is 45°. At the end of each step, the rotor assumes the minimum reluctance position that corresponds to the particular magnetic polarity pattern in the stator. This is a stable equilibrium configuration and is known as the detent position for that step.
- Note: *Reluctance measures the magnetic resistance* in a flux path.

- When the stator currents (phases) are switched for the next step, the minimum reluctance position changes (rotates by the step angle) and the rotor assumes the corresponding stable equilibrium position and the rotor turns through a single step (45° in this example).



Step No.	ϕ_1	ϕ_2
1	1	1
2	0	1
3	-1	1
4	-1	0
5	-1	-1
6	0	-1
7	1	-1
8	1	0

Observe that in one complete rotation of the rotor, the state of each phase sweeps through one complete cycle of the switching sequence in figure in the CW direction.



For CW rotation For CCW rotation

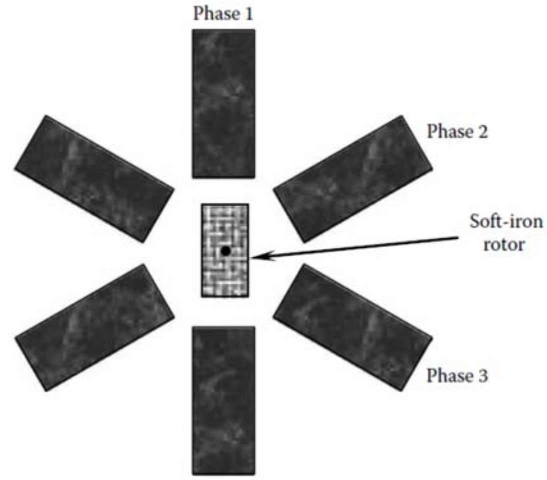
- For CW rotation of the motor, the state of phase 2 (ϕ_2) lags the state of phase 1 (ϕ_1) by two steps.
- For CCW rotation, ϕ_2 leads ϕ_1 by two steps.
- Hence, instead of eight pairs of numbers, just eight numbers with a delay operation would suffice to generate the phase-switching logic.
- This approach is faster and more effective because the switching logic for a stepper motor.

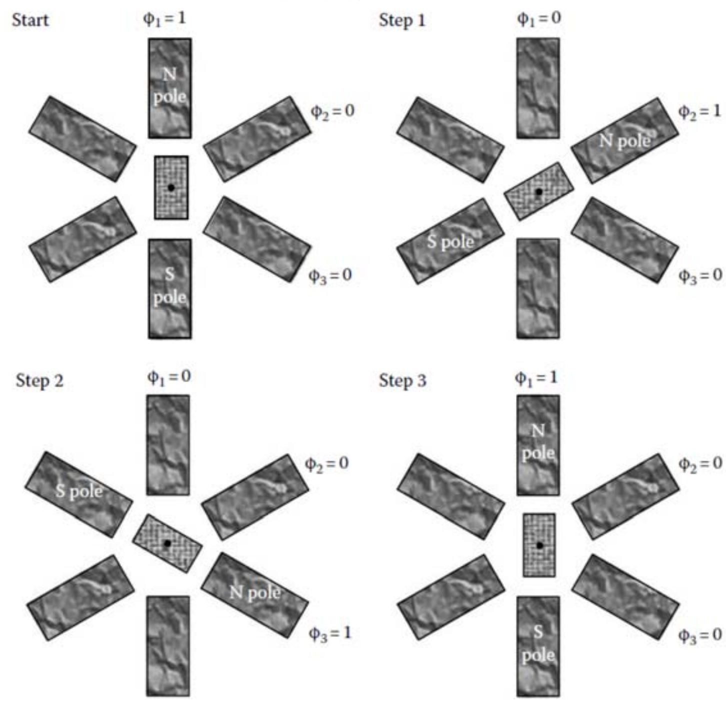
Increasing Resolution:

- A stepping resolution of 45 degrees is too coarse for most applications. We can increase the resolution by adding the rotor pole pairs.
  - o e.g.: Stepping Resolution of 30 degrees
  - o Common types are 30, 15, and 7.5 degrees.
  - o For high precision, this solution is not cost effective.

Variable-Reluctance Stepper Motor

Now consider the VR stepper motor shown.

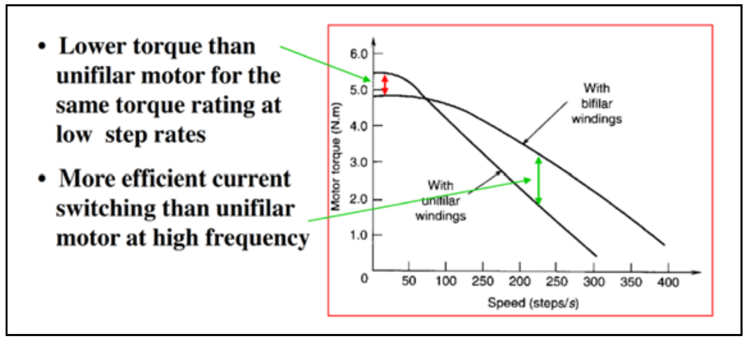




- The rotor is a non-magnetized soft-iron (ferromagnetic) bar.
- The *full-stepping sequence for CW rotation is shown in figure below. The step angle is 60°.* Only one phase is energized at a time in order to execute full stepping.
- With VR steppers, *the direction of the current (the polarity of a stator pole pair) is not reversed in the full-stepping sequence;* only the states 1 and 0 (i.e., on and off) are used for each phase.
- In the *case of half stepping, however, two phases have to be energized simultaneously during some steps*.
- Furthermore, *current reversals are needed in half stepping, thus requiring more elaborate switching circuitry.*
- The *advantage, however, is that the step angle would be halved to 30°, thereby providing improved motion* resolution.
- When *two phases are activated simultaneously,* the *minimum reluctance position is halfway between the corresponding pole pairs (i.e., 30° from the detent position* that is obtained when only one of the two phases is energized), which enables half stepping.
- It follows that, *depending on the energizing sequence of the phases*, *either full stepping or half stepping would be possible*.
- *Micro-stepping provides much smaller step angles achieved by changing the phase currents by small increments (rather than just the states on, off, and reversal)* so that the detent (equilibrium) position of the rotor shifts in correspondingly small angular increments.

Polarity Reversal:
The polarity of a stator pole can be reversed in two ways:

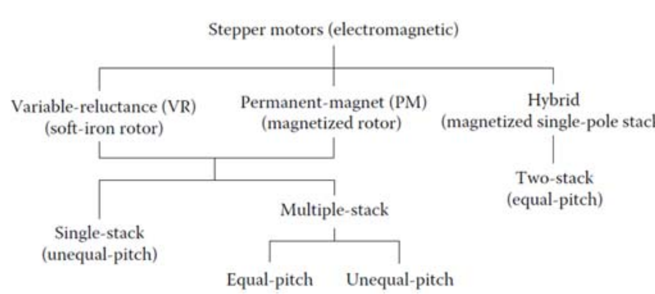
- There is only one set of windings for a group of stator poles. This is the case of unifilar windings. Polarity of the poles is reversed by reversing the direction of current in the winding.
- There are two sets of windings for a group of stator poles. This is the *case of bifilar windings (i.e., double-file or two-coil windings*).



- Only *one set of windings is energized at a time, producing one polarity for this group* of poles. *The other set of windings produces the opposite polarity*.
- *Note*: **<u>One winding with a center tap may be used in place of two windings.</u>** The other two terminals of the coil are given opposite (i.e., positive and negative) voltages.

Stepper Motor Classification:

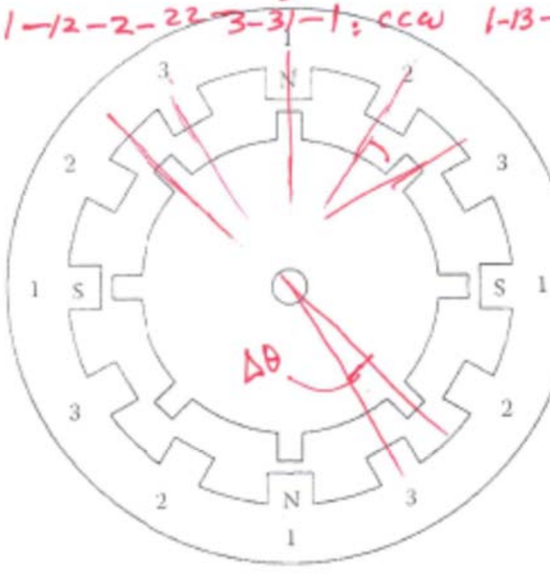
- Most classifications of stepper motors are *based on the nature of the motor rotor.*
- One such classification considers the magnetic character of the rotor. Specifically, as discussed before:
  - o VR stepper motor has a soft-iron rotor, whereas
  - o PM stepper motor has a magnetized rotor.
- Another practical classification that is based on the number of stacks of teeth (or rotor segments) present on the rotor shaft. In particular, an HB stepper motor has two stacks of teeth.
- Further *sub-classifications are possible, depending on the tooth pitch (angle between adjacent teeth) of the stator and the tooth pitch of the rotor*.
- In a *single-stack stepper motor*:
  - o The rotor tooth pitch and the stator tooth pitch generally have to be unequal
  - o So that *not all teeth in the stator are ever aligned with the rotor teeth at any instant*.
  - o It is the *misaligned teeth that exert the magnetic pull*, *generating the driving torque*.
  - o In each motion increment, the rotor turns to the minimum reluctance (stable equilibrium) position corresponding to that particular polarity distribution of the stator.
- In multiple-stack stepper motors:
  - o Operation is possible even when the rotor tooth pitch is equal to the stator tooth pitch, provided that at least one stack of rotor teeth is rotationally shifted (misaligned) from the other stacks by a fraction of the rotor tooth pitch.
  - o In this design, it is this inter-stack misalignment that generates the drive torque for each motion step. It is obvious that unequal-pitch multiple-stack steppers are also a practical possibility. In this design, each rotor stack operates as a separate single-stack stepper motor. The stepper motor classifications described thus far are summarized in Figure 8.8.



Full step ccw : 1-2-3-1 , 1-3-2-1 cw

$\frac{1}{2}$ step : 1-12-2-23-3-31-1 ; ccw 1-13-3-32-2-21-1 cw

Single-Stack Stepper Motors:



phase Winding $p = 3$.

Rotor teeth $n_r = 8$; Soft iron

Stator teeth $n_s = 12$

Poles per phase $m = 4$

Stator pitch $\theta_s = \frac{360}{n_s}$

Rotor pitch $\theta_r = \frac{360}{n_r}$

Step Angle $\Delta\theta$: Smallest Misalignment between stator pole and adjacent Rotor tooth in any stable equillibrium.

$$\left.\begin{aligned} \Delta\theta &= \theta_r - r\theta_s \quad (\theta_r > \theta_s) \\ \Delta\theta &= \theta_s - r\theta_r \quad (\theta_r < \theta_s) \end{aligned}\right\}$$

r is the largest positive integer such that $\Delta\theta > 0$

i.e. Largest feasible r such that a misalignment occurs.

Since $n_s = 12 \rightarrow \theta_s = \frac{360}{12} = 30^\circ$

$n_r = 8 \rightarrow \theta_r = \frac{360}{8} = 45^\circ$

$\Delta\theta = \theta_r - \theta_s = 15^\circ$; Each switching corresponds to rotation θ.

If phase-1 is off, phase-2 is ON ; Rotor will turn 15° ccw

If phase-3 is ON, Rotor will turn 15° cw

If phase-1 is ON & phase 2 is ON 7.5° is also possible.

Fullstepping 1-2-3-1 ccw ; 1-3-2-1 cw

$\frac{1}{2}$ stepping 1-12-2-23-3-31-1 ccw; 1-31-3-23-2-12-1 cw

Se have p phases meaning the whole switching angle is: $P\Delta\theta$; $\theta_r = P\Delta\theta$

P-switches.

