Artificial Neural Networks

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Neural Networks

- Neural networks arise from attempts to model human/animal brains
    - Many models, many claims of biological plausibility
    - We will focus on statistical and computational properties rather than plausibility
- An artificial neural network is a general *function approximator*
- The inner or hidden layers compute *learned auxilliary functions*

Uses of Neural Networks

- Pros
  - Good for continuous input variables.
  - General continuous function approximators.
  - Highly non-linear.
  - Trainable basis functions.
  - Good to use in continuous domains with little knowledge:
    - When you don't know good features.
    - You don't know the form of a good functional model.
- Cons
  - Not interpretable, "black box".
  - Learning is slow.
  - Good generalization can require many datapoints.

Function Approximation Demos

- Home Value of Hockey State https://user-images.githubusercontent.com/22108101/28182140-eb64b49a-67bf-11e7-97aa-046298f721e5.jpg
- Function Learning Examples (open in Safari) http://neuron.eng.wayne.edu/bpFunctionApprox/bpFunctionApprox.html

Applications

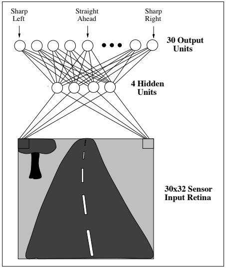
There are many, many applications.

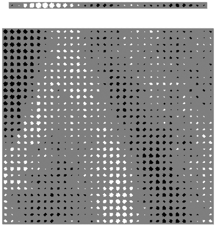
- World-Champion Backgammon Player.
  http://en.wikipedia.org/wiki/TD-Gammon
  http://en.wikipedia.org/wiki/Backgammon
- No Hands Across America Tour.
  http://www.cs.cmu.edu/afs/cs/usr/tjochem/www/nhaa/nhaa_home_page.html
- Digit Recognition with 99.26% accuracy.
- Speech Recognition
  http://research.microsoft.com/en-us/news/features/speechrecognition-082911.aspx
- http://deeplearning.net/demos/

Outline

Outline

No Hands Across America





Non-linear Activation Functions

- Pass input in_j through a non-linear activation function $g(\cdot)$ to get output $a_j = g(in_j)$
- Model of an individual neuron

from Russell and Norvig, AIMA3e

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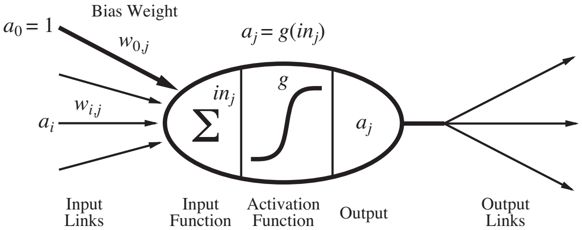

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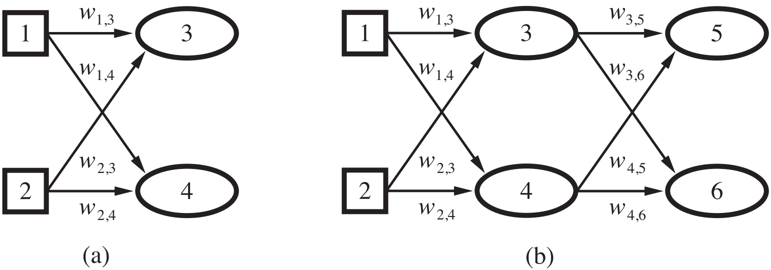
Network of Neurons

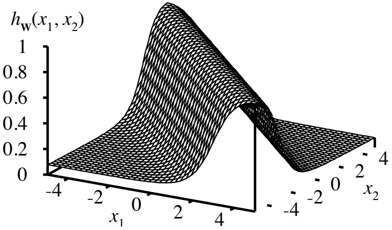


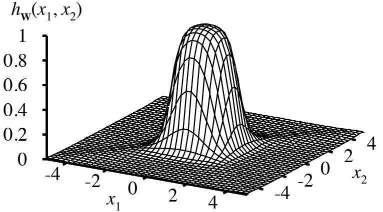
Activation Functions

- Can use a variety of activation functions
  - Sigmoidal (S-shaped)
    - Logistic sigmoid $1/(1 + \exp(-a))$ (useful for binary classification)
    - Hyperbolic tangent $\tanh$
  - Softmax
    - Useful for multi-class classification
  - Rectified Linear Unit (RLU) $max(0, x)$
  - ...
- Should be differentiable for gradient-based learning (later)
- Can use different activation functions in each unit
- See `http://aispace.org/neural/`.

