School of Engineering Science
Simon Fraser University
ENSC327 Communication Systems

Assignment #3 Solution

Q1: From Fourier transform pairs

$$x(t) = m(t)\sin(2\pi f_c t) \Rightarrow X(f) = \frac{1}{2j}\left[M(f - f_c) - M(f + f_c)\right]$$

The first term is in the positive freq, and the 2nd term is in the negative freq. So from Hilbert transform

$$\hat{X}(f) = -j\,\text{sgn}(f)X(f) = -\frac{1}{2}\left[M(f - f_c) + M(f + f_c)\right]$$

From inverse Fourier transform

$$\hat{x}(t) = -\frac{1}{2}\left(m(t)e^{j2\pi f_c t} + m(t)e^{-j2\pi f_c t}\right) = -m(t)\cos(2\pi f_c t)$$

Q2: The SSB wave $s_u(t)$ is defined by

$$s_u(t) = \frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) - \hat{m}(t)\sin(2\pi f_c t)\right] \qquad (1)$$

and its Hilbert transform is defined by (using the results of Q1)

$$\hat{s}_u(t) = \frac{A_c}{2}\left[m(t)\sin(2\pi f_c t) + \hat{m}(t)\cos(2\pi f_c t)\right] \qquad (2)$$

Therefore,

$$s_u(t)\cos(2\pi f_c t) = \frac{A_c}{2}\left[m(t)\cos^2(2\pi f_c t) - \hat{m}(t)\sin(2\pi f_c t)\cos(2\pi f_c t)\right] \qquad (3)$$

$$\hat{s}_u(t)\sin(2\pi f_c t) = \frac{A_c}{2}\left[m(t)\sin^2(2\pi f_c t) + \hat{m}(t)\sin(2\pi f_c t)\cos(2\pi f_c t)\right] \qquad (4)$$

Adding equations (3) and (4) and solving for $m(t)$, we get

$$m(t) = \frac{2}{A_c}\left[s_u(t)\cos(2\pi f_c t) + \hat{s}_u(t)\sin(2\pi f_c t)\right] \qquad (5)$$

Next, we use equations (1) and (2) to write

$$s_u(t)\sin(2\pi f_c t) = \frac{A_c}{2}\left[m(t)\cos(2\pi f_c t)\sin(2\pi f_c t) - \hat{m}(t)\sin^2(2\pi f_c t)\right] \qquad (6)$$

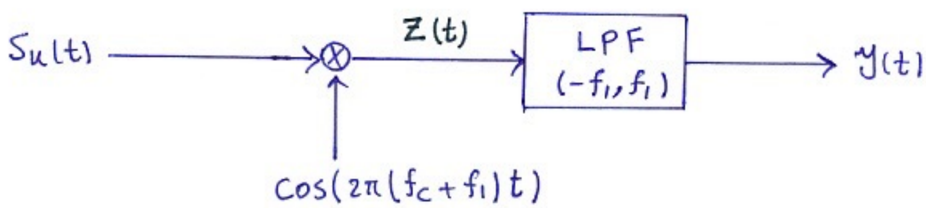
$$\hat{s}_u(t)\cos(2\pi f_c t) = \frac{A_c}{2}\left[m(t)\sin(2\pi f_c t)\cos(2\pi f_c t) + \hat{m}(t)\cos^2(2\pi f_c t)\right] \qquad (7)$$