From Original Equation $\Delta\theta = \theta_r - \gamma\theta_s$

Substituting $\theta_r = P\Delta\theta$ above.

$$\theta_r = r\theta_s + \frac{\theta_r}{P} \quad (\theta_r > \theta_s).$$

For Equation $\Delta\theta = \theta_s - \gamma\theta_r \ (\theta_r < \theta_s)$

$$\theta_s = \gamma\theta_r + \frac{\theta_r}{P} \quad (\theta_r < \theta_s).$$

θ_r = rotor tooth pitch angle

θ_s = stator tooth pitch angle

p = # of phases in stator

γ = Largest feasible positive Integer

pitch angle definitions. $\frac{360^\circ}{n_r} = \frac{r \times 360}{n_s} + \frac{360}{pn_r}$

or $n_s = rn_r + \frac{n_s}{p}$; $(n_s > n_s)$

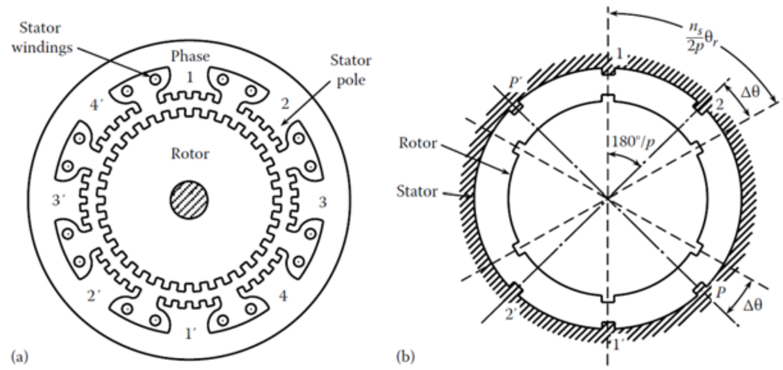
and $n_r = \gamma n_s + \frac{n_s}{p} \ (n_s < n_r)$

Finally # of ~~Revolutions~~ steps

$$n = \frac{360}{\Delta\theta}$$

Read Examples 8.1, 8.2, 8.3

Advantages of Toothed Construction:



The toothed construction of the stator and the rotor of a stepper motor has many advantages.

1. It improves the motion resolution (step angle), which now depends on the tooth pitch. Very small step angles can be achieved as a result.
2. It enhances the concentration of the magnetic field, which generates the motor torque. This means improved torque characteristics.
3. The torque and motion characteristics become smoother (smaller ripples and less jitter) as a result of the distributed tooth construction.

In the case shown in figure above, *the stator teeth are equally spaced but the pitch (angular spacing) is not identical to the pitch of the rotor teeth*.
In the toothed-stator construction, *ns represents the number of teeth rather than the number of poles in the stator*. The number of rotor teeth has to be increased in proportion.

Governing Equations:

Considering $\theta_r > \theta_s$ (i.e. $n_r < n_s$)

$\theta_r - \theta_s$: offset between rotor and stator pitch

# of rotor teeth n_s/mp

$\therefore \Delta\theta = \frac{n_s}{mp}(\theta_r - \theta_s)$; $\theta_r > \theta_s$; p: # of phases; m: # of stator poles per phase

$\Delta\theta = \frac{\theta_r}{p}$; True for toothed construction as well

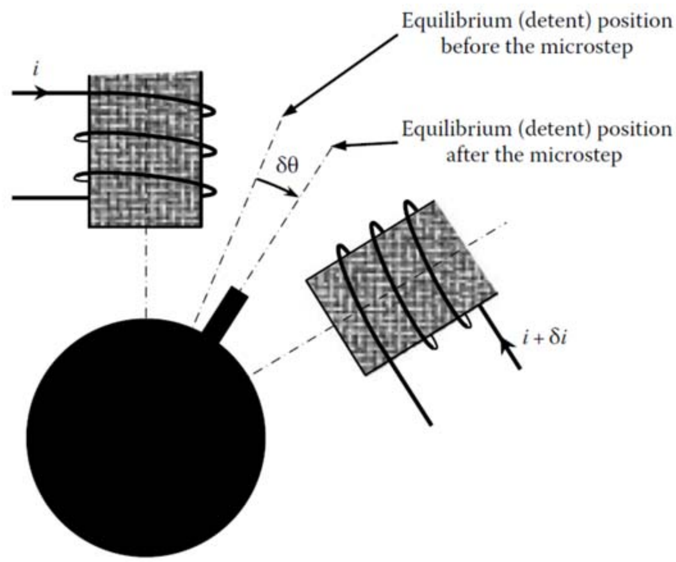
$\theta_r = p\Delta\theta$ and $n_s = m p \theta_s$

$\Delta\theta = \frac{n_s}{mp}(\theta_r - \theta_s) \Rightarrow n_s = n_r + m$ ($n_r < n_s$)

General Formula: $\boxed{n_s = n_r \pm m}$ # of stator Poles per phase

Read Example 8.4.

Micro-Stepping:



- Microstepping is achieved by properly changing the phase currents in small steps, instead of switching them on and off (as in the case of full stepping and half stepping).
- The principle behind this can be understood by considering two identical stator poles (wound with identical windings), as shown in diagram.

- When the *currents through the windings are identical (in magnitude and direction) the resultant magnetic field will lie symmetrically between the two poles*.

- If *the current in one pole is decreased while the other current is kept unchanged, the resultant magnetic field will move closer to the pole with the larger current*.

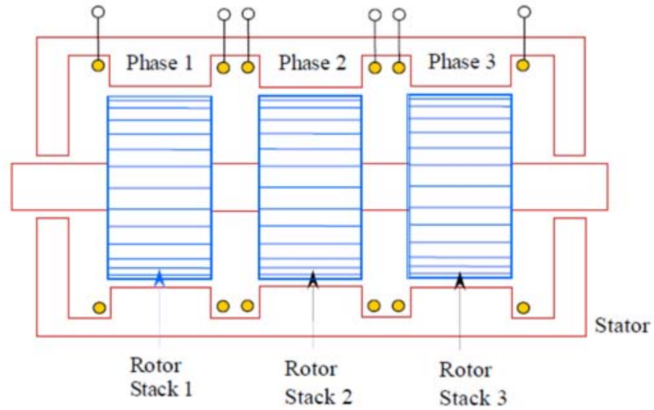
- As the detent position (equilibrium position) depends on the position of the resultant magnetic field, *it follows that very small step angles can be achieved simply by controlling (varying the relative magnitudes and directions of) the phase currents.*

- *Step angles of 1/125 of a full step* or smaller may be obtained through microstepping.

- *For example, 10,000 steps/revolution may be achieved.*

- Note: The step size in a sequence of microsteps is not identical. This is because stepping is done through microsteps of the phase current, which (and the magnetic field generated by it) has (have) a nonlinear relation with the physical step angle.

Multiple-Stack Stepper Motors



- Both equal pitch and unequal pitch constructions are possible.
- Smaller step angles are possible with unequal construction.
- For the equal pitch case, each stator segment has several poles and all poles of each stator segment are wound to the same phase.
- Misalignment can be achieved by two methods:
  1. The teeth in the *stator segments are aligned* but the *teeth in the rotor segments are misaligned* consecutively by 1/3 pitch angle.
  2. The teeth in the *rotor segments are aligned* but the *teeth in the stator segments are misaligned* consecutively by 1/3 pitch angle.

- Both full stepping and half stepping can be achieved.
- For full stepping the step angle is θs/3 = θr/3.
- Stepping sequence 1-2-3-1 would turn the rotor in one direction and
- 1-3-2-1 would turn in the other.
- For half stepping they are 1-12-2-23-3-31-1 and 1-13-3-32-2-21-1
- The full stepping angle if there are s-stacks:

$$\Delta\theta = \frac{\theta_r}{s} = \frac{\theta_s}{s} = \frac{\theta}{s}$$

- Very fine angular resolutions can be achieved with unequal multi-stack stepper motors. Switching logic of these stepper motors is more complex.
- Since the step angle of a non-toothed single stack stepper motor is (θr–θs)

$$\Delta\theta = \frac{\theta_r - \theta_s}{s}$$

- For a tooth-pole multi-stack motor:

$$\Delta\theta = \frac{n_s(\theta_r - \theta_s)}{mps} \qquad \Delta\theta = \frac{\theta_r}{ps}$$

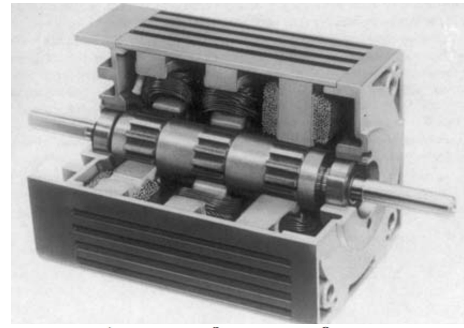
Equal Pitch and Unequal Pitch:

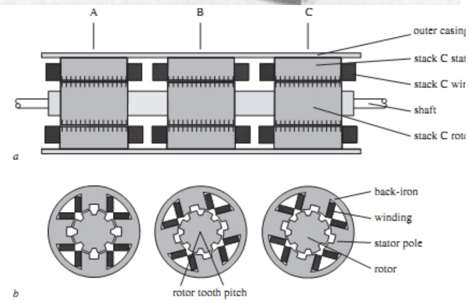
Equal-Pitch Multiple-Stack Stepper:

- For each rotor stack, there is a toothed stator segment around it, whose pitch angle is identical to that of the rotor ($\theta s = \theta r$).
- A stator segment may appear to be similar to that of an equal-pitch single-stack stepper, but this is not the case.
- Each stator segment is wound to a single phase, thus the entire segment can be energized (polarized) or de-energized (depolarized) simultaneously. It follows that, in the equal pitch case,
- Meaning $p = s$; where *p is the # of phases* and *s is the # of rotor stacks*.
- The misalignment that is necessary to produce the motor torque may be introduced in one of two ways:
  1. The teeth in the stator segments are perfectly aligned, but the teeth in the rotor stacks are misaligned consecutively by $1/s \times$ pitch angle.
  2. The teeth in the rotor stacks are perfectly aligned, but the teeth in the stator segments are misaligned consecutively by $1/s \times$ pitch angle.

- Now consider the three-stack case.
  - Suppose that phase 1 is energized
    - Then the *teeth in the rotor stack 1* will *align perfectly with the stator teeth in phase 1 (segment 1)*.
    - But the teeth in the rotor stack 2 will be shifted from the stator teeth in phase 2 (segment 2) by a one-third-pitch angle in one direction, and
    - The teeth in rotor stack 3 will be shifted from the stator teeth in phase 3 (segment 3) by a *two-thirds pitch angle* in the same direction (or a one-third-pitch angle in the opposite direction).
  - It follows that *if phase 1* is now de-energized and phase 2 is energized:
    - The rotor will turn through one-third pitch in one direction.
    - If, *instead, phase 3* is turned on after phase 1, the *rotor will turn through one-third pitch in the opposite direction*.
    - Clearly, the step angle (for full stepping) is a one-third-pitch angle for the three-stack, three-phase construction.
    - The switching sequence 1-2-3-1 will turn the rotor in one direction,
    - And the switching sequence 1-3-2-1 will turn the rotor in the opposite direction.

In general, for a stepper motor with s stacks of teeth on the rotor shaft, the full-stepping step angle is given by: $\Delta\theta = \frac{\theta}{s} = \frac{\theta}{p}$; where $\theta = \theta r = \theta s =$ tooth pitch angle.





- *Note* that the step angle can be decreased by increasing the number of stacks of rotor teeth.
- Increased number of stacks also means more phase windings with associated increase in the magnetic field and the motor torque.
- However, the length of the motor shaft increases with the number of stacks, and can result in flexural (shaft bending) vibration problems (particularly whirling of the shaft), air gap contact problems, large bearing loads, wear and tear, and increased noise.
- As in the case of a single-stack stepper, half stepping can be accomplished by energizing two phases at a time.
- Hence, in the three-stack stepper, for one direction, the half-stepping sequence is 1-12-2-23-3-31-1;
- In the opposite direction, it is 1-13-3-32-2-21-1.

Unequal-Pitch Multiple-Stack Stepper

- Very *fine angular resolutions (step angles) can be achieved* by this design without compromising the length of the motor.
- In an unequal-pitch stepper motor, *each stator segment has more than one phase (p number of phases).*
- Rather than a simple cascading, however, the phases of different stacks are not wound together and can be switched on and off independently. In this manner yet finer step angles are realized, together with an added benefit of increased torque provided by the multistack design.
- For a *single-stack non-toothed-pole stepper*, we have seen that the step angle is equal to $\theta r - \theta s$.
- In a multistack stepper, this misalignment is further subdivided into ***s*** *equal steps using the interstack misalignment.*
- Hence, the overall step angle for an unequal-pitch, multiple-stack stepper motor with *nontoothed poles* is given by:

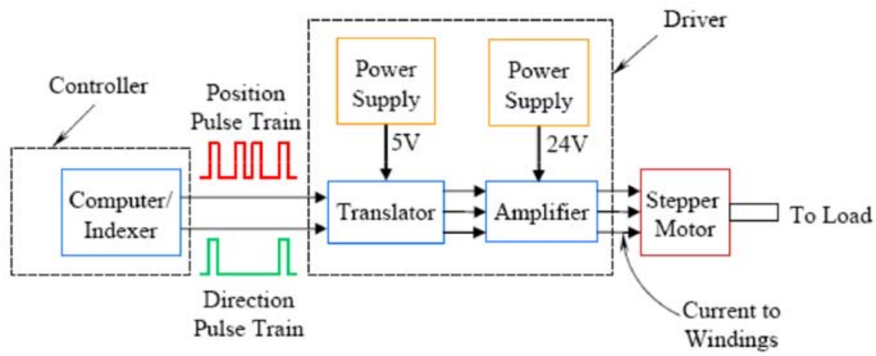
$$\Delta\theta = \frac{\theta_r - \theta_s}{s} \ for \ \theta_r > \theta_s$$

- For a toothed-pole multiple-stack stepper motor, we have:

$$\Delta\theta = \frac{n_s(\theta_r - \theta_{s)}}{mps} \ for \ \theta_r > \theta_s$$

  - *m* is the number of stator poles per phase.
  - p is the number of phases in each stator segment and
  - s is the number of rotor stacks and *s* is the number of rotor stacks.