Function Composition

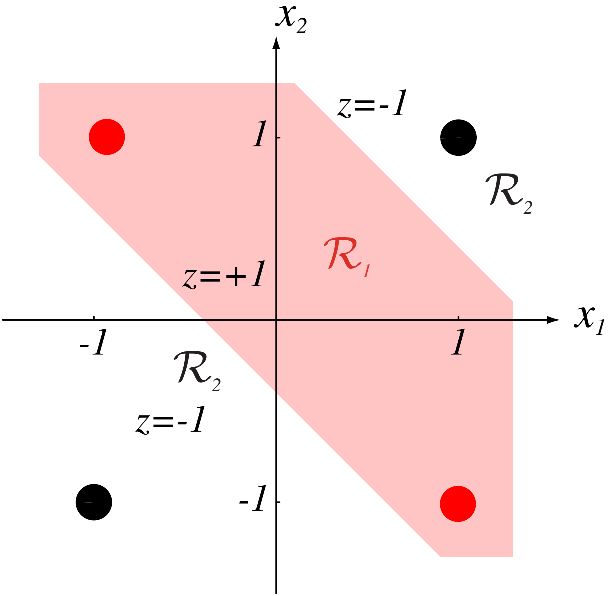
Think logic circuits



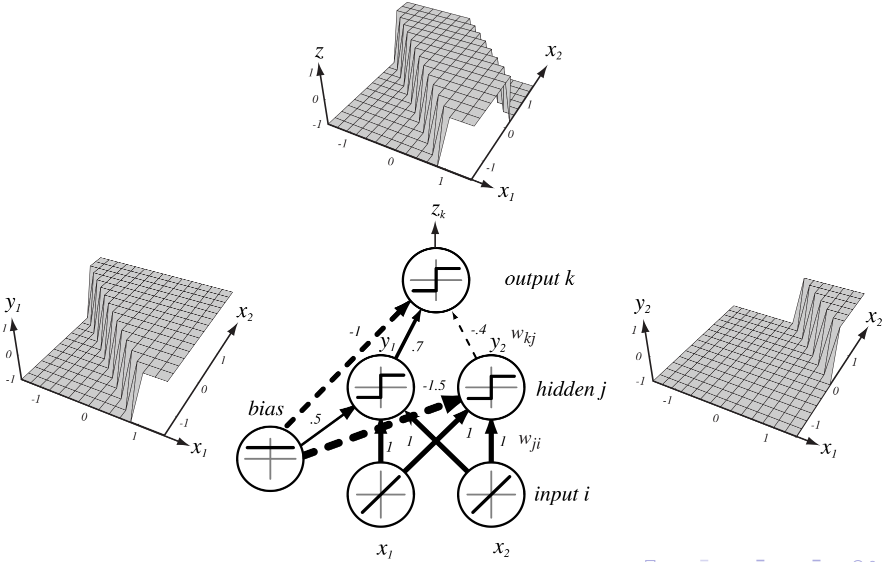


Two opposite-facing sigmoids = ridge. Two ridges = bump.