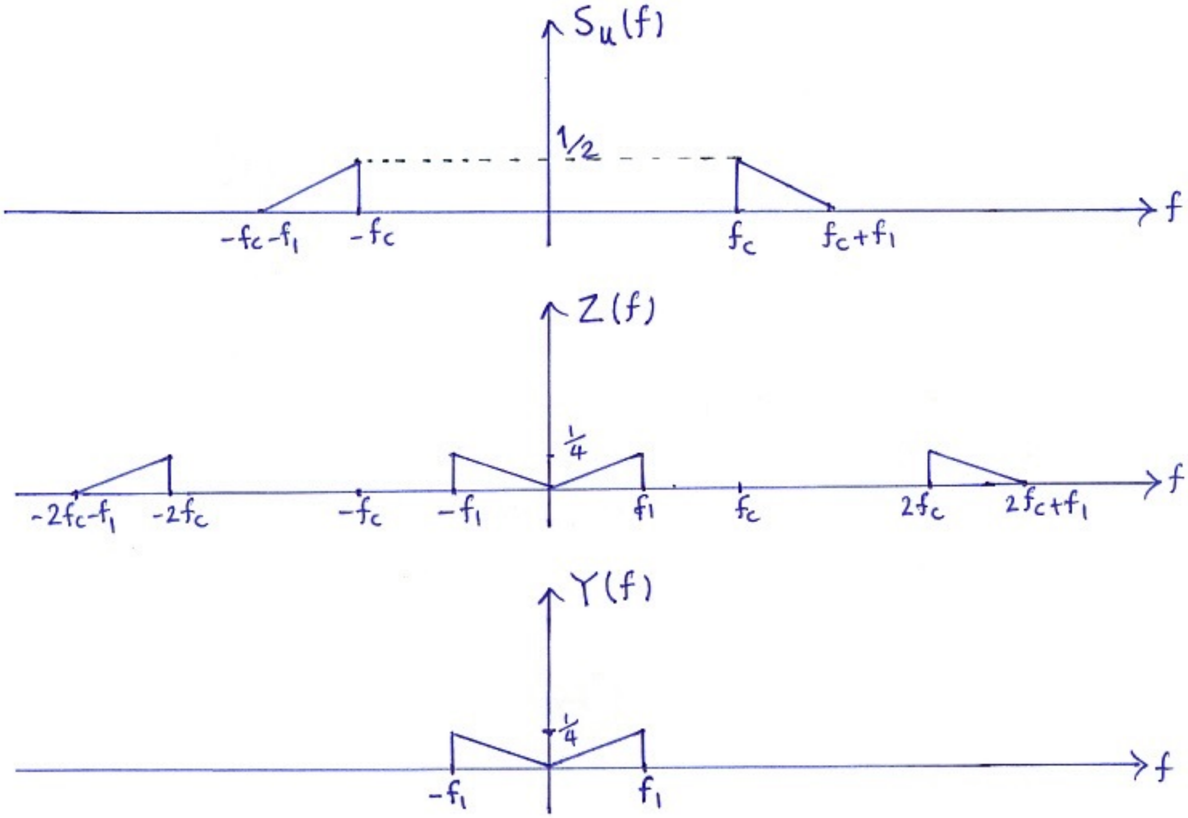
Subtracting Eq. (6) from Eq. (7) and then solving for $\hat{m}(t)$, we get

$$\hat{m}(t) = \frac{2}{A_c}\left[\hat{s}_u(t)\cos(2\pi f_c t) - s_u(t)\sin(2\pi f_c t)\right] \qquad (8)$$

Equations (5) and (8) are the desired results.

Q3:

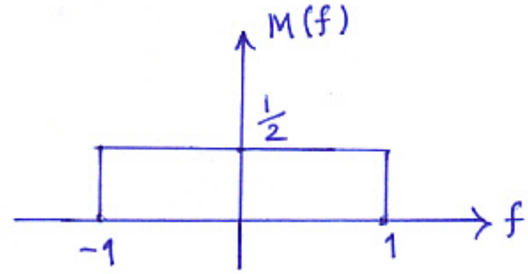




Q4:

$m(t) = \text{Sinc}(2t)$

$M(f) = \frac{1}{2}\text{rect}(\frac{f}{2})$



$\hat{M}(f) = -j\,\text{sgn}(f)\,M(f)$

$= \frac{1}{2}j\,\text{rect}(f+\frac{1}{2}) - \frac{1}{2}j\,\text{rect}(f-\frac{1}{2})$

$\Rightarrow \hat{m}(t) = \frac{1}{2}j\,e^{-j2\pi(\frac{1}{2})t}\,\text{Sinc}(t) - \frac{1}{2}j\,e^{j2\pi(\frac{1}{2})t}\,\text{sinc}(t)$

$= \frac{j}{2}e^{-j\pi t}\,\text{Sinc}(t) - \frac{j}{2}e^{j\pi t}\,\text{Sinc}(t)$

$= \text{Sinc}(t)\left[\frac{e^{-j\pi t} - e^{j\pi t}}{2}\right]j = \text{Sinc}(t)\underbrace{\left(\frac{e^{j\pi t} - e^{-j\pi t}}{2j}\right)}_{\sin(\pi t)}$

$\Rightarrow \hat{m}(t) = \text{Sinc}(t)\,\sin(\pi t)$

Q5:

$$S_{LSB}(t) = \frac{A_c}{2}\left[m(t)\cos 2\pi f_c t + \hat{m}(t)\sin 2\pi f_c t\right]$$

$$m(t) = \cos 2\pi(10^3)t + 2\sin 2\pi(10^3)t$$

we know that:

$$\cos 2\pi f_0 t \xrightarrow{H} \sin 2\pi f_0 t \qquad H: \text{Hilbert transform}$$

$$\sin 2\pi f_0 t \xrightarrow{H} -\cos 2\pi f_0 t$$

$$\Rightarrow S_{LSB}(t) = \frac{100}{2}\Big[\left[\cos 2\pi(10^3)t + 2\sin 2\pi(10^3)t\right]\cos 2\pi(8\times 10^5)t$$

$$+ \left[\sin 2\pi(10^3)t - 2\cos 2\pi(10^3)t\right]\sin 2\pi(8\times 10^5)t\Big]$$

Using:

$$\sin\alpha\cos\beta - \sin\beta\cos\alpha = \sin(\alpha-\beta)$$

$$\cos\alpha\cos\beta + \sin\alpha\sin\beta = \cos(\alpha-\beta)$$

we can write:

$$S_{LSB}(t) = 50\left[\cos 2\pi(799\times 10^3)t - 2\sin 2\pi(799\times 10^3)t\right]$$