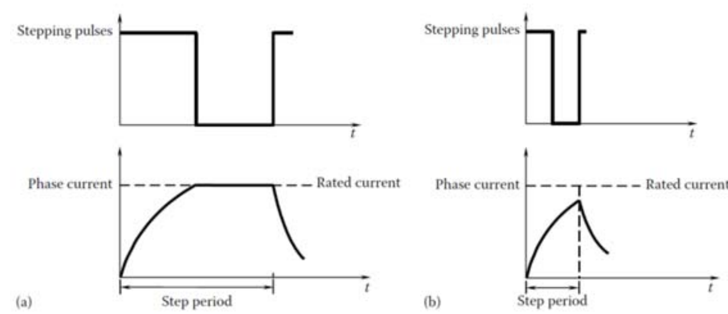
Controller and Driver:



Time Constant and Torque Degradation:

> *Self inductance* is defined as the *induction* of a voltage in a current-carrying wire when the current in the wire itself is changing. In the case of *self-inductance*, the magnetic field created by a changing current in the circuit itself *induces* a voltage in the same circuit. Therefore, the voltage is *self-induced*.

- As the *torque generated by a stepper motor* **is** *proportional to the phase current*.
- It is desirable for a phase winding to reach its maximum current level as quickly as possible when it is switched on.
- Unfortunately, as a result of *self-induction*, the current in the energized phase does not build up instantaneously when switched on.
- As the stepping rate increases, the time period that is available for each step decreases.
- Consequently, *a phase may be turned off before reaching its desired current level in order to turn on the next phase*, thereby *degrading the generated torque as shown in figure*.
- One way to increase the current level reached by a phase winding would be to simply increase the supply voltage as the stepping rate increases.



Low stepping Rate   Fast stepping rate

- Another approach would be to use a chopper circuit (a switching circuit) to switch on and off at high frequency, a supply voltage that is several times higher than the rated voltage of a phase winding.
- Specifically, *a sensing element (typically, a resistor) in the drive circuit detects the current level and when the desired level is reached, the voltage supply is turned* off.
- When the *current level goes below the rated level, the supply is turned on* again. The required switching rate (chopping rate) is governed by the electrical time constant of the motor. The electrical Time constant is given by: $\tau_e = \frac{L}{R}$

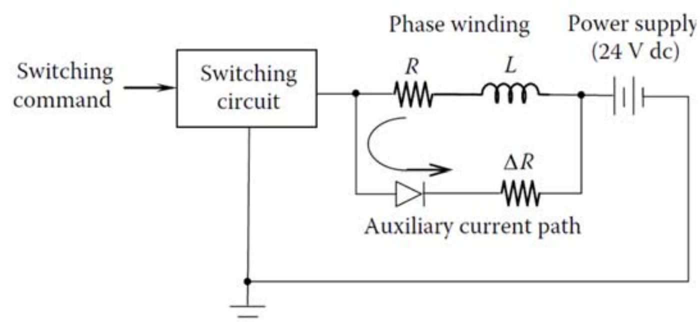
L- Inductance of the energized phase winding R - Resistance of the energized circuit, including winding resistance

The current increase (build-up) equation is given by: $i = \frac{v}{R}\exp\left(1 - \frac{t}{\tau_e}\right);\ \tau_e = \frac{L}{R}$

- The larger the electrical time constant the slower the current buildup.
- The driving torque of the motor decreases due to the lower phase current.
- Also, because of self-induction, the current does not die out instantaneously when the phase is switched off.

- The *torque characteristics of a stepper motor can be improved (particularly at high stepping rates)* and *the harmful effects of induced voltages can be reduced by decreasing the electrical time constant*.

- A convenient way to accomplish this is by increasing the resistance R.



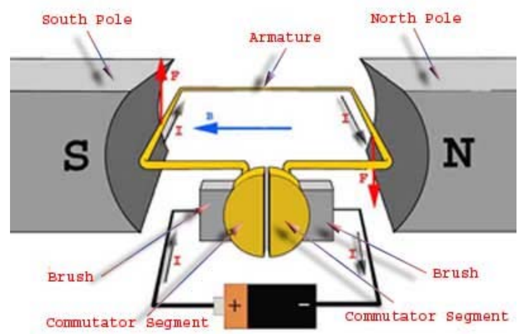
- Note that *we want this increase in R to be effective only during the transient periods (at the instants of switch-on and switch-off).*

- During *the steady period, we like to have a smaller R, which will give a larger current (and magnetic field), producing a higher torque*, and furthermore lower power dissipation (and associated mechanical and thermal problems) and reduction of efficiency.

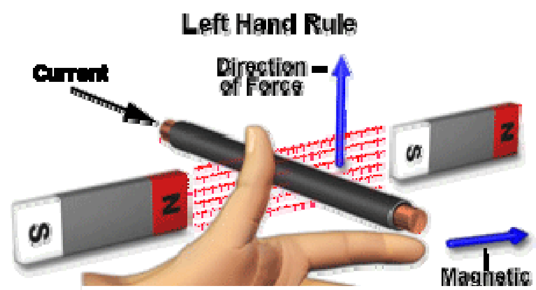
- This can be accomplished by using a diode and a resistor ΔR, connected in parallel with the phase winding, as shown in figure above.

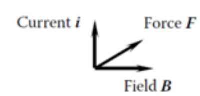
- In this case, *the current will loop through R and ΔR, as shown, during the switch-on and switch-off periods*, thereby decreasing the electrical time constant to: $\tau_e = \frac{L}{R+\Delta R}$

Chapter-9: DC Motors:



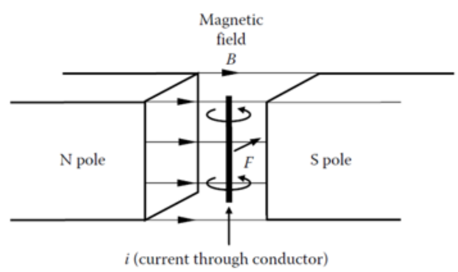
- A dc motor converts dc electrical energy into rotational mechanical energy.
- A major part of the torque generated in the rotor (armature) of the motor is available to drive an external load.
- DC motors are still widely used in numerous engineering applications including robotic manipulators, vehicles, transport mechanisms, disk drives, positioning tables, machine tools, biomedical devices, and servo-valve actuators.
- In view of effective control techniques that have been developed for ac motors, they are rapidly becoming popular in applications where dc motors had dominated. Still, dc motor is the basis of the performance of an ac motor which is judged in such applications.





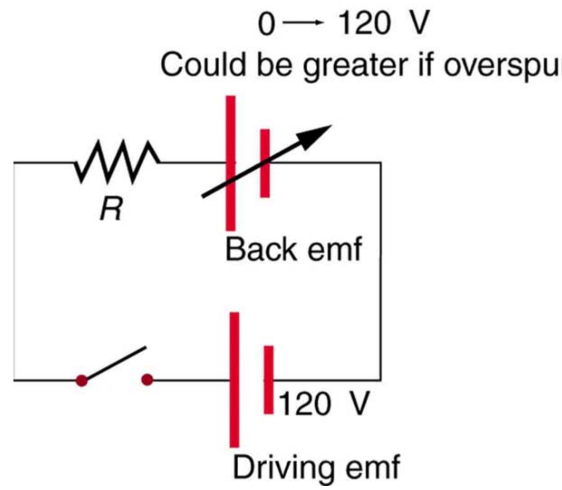
Principle of Operation:

The principle of operation of a dc motor is illustrated in figure shown.



- Consider an *electric conductor placed in a steady magnetic field at right angles to the direction of the field*.
- Flux density B is assumed constant.
- If a *dc current is passed through the conductor*, the *magnetic flux is formed due to the current* loops around the conductor, as shown in the figure.
- Consider a plane through the conductor, parallel to the direction of flux of the magnet.
- On one side of this plane, the current flux and the field flux are additive; on the opposite side, the two magnetic fluxes oppose each other. As a result, an imbalance magnetic force F is generated on the conductor, normal to the plane.
- This force (Lorentz's force) is given by the Lorentz's law: $F = B\,i\,l$
  - B is the flux density of the original field,
  - i is the current through the conductor and
  - l is the length of the conductor
- The active components of i, B, and F are mutually perpendicular and form a right-hand triad, as shown in figure. OR In other words, *in the vector representation of these three quantities*, the vector F can be interpreted as the cross product of the vectors **i** and **B**. Specifically, **F** = **i** × **B**.

- If the conductor is free to move, the generated force moves it at some velocity ***v*** in the direction of the force.
- As a result of this motion in the magnetic field **B**, a voltage is induced in the conductor. *This is known as the back electromotive force or back e.m.f.*, and is given by: $v_b = Blv$
- According to Lenz's law, the flux due to the back e.m.f. v_b opposes the flux due to the original current through the conductor, thereby trying to stop the motion. This is *the cause of electrical damping* in motors.

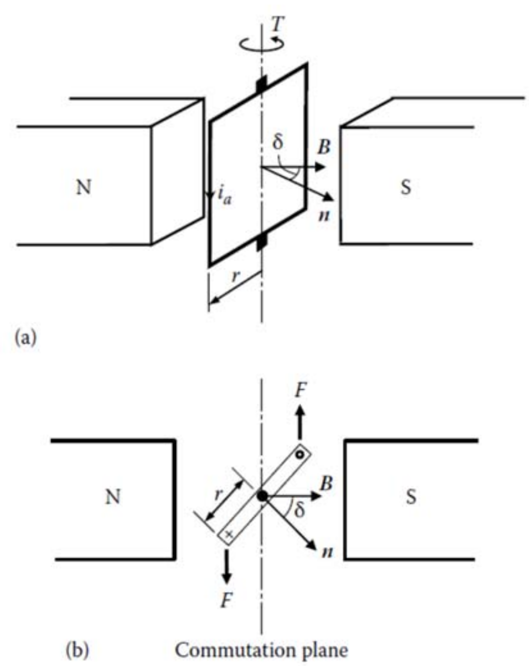


Static Torque Characteristics:
For static torque we *assume that the motor speed is low so that the dynamic effects need not be explicitly included* in the discussion.

- Consider a two-pole permanent magnet stator and a planar coil that is free to rotate about the motor axis, as shown in figure-a.
- The coil (rotor, armature) is energized by current i_a as shown.

- The flux density vector of the stator magnetic field is **B** and the unit vector normal to the plane of the coil is **n**.

- The angle between **B** and **n** is *δ, which is known as the torque angle*.

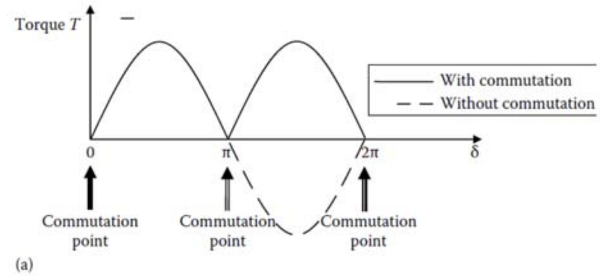


- It should be clear from figure-b that the torque T generated in the rotor is given by T = F × 2r sin δ.