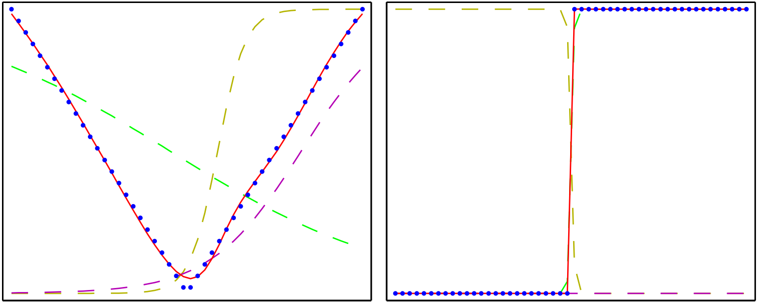
The XOR Problem Revisited



The XOR Problem Solved



Hidden Units Compute Auxilliary Functions

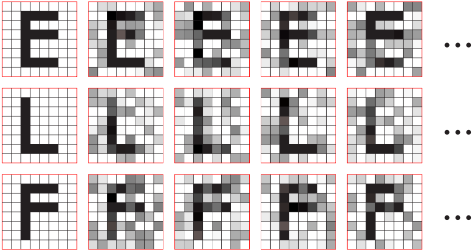


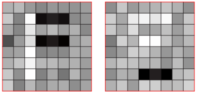
- red dots = network function
- dashed line = hidden unit activation function.
- blue dots = data points

Network function is roughly the sum of activation functions.

Hidden Units As Feature Extractors

sample training patterns





learned input-to-hidden weights

- 64 input nodes
- 2 hidden units
- learned weight matrix at hidden units

Outline

Network Training

- Given a specified network structure, how do we set its parameters (weights)?
  - As usual, we define a criterion to measure how well our network performs, optimize against it
- Training data are $(\boldsymbol{x}_n, \boldsymbol{y}_n)$
- Corresponds to neural net with multiple output nodes
- Given a set of weight values $\boldsymbol{w}$, the network defines a function $\boldsymbol{h}_{\boldsymbol{w}}(\boldsymbol{x})$.
- Can train by minimizing L2 loss:

$$E(\boldsymbol{w}) = \sum_{n=1}^{N} |\boldsymbol{h}_{\boldsymbol{w}}(\boldsymbol{x}_n) - \boldsymbol{y}_n)|^2 = \sum_{n=1}^{N} \sum_{k} (y_k - a_k)^2$$

where k indexes the output nodes

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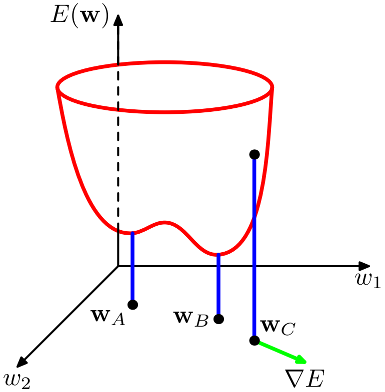
Parameter Optimization



- For either of these problems, the error function $E(\boldsymbol{w})$ is nasty
  - Nasty = non-convex
  - Non-convex = has local minima

Gradient Descent

- The function $h_w(\boldsymbol{x})$ implemented by a network is complicated.
- No closed-form: Use gradient descent.
- It isn't obvious how to compute error function derivatives with respect to *hidden* weights.
  - The credit assignment problem.
- Backpropagation solves the credit assignment problem

Outline

Feed-forward Networks

Network Training

Error Backpropagation

Applications

Error Backpropagation

- Backprop is an efficient method for computing error derivatives $\frac{\partial E}{\partial w_{ij}}$ for *all* weights in the network. Intuition:
  1. Calculating derivatives for weights connected to output nodes is easy.
  2. Treat the derivatives as virtual "error", compute derivative of error for nodes in previous layer.
  3. Repeat until you reach input nodes.
- This procedure propagates backwards the output error signal through the network.
- Stochastic Gradient Descent: Fix input $\boldsymbol{x} \equiv \boldsymbol{x}_n$ and target output $\boldsymbol{y} \equiv \boldsymbol{y}_n$, resulting in error E_n.

