An FM signal has a deviation of 3 kHz and a modulating frequency of 1 kHz. Its total power P_T is 5 W, developed across a 50 Ω resistive load. The carrier frequency is 160 MHz.

(a) Calculate the RMS signal voltage V_T.

(b) Calculate the RMS voltage at the carrier frequency and each of the first three sets of sidebands.

(c) For the first three sideband pairs, calculate the frequency of each sideband.

(d) Calculate the power at the carrier frequency and at each of the sideband frequencies found in part (c).

(e) Determine what percentage of the total signal power is unaccounted for by the components described above.

(f) Sketch the signal in the frequency domain, as it would appear on a spectrum analyzer. The vertical scale should be power in dBm, and the horizontal scale should be frequency.

TABLE 4.1

m	J_0	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	J_{10}	J_{11}	J_{12}	J_{13}	J_{14}	J_{15}	J_{16}	J_{17}	J_{18}	J_{19}	J_{20}
0	1.00																				
0.25	0.98	0.12																			
0.5	0.94	0.24	0.03																		
0.75	0.86	0.35	0.07	0.01																	
1	0.77	0.44	0.11	0.02																	
1.25	0.65	0.51	0.17	0.04	0.01																
1.5	0.51	0.56	0.23	0.06	0.01																
1.75	0.37	0.58	0.29	0.09	0.02																
2	0.22	0.58	0.35	0.13	0.03	0.01															
2.25	0.08	0.55	0.40	0.17	0.05	0.01															
2.4	0.00	0.52	0.43	0.20	0.06	0.02															
2.5	−0.05	0.50	0.45	0.22	0.07	0.02															
2.75	−0.16	0.43	0.47	0.26	0.10	0.03	0.01														
3	−0.26	0.34	0.49	0.31	0.13	0.04	0.01														
3.5	−0.38	0.14	0.46	0.39	0.20	0.08	0.03	0.01													
4	−0.40	−0.07	0.36	0.43	0.28	0.13	0.05	0.01													
4.5	−0.32	−0.23	0.22	0.42	0.35	0.20	0.08	0.03	0.01												
5	−0.18	−0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02	0.01											
5.5	0.00	−0.34	−0.12	0.26	0.40	0.32	0.19	0.09	0.03	0.01											
6	0.15	−0.28	−0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02	0.01										
6.5	0.26	−0.15	−0.31	−0.03	0.28	0.37	0.30	0.18	0.09	0.04	0.01										
7	0.30	−0.01	−0.30	−0.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02	0.01									
7.5	0.27	0.14	−0.23	−0.26	0.02	0.28	0.35	0.28	0.17	0.09	0.04	0.01	0.01								
8	0.17	0.24	−0.11	−0.29	−0.11	0.19	0.34	0.32	0.22	0.13	0.06	0.03	0.01								
8.5	0.04	0.27	0.02	−0.26	−0.21	0.07	0.29	0.34	0.27	0.17	0.09	0.04	0.02	0.01							
8.65	0.00	0.27	0.06	−0.24	−0.23	0.03	0.27	0.34	0.28	0.18	0.10	0.05	0.02	0.01							
9	−0.09	0.25	0.14	−0.18	−0.27	−0.06	0.20	0.33	0.30	0.21	0.13	0.06	0.03	0.01							
10	−0.25	0.04	0.26	0.06	−0.22	−0.23	−0.01	0.22	0.32	0.29	0.21	0.12	0.06	0.03	0.01						
11	−0.17	−0.18	0.14	0.23	−0.01	−0.24	−0.20	0.02	0.23	0.31	0.28	0.20	0.12	0.06	0.03	0.01	0.01				
12	0.05	−0.22	−0.08	0.20	0.18	−0.07	−0.24	−0.17	0.04	0.23	0.30	0.27	0.20	0.12	0.07	0.03	0.01	0.10			
13	0.21	−0.07	−0.22	0.00	0.22	0.13	−0.12	−0.24	−0.14	0.07	0.23	0.29	0.26	0.19	0.12	0.07	0.03	0.01	0.01		
14	0.17	0.13	−0.15	−0.18	0.08	−0.15	−0.23	−0.11	0.08	0.24	0.29	0.25	0.19	0.12	0.07	0.03	0.02	0.01			
15	−0.01	0.20	0.04	−0.19	−0.12	0.13	0.21	0.03	−0.17	−0.22	−0.09	0.10	0.24	0.28	0.25	0.18	0.12	0.07	0.03	0.02	0.01
16	−0.17	0.09	0.19	−0.04	−0.20	−0.06	0.17	0.18	−0.01	−0.19	−0.21	−0.07	0.11	0.24	0.27	0.24	0.18	0.11	0.07	0.03	0.02
17	−0.17	−0.10	0.16	0.14	−0.11	−0.19	0.00	0.19	0.15	−0.04	−0.20	−0.19	−0.05	0.12	0.24	0.27	0.23	0.17	0.11	0.07	0.04

Solution

(a) The signal power does not change with modulation, and neither does the voltage, which can easily be found from the power equation.

$$P_T = \frac{V_T^2}{R_L}$$

$$V_T = \sqrt{P_T R_L}$$

$$= \sqrt{5\text{ W} \times 50\ \Omega}$$

$$= 15.8\text{ V (RMS)}$$

(b) The modulation index must be found in order to use Bessel functions to find the carrier and sideband voltages.