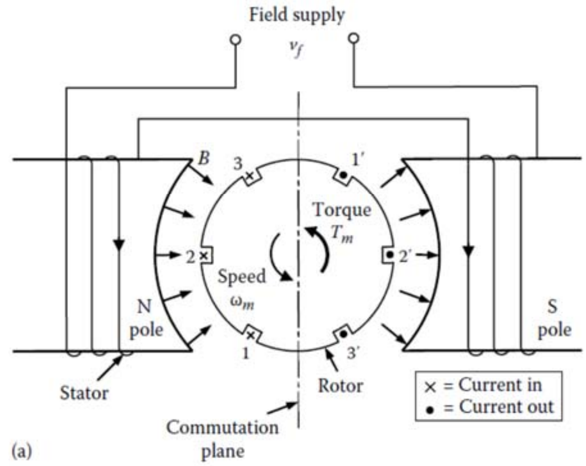
- Which becomes T = B$i_a l$ × 2r sin δ, or
- $T = A\, i_a B\ Sin\ \delta$;
    - l is the axial length of the rotor
    - r is the radius of the rotor
    - A is the face area of the planar rotor

Continuing from last page:

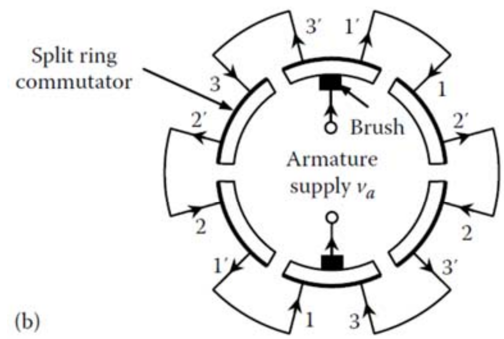
- Suppose that the rotor rotation starts by coinciding with the commutation plane, where $\delta = 0$ or π, and the rotor rotates through an angle of 2π.
- The corresponding torque profile is shown in figure on top.

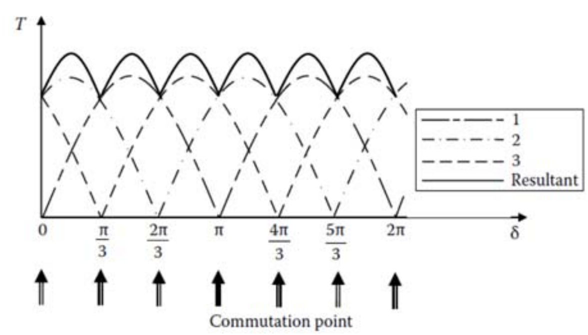


- Next suppose *that the rotor has three planar coil segments placed at 60° apart*, *and denoted by 1, 2, and 3, as in figure #2.*
- Note that current switching occurs at every 60° rotation, and in a given instant two coil segments are energized.



- Figure shows the torque profile of each coil segment and the overall torque profile due to the three-segment rotor in Figure#3.
- Note that the torque profile has improved (i.e., larger torque magnitude and smaller variation) as a result of the multiple coil segments, with shorter commutation angles.
- The *torque profile can be further improved by incorporating still more coil segments, with correspondingly shorter commutation angles*, *but the design of the split-ring and brush arrangement becomes more challenging then*.





- Hence, there is a design limitation to achieving uniform torque profiles in a dc motor.
- It should be clear from Figure#2 *that if the stator field can be made radial, then B is always perpendicular to n* and *hence sin δ becomes equal to 1*. In that case, the torque profile is uniform, under ideal conditions.

Brushless DC Motors:
Before we get into Brushless DC Motors, we should look at shortcomings of the slip-ring and brush mechanisms:

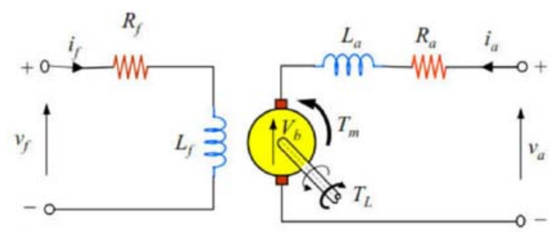
- Rapid wear out, mechanical loading,
- Heat generation due to sliding friction,
- Contact bounce,
- Excessive noise, and
- Electrical sparks with the associated dangers in hazardous (e.g., chemical) environments,
- Problems of oxidation,
- Problems in applications that require wash down (e.g., in food processing), and voltage ripples at current switching instants.
- Conventional remedies to these problems—such as the use of improved brush designs and modified brush positions to reduce sparking—are inadequate in more demanding and sophisticated applications.
- Cooling of the coils is typically needed in long-period operation of heavy-duty motors which may be achieved through forced convection of air or water.
- In addition, the required maintenance (to replace brushes and resurface the split-ring commutator) can be rather costly and time consuming.
- *Electronic communication, as used in brushless dc motors, is able to overcome these problems*.

Permanent-Magnet Motors:

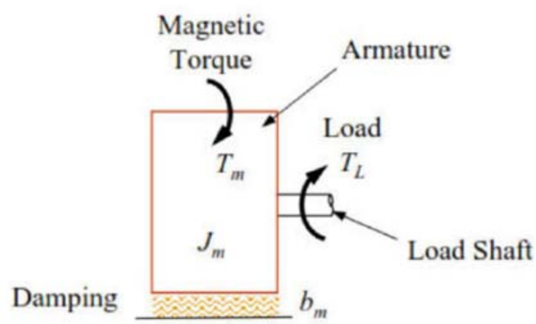
- Brushless dc motors have permanent-magnet rotors.
- Since in them the polarities of the rotor cannot be switched as the rotor crosses a commutation plane, commutation is accomplished by electronically switching the current in the stator winding segments.
- Note that this is the reverse of what is done in brushed commutation,
  - where the stator polarities are fixed and
  - The rotor polarities are switched when crossing a commutation plane.
- The stator windings of a brushless dc motor can be considered the armature windings, whereas for a brushed dc motor, rotor is the armature.
- The torque–speed characteristics of dc motors are different from those of stepper motors or ac motors.
- Brushless DC motors are commonly used for smooth torque transitions and speed control whereas, stepper motors are commonly used for stepping precision motion control.

DC Motor Equations:

- Consider a dc motor with separate windings in the stator and the rotor.
- Each coil has a resistance (R) and an inductance (L).
- When a voltage (v) is applied to the coil, a current (i) flows through the circuit, thereby generating a magnetic field.
- Forces are produced in the rotor windings, and an associated torque (T_m), which turns the rotor.
- The rotor speed (ω_m) causes the magnetic flux linkage with the rotor coil from the stator field to change at a corresponding rate, thereby generating a voltage (back e.m.f.) in the rotor coil.
- Equivalent circuits for the stator and the rotor of a conventional dc motor are shown.
- Since the field flux is proportional to the field current i_f, *we can express the magnetic torque of the motor as*: $T_m = k\, i_f i_a = k_m i_a$



Stator (Field Circuit)   Rotor (Armature Circuit)



using Equation $F = Bil$ and $V_b = Blv$

Back emf generated in Armature is (ideal/steady)

$$V_b = K' i_f \omega_m = K'_m \omega_m \; ; \quad K i_f = K_m$$

i_f = field Current

i_a = Armature Current

ω_m = angular speed of Motor

k, k' : Motor Constants : dependant on dimensions, turns, μ, Reluctance etc.

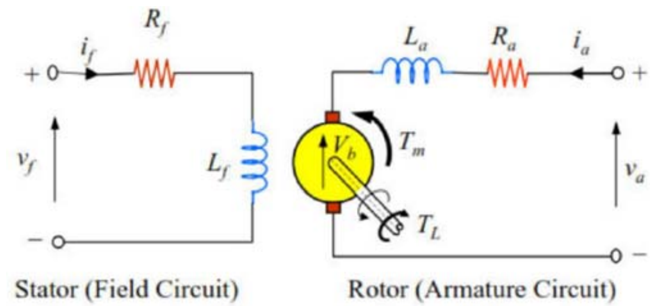
Note that: Ideal Electrical to Mechanical Conditions $\Rightarrow$ $\boxed{T_m \times \omega_m = V_b * i_a}$

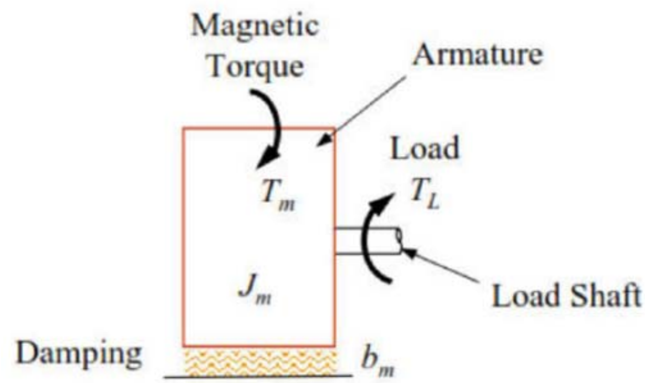
Newtons-m $\frac{rad}{sec}$ Volts   Amperes

Under ideal Conditions $K = K'$

or $K_m = K'_m$

Obtaining equations for Field Circuit, Armature Circuit and Mechanical Dynamic:





Field Circuit Equations:

- Assuming stator Magnetic field is NOT affected by the rotor Magnetic field.

⇒ STATOR 'L' not affected by Rotor and No EDDY CURRENTS in STATOR

Stator Supply Voltage → $V_f = R_f i_f + L_f \frac{di_f}{dt}$

field Winding Resistance, Field Winding Inductance

Armature Circuit

Equation for Armature (Rotor) circuit is

Armature Supply Voltage → $V_a = R_a i_a + L_a \frac{di_a}{dt} + V_b$

Armature Winding Resistance

Leakage inductance in Armature Windings

Mechanical Dynamics:

using Newton's 2nd Law to Rotor

- Motor drives some Load, requiring Torque T_L to operate
- Frictional Resistance in Armature can be modelled as

Moment of Inertia of Rotor → $J_m \cdot \frac{d\omega_m}{dt} = T_m - T_L - b_m \omega_m$

equivalent Mechanical damping constant for Rotor

Assumptions:
In obtaining equations for this dynamic model for the system, we have made several assumptions and approximations. In particular, we have either approximated or neglected the following factors:

1. Coulomb friction and associated *dead-band effects*.

2. Magnetic hysteresis (particularly in the stator core, but in the armature as well if not a brushless motor)

3. Magnetic saturation (in both stator and the armature)

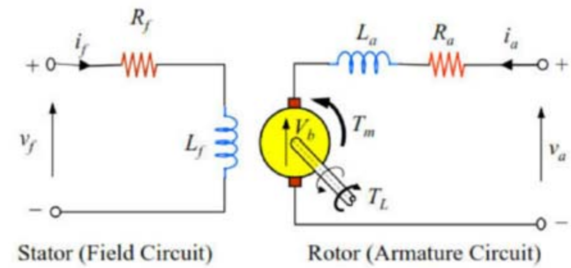
4. Eddy current effects (laminated core reduces this effect)

5. Nonlinear constitutive *relations for magnetic induction (in which case inductance L is not constant*)

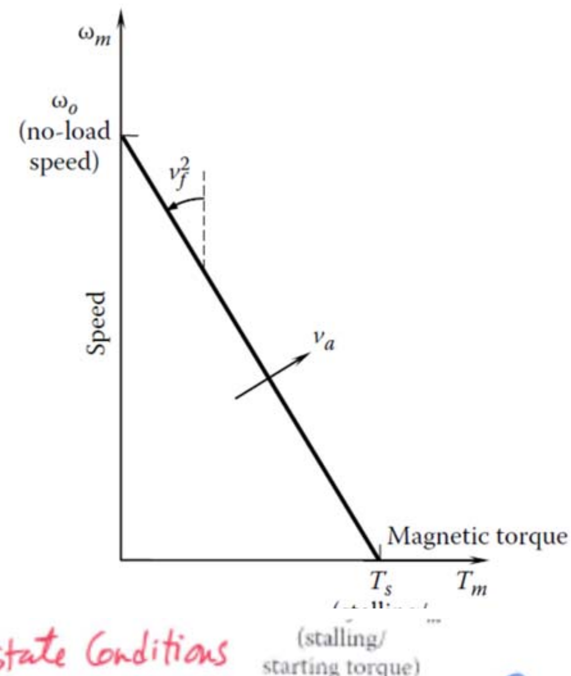
6. In split-ring and brush commutation, brush contact electrical resistance and friction, finite width contact of brushes, and other types of noise and nonlinearities

7. The effect of the rotor magnetic flux (armature flux) on the stator magnetic flux (field flux)

Steady-State Characteristics:



- In selecting a motor for a given application, its steady-state characteristics are a major determining factor.
- Steady-state torque–speed curves are employed for this purpose.
- The rationale is that, if the motor is able to meet the steady-state operating requirements, with some design conservatism, *it should be able to tolerate some deviations under transient conditions of short duration*.
- In the separately excited case shown in top figure, where the armature circuit and field circuit are excited by separate and independent voltage sources, it can be shown that the steady-state torque–speed curve is a straight line.



starting with Last page $V_f = R_f i_f + L_f \frac{di_f}{dt}$ — (1)

and $V_a = R_a i_a + L_a \frac{di_a}{dt} + V_b$ — (2)

Take derivative and Set = 0. ⇒ steady state Conditions

Note i_f is constant for fixed Supply V_f

Also previously $T_m = K i_f i_a = K_m i_a$ (3) and $V_b = K' i_f \omega_m = K'_m \omega_m$ (4)

Substituting (4) & (5) into (2) We get $\boxed{V_a = \frac{R_a}{K i_f} \cdot T_m + K' i_f \omega_m}$