Error at the output nodes

- First, feed training example $\mathbf{x}_n$ forward through the network, storing all node activations a_i
- Calculating derivatives for weights connected to output nodes is easy.
  - like logistic regression with input "features" a_i
- For output node j with activation $a_j = g(in_j) = g(\sum_i w_{ij} a_i)$:

$$\frac{\partial E_n}{\partial w_{ij}} = \frac{\partial}{\partial w_{ij}} \frac{1}{2}(y_j - a_j)^2 = -a_j \times g'(in_j) \times (y_j - a_j)$$

- 0 if no error, or if input a_i from node i is 0.
- **Modified Error**: $\Delta[j] \equiv g'(in_j)(y_j - a_j)$.
- Gradient Descent Weight Update:

$$w_{ij} \leftarrow w_{ij} + \alpha \times a_i \times \Delta[j]$$

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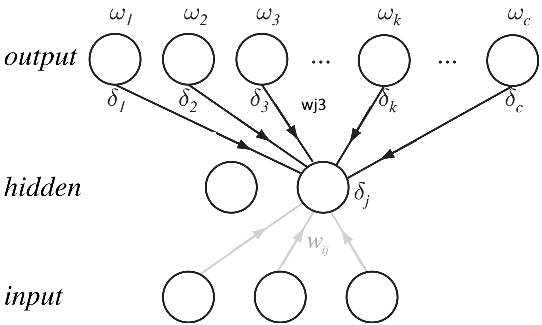
Error at the hidden nodes

- Consider a hidden node i connected to downstream nodes in the next layer.
- The **modified error** signal $\Delta[i]$ is node activation derivative, times the *weighted sum of contributions to the connected errors.*
- In symbols,

$$\Delta[i] = g'(in_i) \sum_j w_{ij} \Delta[j].$$

Backpropagation Picture



The error signal at a hidden unit is proportional to the error signals at the units it influences:

$$\Delta[j] = g'(in_j) \sum_k w_{jk} \Delta[k].$$

The Backpropagation Algorithm

1. Apply input vector $\boldsymbol{x}_n$ and forward propagate to find all inputs in_i and activation output levels a_i.
2. Evaluate the error signals $\Delta[j]$ for all output nodes.
3. Backpropagate the $\Delta[j]$ to obtain error signals $\Delta[i]$ for each hidden node i.
4. Update each weight vector w_{ij} using

$$w_{ij} := w_{ij} + \alpha \times a_i \times \Delta[j].$$

Demo AIspace `http://aispace.org/neural/`.

Other Learning Topics

- Regularization: L2-regularizer (weight decay).
- Prune Weights: the Optimal Brain Method.
- Experimenting with Network Architectures is often key.

Outline

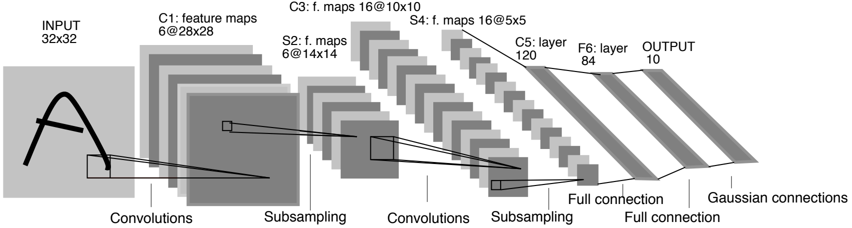
Applications of Neural Networks

- Many success stories for neural networks
  - Credit card fraud detection
  - Hand-written digit recognition
  - Face detection
  - Autonomous driving (CMU ALVINN)

Hand-written Digit Recognition

- MNIST - standard dataset for hand-written digit recognition
    - 60000 training, 10000 test images

LeNet-5



- LeNet developed by Yann LeCun et al.
  - Convolutional neural network
    - Local receptive fields (5x5 connectivity)
    - Subsampling (2x2)
    - Shared weights (reuse same 5x5 “filter”)
    - Breaking symmetry
- See http://www.codeproject.com/KB/library/NeuralNetRecognition.aspx

- The 82 errors made by LeNet5 (0.82% test error rate)

Conclusion

- Feed-forward networks can be used for regression or classification
- Learning is more difficult, error function not convex
  - Use stochastic gradient descent, obtain (good?) local minimum
- Backpropagation for efficient gradient computation