$$m_f = \frac{\delta}{f_m}$$

$$= \frac{3\text{ kHz}}{1\text{ kHz}}$$

$$= 3$$

From the Bessel function table, the coefficients for the carrier and the first three sideband pairs are:

$$J_0 = -0.26 \qquad J_1 = 0.34 \qquad J_2 = 0.49 \qquad J_3 = 0.31$$

These are normalized voltages, so they will have to be multiplied by the total RMS signal voltage to get the RMS sideband and carrier-frequency voltages. For the carrier,

$$V_c = J_0 V_T$$

J_0 has a negative sign. This simply indicates a phase relationship between the components of the signal. It would be required if we wanted to add together all the components to get the resultant signal. For our present purpose, however, it can simply be ignored, and we can use

$$V_c = |J_0| V_T$$

$$= 0.26 \times 15.8\text{ V}$$

$$= 4.11\text{ V}$$

Similarly, we can find the voltage for each of the three sideband pairs. Note that these are voltages for individual components. There will be a lower and an upper sideband with each of these calculated voltages.

$$\begin{aligned} V_1 &= J_1 V_T \\ &= 0.34 \times 15.8 \text{ V} \\ &= 5.37 \text{ V} \end{aligned}$$

$$\begin{aligned} V_2 &= J_2 V_T \\ &= 0.49 \times 15.8 \text{ V} \\ &= 7.74 \text{ V} \end{aligned}$$

$$\begin{aligned} V_3 &= J_3 V_T \\ &= 0.31 \times 15.8 \text{ V} \\ &= 4.9 \text{ V} \end{aligned}$$

(c) The sidebands are separated from the carrier frequency by multiples of the modulating frequency. Here, $f_c = 160$ MHz and $f_m = 1$ kHz, so there are sidebands at each of the following frequencies:

$$\begin{aligned} f_{USB_1} &= 160 \text{ MHz} + 1 \text{ kHz} \\ &= 160.001 \text{ MHz} \end{aligned}$$

$$\begin{aligned} f_{USB_2} &= 160 \text{ MHz} + 2 \text{ kHz} \\ &= 160.002 \text{ MHz} \end{aligned}$$

$$\begin{aligned} f_{USB_3} &= 160 \text{ MHz} + 3 \text{ kHz} \\ &= 160.003 \text{ MHz} \end{aligned}$$

$$\begin{aligned} f_{LSB_1} &= 160 \text{ MHz} - 1 \text{ kHz} \\ &= 159.999 \text{ MHz} \end{aligned}$$

$$\begin{aligned} f_{LSB_2} &= 160 \text{ MHz} - 2 \text{ kHz} \\ &= 159.998 \text{ MHz} \end{aligned}$$

$$\begin{aligned} f_{LSB_3} &= 160 \text{ MHz} - 3 \text{ kHz} \\ &= 159.997 \text{ MHz} \end{aligned}$$

(d) Since each of the components of the signal is a sinusoid, the usual equation can be used to calculate power. All the components appear across the same 50 Ω load.

$$P_c = \frac{V_c^2}{R_L} = \frac{4.11^2}{50} = 0.338 \text{ W}$$

$$P_1 = \frac{V_1^2}{R_L} = \frac{5.37^2}{50} = 0.576 \text{ W} \qquad P_2 = \frac{V_2^2}{R_L} = \frac{7.74^2}{50} = 1.2 \text{ W} \qquad P_3 = \frac{V_3^2}{R_L} = \frac{4.9^2}{50} = 0.48 \text{ W}$$

(e) To find the total power P_T in the carrier and the first three sets of sidebands, it is only necessary to add the powers calculated above, counting each of the sideband powers twice, because each of the calculated powers represents one of a pair of sidebands. We only count the carrier once, of course.

$$\begin{aligned} P_T &= P_c + 2(P_1 + P_2 + P_3) \\ &= 0.338 + 2(0.576 + 1.2 + 0.48) \text{ W} \\ &= 4.85 \text{ W} \end{aligned}$$

This is not quite the total signal power, which was given as 5 W. The remainder is in the additional sidebands. To find how much is unaccounted for by the carrier and the first three sets of sidebands, we can subtract. Call the difference P_x:

$$\begin{aligned} P_x &= 5 - 4.85 \\ &= 0.15 \text{ W} \end{aligned}$$

As a percentage of the total power this is

$$\begin{aligned} P_x (\%) &= \left(\frac{0.15}{5}\right)100 \\ &= 3\% \end{aligned}$$

(f) All the information we need for the sketch is on hand, except that the power values have to be converted to dBm using the equation

$$P\ (\text{dBm}) = 10 \log \frac{P}{1\ \text{mW}}$$

This gives

$$P_c\ (\text{dBm}) = 10 \log 338 = 25.3\ \text{dBm}$$

$$P_1\ (\text{dBm}) = 10 \log 576 = 27.6\ \text{dBm}$$

$$P_2\ (\text{dBm}) = 10 \log 1200 = 30.8\ \text{dBm}$$

$$P_3\ (\text{dBm}) = 10 \log 480 = 26.8\ \text{dBm}$$

The sketch is shown in Figure 4.8.