Under steady state in field circuit from (1). $V_f = i_f R_f$

Torque Speed may be expressed as:

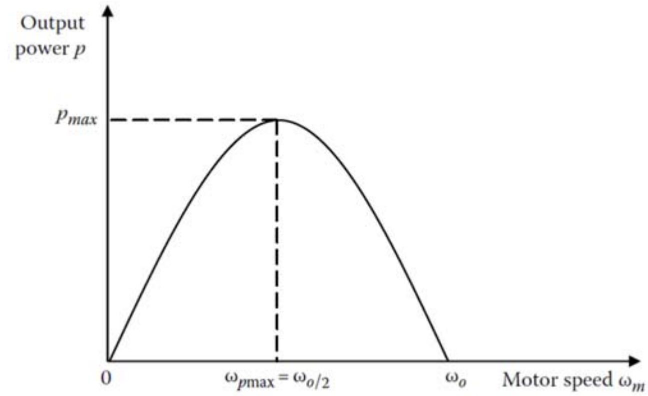
$$\omega_m + \frac{R_a R_f^2}{K^2 V_f^2} T_m = \frac{R_f V_a}{K V_f} \quad \text{or} \quad \omega_m + \frac{R_a}{K_m^2} T_m = \frac{V_a}{K_m}$$

→ V_a, V_f : Constant $\longrightarrow$ defining T_s (Stalling/starting) Torque, ω_0 (no-load speed & no-damping $b_m = 0$)

$$\frac{\omega_m}{\omega_0} + \frac{T_m}{T_s} = 1$$

Read Example 9.1 Pg. 667 text

Output Power:



Output Power of a motor is given by

$$P = T_m \omega_m \quad (1)$$

using

$$\frac{\omega_m}{\omega_0} + \frac{T_m}{T_S} = 1 \quad (2)$$

T_m: Magnetic Torque

T_S: stalling Torque or starting

ω_m: [mo]tor [an]gular speed

ω_0: no load speed

Substitute (2) in (1)

$$P = T_s\left(1 - \frac{\omega_m}{\omega_0}\right)\omega_m \quad (3)$$

shown in graphic form as quadratic shape

Pt. of Max Power: differentiate (3) w.r.t speed & = 0.

$$\therefore \frac{dP}{d\omega_m} = T_s\left(1 - \frac{\omega_m}{\omega_0}\right) - \frac{T_s}{\omega_0}\omega_m = T_s\left(1 - 2\frac{\omega_m}{\omega_0}\right) = 0$$

$$\therefore \omega_{p(max)} = \frac{\omega_0}{2}$$

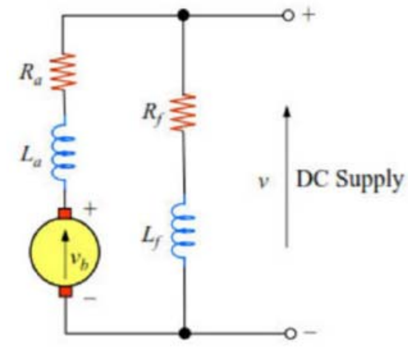
; Max Power is given by $\frac{1}{2}$ no-load speed.

& The max Power value is

$$P_{max} = \frac{1}{4} T_s \omega_0$$

Combined Excitation of Motor Windings:

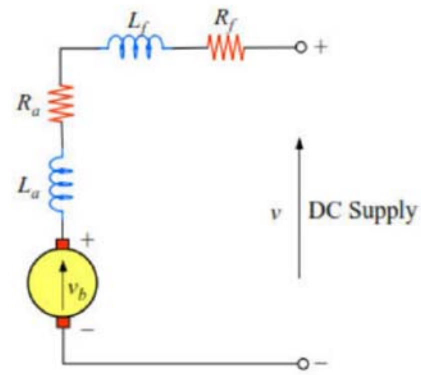
The shape of the steady-state speed–torque curve will change if a common voltage supply is used to excite both *the field windings* and *the armature windings*. Here, the two windings have to be connected together.



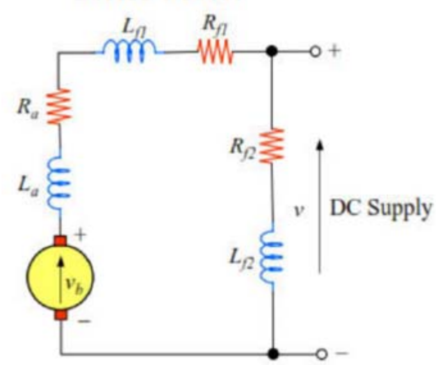
Shunt Motor

There are three common ways the windings of the rotor and the stator are connected.

1. Shunt-wound motor
2. Series-wound motor
3. Compound-wound motor



Series Motor



Compound Motor

- *In a shunt-wound motor*, the armature windings and the field windings are connected in parallel.
- *In the series-wound motor*, they are connected in series.
- *In the compound-wound motor*, part of the field windings is connected with the armature windings in series and *the other part is connected in parallel.*
- Note that in a shunt-wound motor at steady state, the back e.m.f. v_b depends directly on the supply voltage.
- Since the *back e.m.f. v_b is proportional to the speed*, it follows that *speed controllability is good with the shunt-wound configuration.*
- In a series-wound motor,
  - *The relation between v_b and the supply voltage is coupled through both the armature windings and the field windings*.
  - Hence its *speed controllability is relatively poor*.
  - But in this case, a relatively large current flows through both windings at low speeds of the motor (when the back e.m.f. is small), *giving a higher starting torque*.
  - Also, the *operation is approximately at constant power in this case*. These properties are summarized in Table.
  - Since both speed controllability and higher starting torque are desirable characteristics, *compound-wound motors* are used to obtain a performance in between the two extremes.

TABLE 9.1 Influence of the Winding Configuration on the Steady-State Characteristics of a DC Motor

DC Motor Type	Field Coil Resistance	Speed Controllability	Starting Torqu
Shunt-wound	High	Good	Average
Series-wound	Low	Poor	High
Compound-wound	Parallel high, series low	Average	Average

Speed Regulation:
Variation in the operating speed of a motor due to changes in the external load is measured by the percentage speed regulation. Specifically,

$$Percent\ Speed\ Regulation = \frac{\omega_o - \omega_f}{\omega_f} * 100\%$$

$$where, \boldsymbol{\omega_o}\ is\ the\ no-load\ speed\ ;\ \boldsymbol{\omega_f}\ is\ the\ full-load\ speed$$

- This is a measure of the speed stability of a motor;
  - The *smaller the percentage speed regulation*, *the more stable the operating speed under varying load conditions* (particularly in the presence of load disturbances).
  - In the shunt-wound configuration, *the back e.m.f.*, and hence the rotating speed, depends:
    - Directly on the supply voltage.
    - Consequently, the armature current and the related motor torque have virtually no effect on the speed.
    - The *percentage speed regulation is relatively small for shunt-wound motors*, resulting in improved speed stability.

HWK - Read Example 9.3

Electrical Damping Constant:
Newton's second law governs the dynamic response of a motor. Looking at previous equation: Jm as Motor Moment of Inertia is given by :

$$J_m \frac{d\omega_m}{dt} = T_m - T_L - b_m\omega_m$$

Mechanical damping Constant

- b_m denotes the mechanical (viscous) damping constant and represents mechanical dissipation of energy.
- As is intuitively clear, mechanical damping torque opposes motion - hence the negative sign in the $b_m\omega_m$ term in equation.
- The *magnetic torque* T_m of the motor is also dependent on speed ω_m.
- In particular, the back e.m.f., which is governed by ω_m, produces a magnetic field, which tends to oppose the motion of the motor rotor.
- This acts as a damper, and the corresponding damping constant is given by

$$b_e = -\frac{\partial T_m}{\partial \omega_m}\ ;$$

Electrical damping Constant

Control of DC Motors:

- Both speed and torque of a dc motor may have to be controlled for proper performance in a given application of a dc motor.
- By using *proper winding arrangements, a dc motor can be operated over a wide range of speeds and torques*.
- Because of this adaptability, dc motors are particularly suitable as variable-drive actuators.
- Since variable-speed control of a dc motor is quite convenient and straightforward, dc motors have dominated in industrial control applications for many decades.
- Following *a specified motion trajectory is called servoing, and servomotors (or servo actuators) are used for this purpose*. The vast *majority of servomotors are dc motors with feedback control of motion*.
- Servo control is essentially a motion control problem, which involves the control of position and speed.
- There are applications, however, that require torque control, directly or indirectly, but they usually require more sophisticated sensing and control techniques.

Armature Control and Field Control

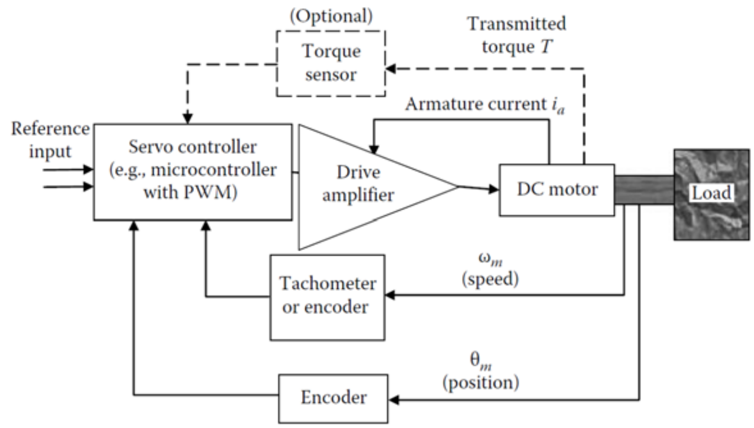
Armature Control

- Control over the voltage of the armature to control speed.
- To keep speed constant, voltage is kept constant.

Field Control

- Control current of the armature and then control back e.m.f. v_b

DC Servomotors:



What is a Servomotor?

- Motors with motion feedback control.
- Able to follow a specified motion trajectory.

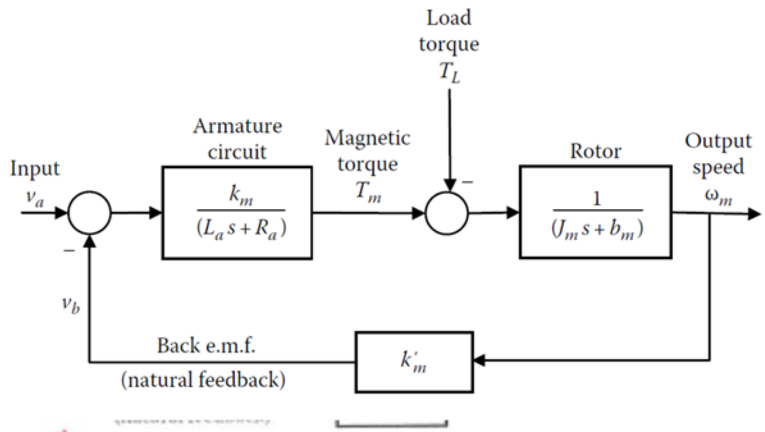
Requirements in DC servomotor system:

- Both angular position and speed might be measured (*using shaft encoders, tachometers, resolvers, rotary-variable differential transformers (RVDTs*), potentiometers, and compared with the desired position and speed.
- The error signal (= [desired response] − [actual response]) is *conditioned and compensated using analog circuitry or is processed by a digital hardware processor or microcontroller, and is supplied to drive the servomotor toward the desired response*.
- Need *feedback and velocity for accurate position control*.
- Velocity feedback alone might be adequate for speed control, but position error can build up.
  - If *only position feedback* is used, a large error in velocity is possible, even when the position error is small.
  - Under certain conditions (e.g., high gains, large time delays), with position feedback alone, the control system *may become marginally stable or even unstable*.
  - DC servo systems *historically employed tachometer feedback* (velocity feedback) in addition to other types of feedback, primarily position feedback.

- In schematic above:
  - The actuator in this case is a dc motor.
  - *The sensors include a tachometer to measure angular speed,*
    - *A potentiometer to measure angular position, and*
    - *A strain gauge torque sensor, which is optional.*
  - A *single optical encoder* for *both angular position and speed*. *This avoids the need for an analog-to-digital converter (ADC) for digital control*. *An ADC is still needed when an analog tachometer and torque sensor are used*.
  - *Driven system process is represented by the load block in the figure*.
  - Signal-conditioning (filters, amplifiers, etc.) and compensating (lead, lag, etc.) circuitry are represented by a single block.

Armature Control dc Motor:

- Armature voltage v_a is the control input, while keeping the conditions in the field circuit constant.
- The field current i_f (or, the magnetic field in the stator) is assumed constant.



Equations between Torque and Current can be re-written as:

$T_m = K_m i_a$; $V_b = K'_m \omega_m$; $K_m = K'_m$: ideal Mechanical to electrical Conversion.

