ENSC-327Midetermm Formula sheet

Trigonometric Identities:

$$\cos(u) = \frac{e^{ju} + e^{-ju}}{2} \qquad \cos^2(u) - \sin^2(u) = \cos(2u)$$

$$\sin(u) = \frac{e^{ju} - e^{-ju}}{2j} \qquad 2\sin(u)\cos(u) = \sin(2u)$$

$$\cos(u)\cos(v) = \frac{1}{2}\cos(u-v) + \frac{1}{2}\cos(u+v)$$

$$\sin(u)\cos(v) = \frac{1}{2}\sin(u-v) + \frac{1}{2}\sin(u+v)$$

$$\sin(u)\sin(v) = \frac{1}{2}\cos(u-v) - \frac{1}{2}\cos(u+v)$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

Fourier Transform and Inverse Fourier Transform:

$$G(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi ft} dt, \qquad g(t) = \int_{-\infty}^{\infty} G(f) e^{j2\pi ft} df$$

Fourier Series Expansion for periodic signals:

$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{j2\pi n f_0 t} \quad \text{where} \quad X_n = \int_{-\infty}^{\infty} x(t) e^{-j2\pi n f_0 t} dt$$

Properties of Fourier Transform:

Name	Time-domain operation (signals assumed real)	Frequency-domain operation
Superposition	$a_1 x_1(t) + a_2 x_2(t)$	$a_1 X_1(f) + a_2 X_2(f)$
Time delay	$x(t - t_0)$	$X(f)\exp(-j2\pi t_0 f)$
Scale change	$x(at)$	$\lvert a\rvert^{-1} X(f/a)$
Time reversal	$x(-t)$	$X(-f) = X^*(f)$
Duality	$X(t)$	$x(-f)$
Frequency translation	$x(t)\exp(j2\pi f_0 t)$	$X(f - f_0)$
Modulation	$x(t)\cos(2\pi f_0 t)$	$\frac{1}{2}X(f - f_0) + \frac{1}{2}X(f + f_0)$
Convolution[4]	$x_1(t) * x_2(t)$	$X_1(f) X_2(f)$
Multiplication	$x_1(t) x_2(t)$	$X_1(f) * X_2(f)$
Differentiation	$\frac{d^n x(t)}{dt^n}$	$(j2\pi f)^n X(f)$
Integration	$\int_{-\infty}^{t} x(\lambda)\, d\lambda$	$X(f)/(j2\pi f) + \frac{1}{2}X(0)\delta(f)$

Parseval's theorem:

$$\int_{-\infty}^{\infty} |g(t)|^2 dt = \int_{-\infty}^{\infty} |G(f)|^2 df$$

- Conjugation rule:

$$g(t) \leftrightarrow G(f) \rightarrow g^*(t) \leftrightarrow G^*(-f)$$

Hilbert Transform:

$$\hat{x}(t) \overset{\Delta}{=} x(t) * \frac{1}{\pi t}$$

$$\hat{X}(f) = -j\,\mathrm{sgn}(f)X(f)$$

Single Sideband Modulation:

$$s_{ssb}(t) = \frac{A_c}{2}\left(m(t)\cos(2\pi f_c t) \mp \hat{m}(t)\sin(2\pi f_c t)\right)$$

USB

LSB

PM

$$s(t) = A_c \cos(2\pi f_c t + k_p m(t))$$

FM

$$s(t) = A_c \cos(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau)$$

TABLE A6.2 *Fourier-Transform Pairs*

Time Function	*Fourier Transform*
$\mathrm{rect}\left(\frac{t}{T}\right)$	$T\,\mathrm{sinc}\,(fT)$
$\mathrm{sinc}\,(2Wt)$	$\frac{1}{2W}\mathrm{rect}\left(\frac{f}{2W}\right)$
$\exp(-at)u(t), \quad a > 0$	$\frac{1}{a + j2\pi f}$
$\exp(-a\lvert t\rvert), \quad a > 0$	$\frac{2a}{a^2 + (2\pi f)^2}$
$\exp(-\pi t^2)$	$\exp(-\pi f^2)$
$\begin{cases} 1 - \frac{\lvert t\rvert}{T}, & \lvert t\rvert < T \\ 0, & \lvert t\rvert \geq T \end{cases}$	$T\,\mathrm{sinc}^2(fT)$
$\delta(t)$	1
1	$\delta(f)$
$\delta(t - t_0)$	$\exp(-j2\pi f t_0)$
$\exp(j2\pi f_c t)$	$\delta(f - f_c)$
$\cos(2\pi f_c t)$	$\frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)]$
$\sin(2\pi f_c t)$	$\frac{1}{2j}[\delta(f - f_c) - \delta(f + f_c)]$
$\mathrm{sgn}(t)$	$\frac{1}{j\pi f}$
$\frac{1}{\pi t}$	$-j\,\mathrm{sgn}(f)$
$u(t)$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$
$\sum_{i=-\infty}^{\infty} \delta(t - iT_0)$	$\frac{1}{T_0}\sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_0}\right)$