Torque Constant

Back emf Constant.

Prev. Equation Armature Control: $V_a = R_a i_a + L_a \frac{di_a}{dt} + V_b$

In Laplace domain: $V_a - V_b = (L_a s + R_a) i_a$

Prev. Eqn in Mechanical domain: $J_m \frac{d\omega_m}{dt} = T_m - T_L - b_m \omega_m$

Rotor Moment of Inertia

equivalent Mechnical damping constant

* In Laplace domain: $T_m - T_L = (J_m s + b_m)\,\omega_m$ ← Motor output

* If need θ_m Motor Position : obtained by passing ω_m through $1/s$ integration block

* $T_L \uparrow$ with ω_m ; Need Large Torque to drive Load to higher Speed.

* Figure is NOT a feedback Control System.

* Back emf : is a Natural feedback and is Characteristic of the DC Motor — NOT AN EXTERNAL Control feed back loop.

using Superposition: over-all relation:

$$\omega_m = \frac{K_m}{\Delta(s)} V_a - \frac{(L_a s + R_a)}{\Delta(s)} T_L$$

Characteristic Polynomial of System

$$\Delta s = (L_a s + R_a)(J_m s + b_m) + K_m K'_m$$

Equal Under Ideal Cond.

Armature Time Const.
$\tau_a = L_a / R_a$
Mechanical Time Const.
$\tau_m = J_m / b_m$

Example 9.5

Determine an expression for the dominant time constant of an armature-controlled dc motor. What is the speed behavior (response) of the motor to a unit step input in armature voltage, in the absence of a mechanical load?

Sol: starting with Δs Equation

$$\Delta s = (L_a s + R_a)(J_m s + b_m) + k_m k_m'$$

neglecting Electrical Time Constant

$$\Delta s = R_a(J_m s + b_m) + K_m K_m'$$

$$= k'(\tau s + 1);$$

τ being overall dominant time Constant

$$\tau = \frac{R_a J_m}{(R_a b_m + K_m K_m')}$$

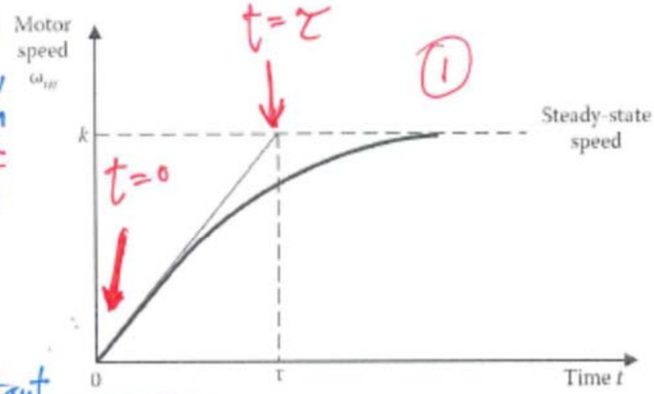
If $T_L = 0$; Motor transfer relation

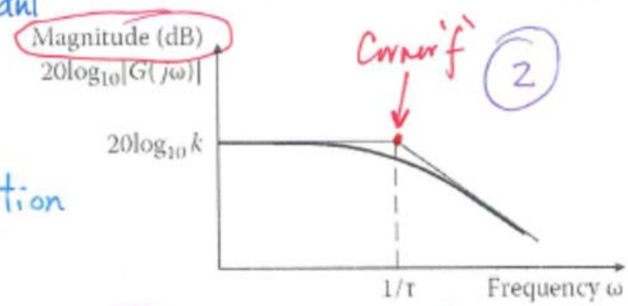
$$\omega_m = \frac{K}{\tau s + 1} v_a$$

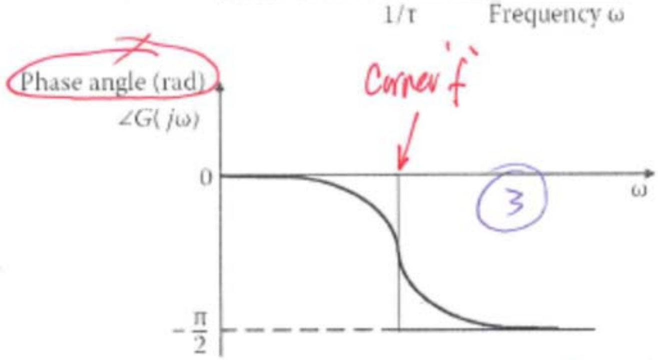
K, dc gain $= \dfrac{K_m}{R_a b_m + K_m K_m'}$

Synthesize

$$\tau \frac{d\omega_m}{dt} + \omega_m = K v_a$$







$\Rightarrow$ $\omega_m(t) = K(1 - e^{-t/\tau})$ $\Rightarrow$ Speed Response to Unit step change in V_a; & Zero initial Conditions.

!! non-oscillatory Response: Mathematically but practically will have in practise

differentiate, then $t=0 \Rightarrow \frac{d\omega_m}{dt}(0) = \frac{K}{\tau}$ $\rightarrow$ See ① above.

Alternately using $\omega_m = \dfrac{K}{(\tau s + 1)} V_a \longrightarrow G(j\omega) = \dfrac{K}{(\tau j\omega + 1)}$

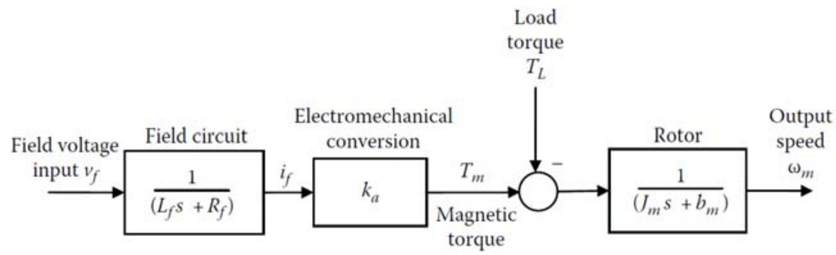
DC Gain $\Rightarrow 20 \log_{10} K$; Corner frequency $= \omega_c = \dfrac{1}{\tau}$

Page - 141 | ENSC387 - Introduction to Electro-Mechanical Sensors and Actuators: Simon Fraser University – Engineering Science

Read Example 9.6, 9.7

Field Control

In field-controlled dc motors, the armature voltage is kept constant, and the field voltage is used as the control input. It is assumed that armature current i_a (and the rotor magnetic field) is maintained constant. (Note: Leakage inductance in the armature circuit, and the associated voltage drop is negligible as well.) Ka: Electromechanical constant



Electromechanical

$\therefore T_m = K_a i_f$

Grabbing Previous Equation Elec. + Mechanical

$$V_f = R_f i_f + L_f \frac{di_f}{dt} \quad \text{and} \quad J_m \frac{d\omega_m}{dt} = T_m - T_L - b_m \omega_m$$

in Laplace form →

$$V_f = (L_f s + R_f) i \quad \text{and} \quad T_m - T_L = (J_m s + b_m)\omega_m$$

Figure above represents Equations in the open-Loop block diagram.

Note: i_a: assumed Constant (not in Practise); V_a: Kept Constant, L_a: neglected, i_a depends on back emf: $V_b \propto$ motor Speed and i_f

Assuming Linearity and Superposition principle

Constant gain

$$\omega_m = \frac{K_a}{(L_f s + R_f)(J_m s + b_m)} V_f - \frac{1}{(J_m s + b_m)} T_L$$

Electrical Time Constant: $\tau_f = \frac{L_f}{R_f}$

Mechanical Time Constant: $\tau_m = \frac{J_m}{b_m}$

True Time Constants ∵ Mech and Elec. uncoupled

characteristic Polynomial is $\Delta s = (L_f s + R_f)(J_m s + b_m)$

Feedback Control of DC Motors

- Open-loop operation of a dc motor, *for armature control and for field control*, *can lead to excessive error and even instability*:
  - Because of the unknown load input, and
  - Due to the *integration effect when position* (not speed) is the desired output (as in positioning applications).
- Feedback control is necessary under these circumstances:
  - In feedback control, *the motor response (position, speed, or both) is measured using an appropriate sensor* and
  - Fed back into the motor controller, which generates the control signal for the drive hardware of the motor.
- An optical encoder (Ch6) can be used to sense both position and speed and a tachometer may be used to measure the speed alone (Ch5). The following three types of feedback control are important:
  1. Velocity feedback
  2. Position plus velocity feedback
  3. Position feedback with a multi-term controller

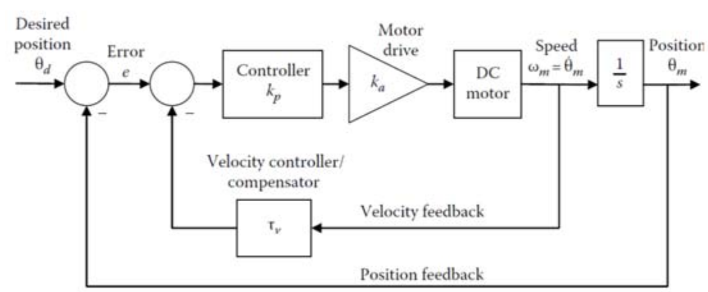
Velocity Feedback Control

- Motor speed is sensed using a device (*tachometer or an optical encoder*):
  - *Fed back to the controller,*
  - *Which compares it with the desired speed, and*
  - *The error is used to correct the deviation.*
- Additional *dynamic compensation* (e.g., lead compensation or lag compensation) may be *needed to improve the accuracy* and *the effectiveness of the controller*, and can be provided using either analog circuits or digital processing.
- The *error signal is passed through the compensator in order to improve the performance* of the control system.

Position Plus Velocity Feedback Control:

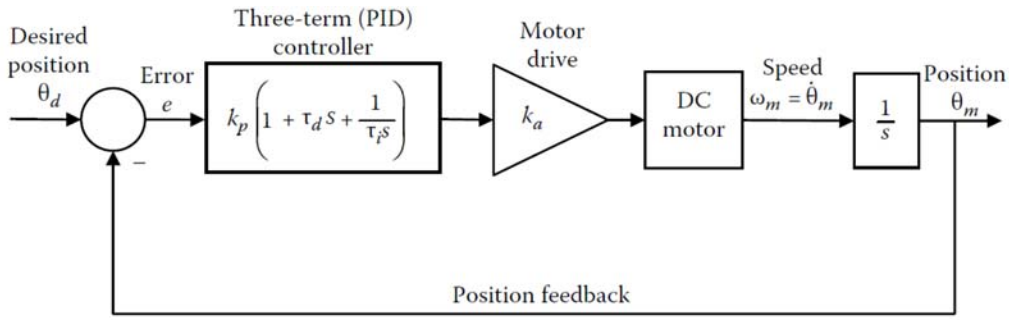
In position control, the motor angle θ_m is the output & the characteristic polynomial is s (τs + 1).

- This is *a marginally stable system*, in view of the pole at the origin (s = 0).
- *If a slight disturbance* or *model error is present, it will be integrated out, which can lead to a diverging error in the motor angle*.
- In particular, the load torque T_L is an input to the system, and is unknown and therefore a possible disturbance.



- In view of the *free integrator associated with the position output*, the *resulting unstable behavior cannot be corrected using velocity feedback alone*. Position feedback is needed to remedy the problem and therefore both position and velocity feedback are needed.

Position Feedback with PID Control

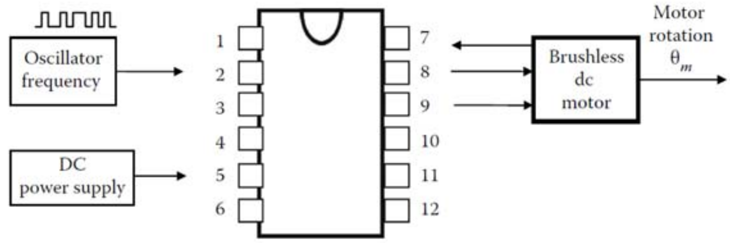


- A popular method of controlling a dc motor is to use just position feedback, and then compensate for the error using a three-term controller having:
  - Proportional,
  - Integral, and
  - Derivative (PID) actions.

- A block diagram for this control system is shown in figure above.

- Each term of the PID controller *provides specific benefits and some undesirable side effects* as well.
  - In particular, *proportional* action improves the speed of response and reduces the steady-state error but it tends to increase the level of overshoot (i.e., system becomes less stable).
  - *Derivative action adds damping*, just like velocity feedback, thereby making the system more stable (less overshoot). In doing so, it does not degrade the speed of response, however, which is a further advantage. But, the *derivative action amplifies high-frequency noise and disturbances*.
  - Strictly speaking, *a pure derivative action is not physically realizable using analog hardware*.
  - The *integral action reduces the steady-state error (typically reduces it to zero*), but *it tends to degrade the system stability and the speed of response*.

Phase-Locked Control



- *The position command is a frequency input, which is generated according to the desired (specified) motion (rotation) of the motor, using a microcontroller. This identifies a reference signal in the form of a pulse train.*

The rotation of the motor (and hence the commutating instant) is sensed using Hall-effect current, which also corresponds to pulse train (actual motor rotation θ_m).

Pin designation
1. Phase-lock indicator
2. Oscillator input
3. Disable
5. DC power input
6. Ground
7. Hall-effect sensing for coil A
8. Hall-effect sensing for coil B
9. Current to stator coil A
10. Current to stator coil B

- *The reference pulse train and the actual rotation pulse train signal are compared by a phase detector in the control IC package.*
- *The objective is to maintain a fixed phase difference (ideally, a zero phase difference) between the reference pulse signal and the actual position pulse signal. Under these conditions, the two signals are synchronized or phase-locked together.*
- *Any deviation from the locked conditions generates an error signal, which brings the motor motion back in phase with the reference command.*
- *In this manner, deviations due to external disturbances, such as load changes on the motor, are also corrected.*
- *In phase-locked control, the phase angle of the output is locked to the phase angle of the command signal.*
- *Very accurate position control can be realized by driving the phase difference to zero. In more sophisticated phase-locked servos, the frequency differences are also detected and compensated. This is analogous to the classic PPD control.*
- *Phase-locked servos are velocity control devices as well, because velocity is proportional to the pulse frequency.*
  - *When the two pulse signals are synchronized, the velocity error also approaches zero, subject to the available resolution of the control system components.*
  - *Typically, speed error levels of ± 0.002% or less are possible using phase-locked servos.*
  - *The overall cost of a phase-locked servo system is usually less than that of a conventional analog servo system, because less-expensive solid-state IC devices replace bulky analog control circuitry.*

Phase Difference Sensing

One method of determining the phase difference of two pulse signals is by detecting the edge transitions (as in Ch6). An alternative method is to take the product of the two signals and then low-pass filter

Let Reference pulse be $u_o \sin\phi_u$ and response pulse be $y_o \sin\phi_y$

$\theta_u = \omega t + \phi_u$ and $\theta_y = \omega t + \phi_y$; $\omega \Rightarrow$ frequency (same) of 2 signals
$\phi \Rightarrow$ phase angle.

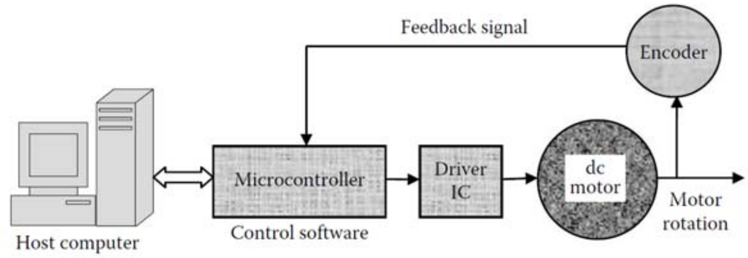
Product: $P = u_o y_o \sin\theta_u \sin\theta_y = \frac{1}{2} u_o y_o [\cos(\theta_u - \theta_y) - \cos(\theta_u + \theta_y)]$

$= \frac{1}{2} u_o y_o \cos(\theta_u - \theta_y) - \frac{1}{2} u_o y_o \cos(2\omega t + \phi_u + \phi_y)$ gone

Low pass filter $\Rightarrow 2\omega t \rightarrow$ gone

$\Rightarrow e = \frac{1}{2} u_o y_o \cos(\phi_u - \phi_y)$

Motor Driver and Feedback Control:



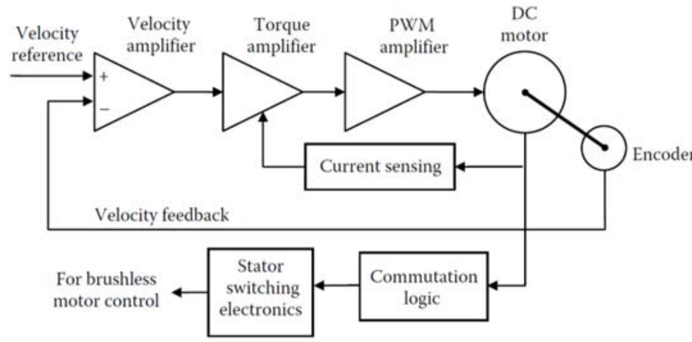
- The feedback control system of a dc motor typically consists of a microcontroller, *which provides drive commands (rotation and direction) to the driver*.
- The driver is a hardware unit, typically an IC package, which generates the necessary current to energize the windings of the motor. The *motor torque can be controlled by controlling the current generated by the driver*.
- By *receiving feedback from a motion sensor (encoder, tachometer, etc.)*, the *microcontroller can control the angular position and the speed of the motor*.

Driver Hardware:

- Main hardware component of the motor drive system is the driver IC package.
- In traditional motion control applications, there are amplifiers called *drive amplifiers or servo amplifiers*, *which are included in the drive hardware*.
- The name servo amplifier is used specifically when feedback signals are received by it for proper servoing (*following a motion trajectory*).
- Two basic types of drive amplifiers are commercially available:
  1. *Linear amplifier*
  2. *PWM amplifier*

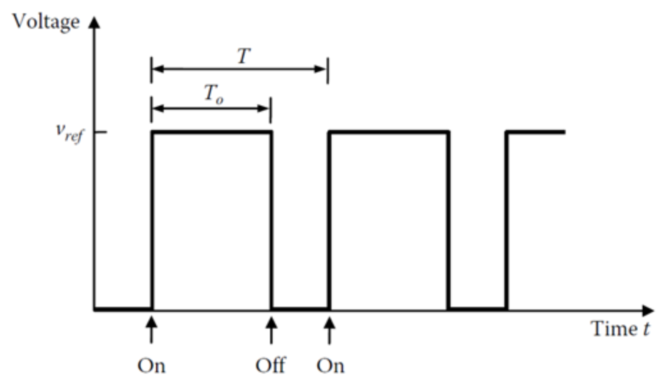
- A linear amplifier generates a voltage output:
  - Which is proportional to the input provided to it.
  - Since the output voltage is proportioned by dissipative means (using resistor circuitry), *this is a wasteful and inefficient approach.*
  - Fans and heat sinks have to be provided to remove the generated heat, particularly in continuous, long-term operation.
- **Example:**
  - To understand the inefficiency associated with a linear amplifier, suppose that the operating output range of the amplifier is 0–20 V, and that the amplifier is powered by a 20 V power supply. Under a particular operating condition, suppose that the motor is applied 10 V and draws a current of 4 A. The power used by the motor then is 10×4 W = 40 W.
  - Still, the power supply provides 20 V at 5 A, thereby consuming 100 W. This means, 60 W of power is dissipated, and the efficiency is only 40%. *The efficiency can be made close to 100% using modern PWM amplifiers*, which *are non-dissipative devices*, and *depend on high-speed switching at constant voltage to control the power supplied to the motor*.

- *Integrated microelectronic design makes them compact accurate, and inexpensive*. The components of a typical PWM-drive system are shown in the diagram.



- Other signal-conditioning hardware (e.g., filters) and auxiliary components such as isolation hardware, safety devices including tripping hardware, and cooling fan are not shown in the figure, but note the following components, connected in series:
  1. *A velocity amplifier (a differential amplifier)*
  2. *A torque amplifier*
  3. *A PWM amplifier*

- The *reference velocity signal* and *the feedback signal (from an encoder or a tachometer)* are used by the velocity amplifier.
- The *resulting difference (error signal) is conditioned and amplified by the torque amplifier to generate a current corresponding to the required torque* (corresponding to the driving speed).
- The *motor current is sensed and fed back to this amplifier, to improve the torque performance of the motor*.
- The *output from the torque amplifier is used as the modulating signal to the PWM amplifier*.
- The *PWM is accomplished by varying the duty cycle of the generated pulse signal, through switching control*.



- Chopper circuits that use discrete thyristor elements (a solid-state switch that is also known as *silicon-controlled rectifier or SCR)* were commonly used to generate PWM signals to control dc motors.
- Since a *chopper circuit takes dc power and switches it to different levels at some frequency, it is like converting dc to ac*. Hence, it called an inverter circuit.

Looking @ Diagram Duty Cycle = $\frac{T_o}{T} * 100\%$; T_o = ON Period
T = Pulse Period
Keeping V_{ref} and Pulse frequency $\frac{1}{T}$ FIXED ; VARY T_o
∴ PWM achieved by chopping V_{ref} over part of switching period.
Meaning Average voltage is varied → Proportional to T_o.

Duty cycle = $\frac{\text{Average Output}}{\text{Peak output}} * 100\%$

Page - 147 | ENSC387 - Introduction to Electro-Mechanical Sensors and Actuators: Simon Fraser University – Engineering Science

Practise 9.1 → 9.11 chapter 9. 9.2, 9.3, 9.5, 9.8, 9.11

Motor Selection Criteria

- Mechanical data:
  - Rated torque
  - Mechanical time constant
  - No load speed/ Full load speed
  - Maximum acceleration at peak torque
  - Rated output power
  - Frictional torques
  - Damping
  - Moment of inertia
  - Dimensions
- Electrical data:
  - Electrical time constant
  - Input power
  - Armature resistance and inductance
  - Field Resistance and inductance
  - Compatible drive circuit data (V, I, etc.)
- General Data
  - Brush life
  - Motor life
  - Efficiency (input/output power)
  - Operating temperature, humidity, etc
  - Mounting configuration

Induction Motors

- Because of the rapid improvement, ac motors have managed to replace dc motors in many industrial applications until the revival of the dc motor, particularly as a servomotor in control system applications.
- AC motors are generally more attractive than conventional dc motors, in view of their robustness, lower cost, simplicity of construction, and easier maintenance, especially in heavy duty (high-power) applications (e.g., rolling mills, presses, vehicle drives, elevators, cranes, material handlers, and operations in paper, metal, petrochemical, cement, and other industrial plants).

Advantages: Some advantages of ac motors are as follows:

- Cost-effectiveness
- Convenient power source (standard power grid providing single-phase and three-phase ac supplies)
- No commutator and brush mechanisms needed in many types of ac motors
- Low power dissipation, low rotor inertia, and lightweight in some designs
- Virtually no electric spark generation or arcing (less hazardous in chemical environments)
- Capability of accurate constant-speed operation without needing servo control (with synchronous ac motors)
- No drift problems in ac amplifiers in drive circuits (unlike linear dc amplifiers)
- High reliability, robustness, easy maintenance, and long life

Disadvantages: The primary disadvantages of ac motors include the following:

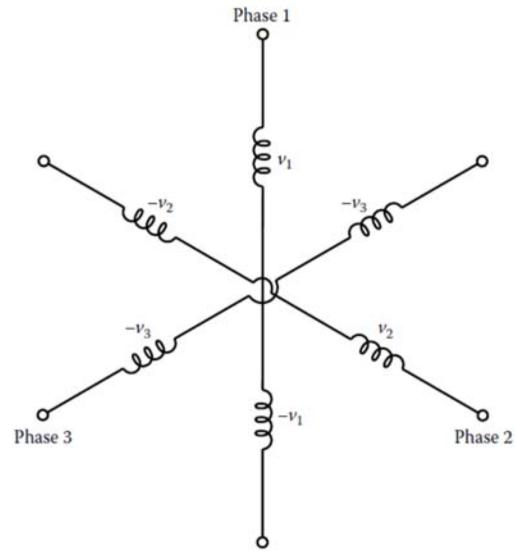
- Low starting torque (synchronous motors have zero starting torque)
- *Need of auxiliary starting devices for ac motors with zero starting torque*
- *Difficulty of variable-speed control or servo control (this problem hardly exists now in view of modern solid-state and variable-frequency drives with devices having field feedback compensation)*
- Instability in low speed operation

We discuss two basic types of ac motors:

1. Induction motors (asynchronous motors)
2. Synchronous motors

Rotating Magnetic Field:

- The operation of an ac motor can be explained using the concept of a rotating magnetic field.
- A rotating field is generated by a set of windings uniformly distributed around a circular stator and excited by ac signals with uniform phase differences.
- To illustrate this, consider a standard three-phase supply. The voltage in each phase is 120° out of phase with the voltage in the next phase. The phase voltages can be represented by:



$$V_1 = a\cos\omega_p t$$
$$V_2 = a\cos\left(\omega_p t - \frac{2\pi}{3}\right)$$
$$V_3 = a\cos\left(\omega_p t - \frac{4\pi}{3}\right)$$

ω_p = frequency of each phase of AC Signal

V_1 leads V_2 by $2\pi/3$

V_1 leads V_3 by $4\pi/3$

3ϕ - 2 windings/phase

phase difference between 2 adjacent windings is $\frac{\pi}{3}$

→ Consider $\Delta t = \frac{\pi}{3\omega_p}$ interval

- status of $-V_3$ @ end of time interval Δt is identical to status of V_1 in the beginning of time interval
- Similarly status of V_2 after Δt become that of $-V_3$ beginning

* Voltage STATUS of one segment becomes identical to adjacent segment in Δt.

* $$\omega_f = \frac{\omega_p}{n}$$

ω_f ← Angular Speed of Rotating Magnetic field.

ω_p ← frequency of AC signal.

n ← # of pairs of windings used per phase.

$n=1 \Rightarrow$ 2 coils/phase, $n=2 \Rightarrow$ 4 pairs/phase

$\therefore n \Rightarrow$ # of pole pairs per phase in stator.

Example 9.10

Another way to interpret the concept of a rotating magnetic field is to consider the resultant field due to the individual magnetic fields in the stator windings. Consider a single set of three-phase windings arranged geometrically as in Figure 9.32. Suppose that the magnetic field due to phase 1 is denoted by $a \sin \omega_p t$. Show that the resultant magnetic field has an amplitude of $3a/2$ and that the field rotates at speed ω_p.

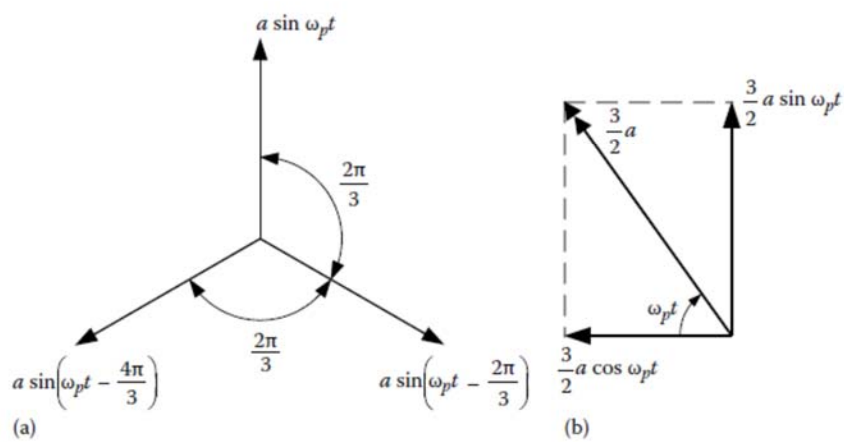
Solution

The magnetic field vectors in the three sets of windings are shown in Figure 9.33a. These can be resolved into two orthogonal components, as shown in Figure 9.33b. The component in the vertical direction (upward) is

$$a\sin\omega_p t - a\sin\left(\omega_p t - \frac{2\pi}{3}\right)\cos\frac{\pi}{3} - a\sin\left(\omega_p t - \frac{4\pi}{3}\right)\cos\frac{\pi}{3}$$

$$= a\sin\omega_p t - \frac{a}{2}\left[\sin\left(\omega_p t - \frac{2\pi}{3}\right) + \sin\left(\omega_p t - \frac{4\pi}{3}\right)\right] = a\sin\omega_p t - a\sin(\omega_p t - \pi)\cos\frac{\pi}{3}$$

$$= a\sin\omega_p t + \frac{a}{2}[\sin\omega_p t] = \frac{3a}{2}\sin\omega_p t$$



Note: In deriving this result, we have used the following trigonometric identities:

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \text{ and } \sin(A-\pi) = -\sin A$$

The horizontal component of the magnetic fields, which is directed to the left, is

$$a\sin\left(\omega_p t - \frac{4\pi}{3}\right)\sin\frac{\pi}{3} - a\sin\left(\omega_p t - \frac{2\pi}{3}\right)\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}a\left[\sin\left(\omega_p t - \frac{4\pi}{3}\right) - \sin\left(\omega_p t - \frac{2\pi}{3}\right)\right]$$

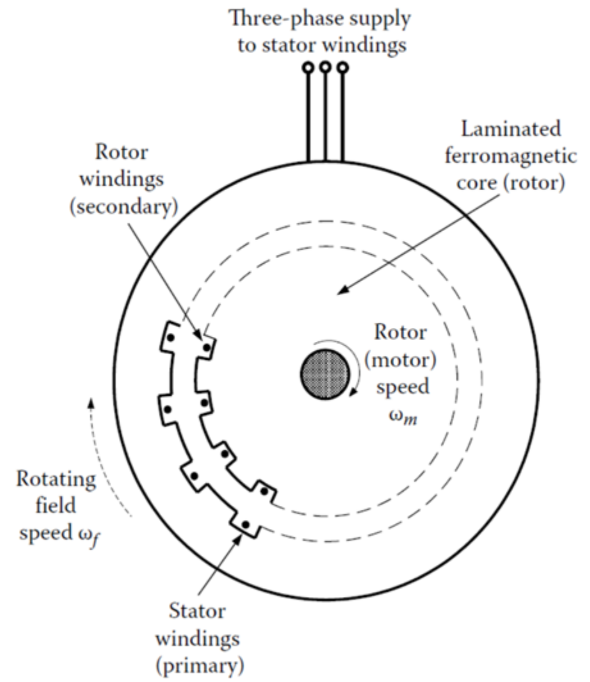
$$= \sqrt{3}a\cos(\omega_p t - \pi)\sin\left(-\frac{\pi}{3}\right) = \frac{3a}{2}\cos\omega_p t$$

Here, we have used the following trigonometric identities:

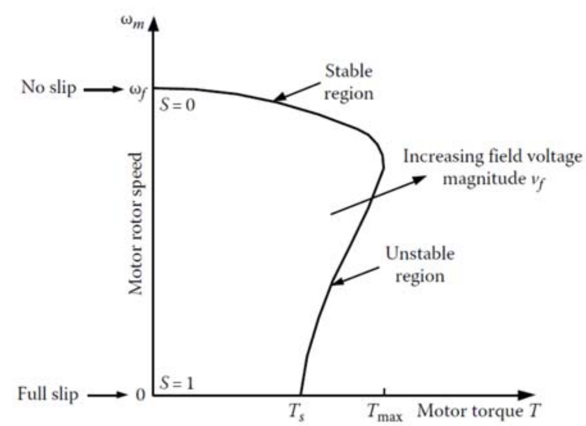
$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}, \quad \cos(A-\pi) = -\cos A, \; \sin(-A) = -\sin A$$

The resultant of the two orthogonal components is a vector of magnitude $3a/2$, making an angle $\omega_p t$ with the horizontal component, as shown in Figure 9.33b. It follows that the resultant magnetic field has a magnitude of $3a/2$ and rotates in the clockwise direction at speed ω_p rad/s.

Induction Motor Characteristics:



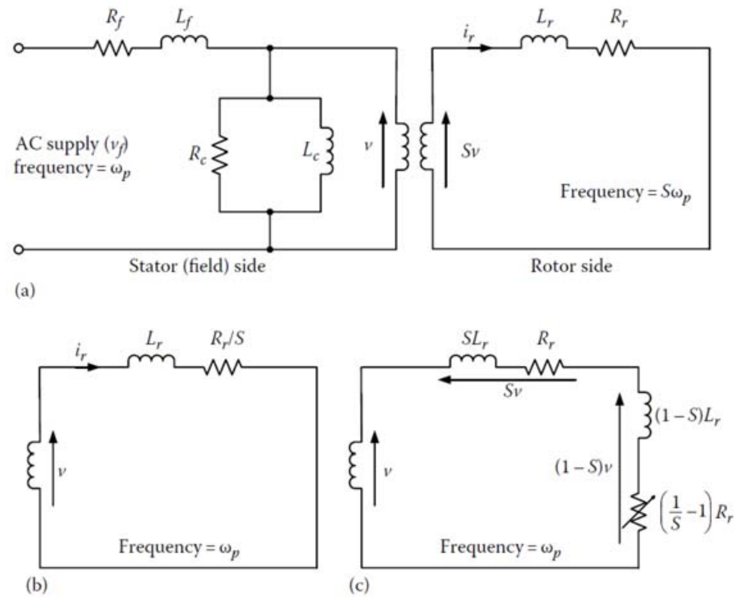
- The *stator windings of an induction motor generate a rotating magnetic field.*
- The *rotor windings are purely secondary windings*, which are not energized by an external voltage and are *used for inducing a magnetic field.*
- For this reason, *no commutator-brush devices are needed in induction motors*.
- The core of the rotor is made of ferromagnetic laminations in order to concentrate the magnetic flux and to minimize dissipation (primarily due to eddy currents).
- As *the rotor speed increases, initially the motor torque also increases* (rather moderately) because of secondary interactions between the stator circuit and the rotor circuit.
- This *increase in torque happens even though the relative speed of the rotating field with respect to the rotor decreases, which reduces the rate of change of flux linkage and hence the direct transformer action*. (*Note: the relative speed is termed the slip rate.*) In this manner, at some speed the maximum torque will be reached.
- Further increase in rotor speed (i.e., a decrease in slip rate) sharply decreases the motor torque, until at *synchronous speed (i.e., zero slip rate) the motor torque becomes zero*.



- This behavior of an induction motor is illustrated by the typical characteristic curve given.
- From the starting torque T_s to the maximum torque (which is known as the breakdown torque) T_{Max}, the motor behavior is unstable.
- *The portion of the curve from T_{Max} to the zero torque (or, no-load or synchronous condition) represents the region of stable operation*.
- *Under normal operating conditions, an induction motor should operate in this region*.
- The fractional slip S for an induction motor is given by: $S = \frac{\omega_f - \omega_m}{\omega_f}$
- If the *rotor speed is increased beyond the synchronous speed* (i.e., S < 0), *the motor becomes a generator*. Note: *When the stator windings are symmetrically distributed around the rotor, as in the foregoing analysis, the motor is called a symmetrical machine (e.g., a symmetrical induction motor)*.

Torque–Speed Relationship:

- It is instructive to determine the torque–speed relationship for an induction motor.
- This relationship provides insight into possible control methods for induction motors.
- The equivalent circuits of the stator and the rotor for one phase of an induction motor are shown.



→ L_f = Stator leakage Inductance

→ R_c = stator core iron loss resistance ; L_r = Rotor Leakage Inductance.

→ L_c = stator core (Magnetizing) Inductance; R_r = Rotor Coil Resistance

→ AC Voltage Supply V_f, ω_p = Line frequency; i_r: rotor Current gen. back emf

→ Induced Voltage (on Secondary Rotor Windings) Changes with Slip "S" which is proportional to $(\omega_f - \omega_m)$

∴ Induced voltage in Rotor Windings is Sv

→ At standstill ($S=1$); frequency of induced voltage is ω_p

→ At Synchronous speed ($S=0$); frequency $= 0$ ∴ field is fixed and Const rel. to rotor.

Using frequency domain representation.

Rotor Current $i_r = \dfrac{Sv}{R_r + jS\omega_p L_r} = \dfrac{v}{\frac{R_r}{S} + j\omega_p L_r}$

Motor Torque gen. in Rotor

Rotor Speed of Motor

Rotor Current Magn.

Available Mechanical power: $T_m \omega_m = p\, i_r^2 \left(\frac{1}{S} - 1\right) R_r$

\# of phases

$\boxed{i_r \text{ Can be obtained as: } \dfrac{v}{\sqrt{R_r^2/S^2 + \omega_p^2 L_r^2}}}$; By Substitution we get Mechanical Value

$$\boxed{T_m = p v^2 \frac{S(1-S)}{\omega_m} \cdot \frac{R}{R_r^2 + S^2 \omega_p^2 L_r^2}}$$

\# of pole-pairs: $n = \dfrac{\omega_p}{\omega_m}(1-S)$

which gives $T_m = \dfrac{p n v^2 S R_r}{\omega_p (R_r^2 + S^2 \omega_p^2 L_r^2)}$.

If the resistance and the leakage inductance in the stator are neglected, v is approximately equal to the stator excitation voltage v_f. This gives the torque–slip relationship:

Motor Torque $\propto$ Square of Supply Voltage

$$T_m = \frac{P n v_f^2 S R_r}{\omega_p (R_r^2 + S^2 \omega_p^2 L_r^2)} = \frac{P v_f^2 S R_r}{\omega_f (R_r^2 + S^2 n^2 \omega_f^2 L_r^2)}$$

Using the fact that $n = \frac{\omega_p}{\omega_m}(1-S)$, we can write the above equation in terms of ω_m (Rotor Speed).

ω_m: Motor Speed is related to slip as:

$$S = \frac{\omega_f - \omega_m}{\omega_f} = \frac{\omega_p - n\,\omega_m}{\omega_p}$$

Motor Speed

frequency of Exciting Voltage