Charge-Coupled Device

- A charge-coupled device (CCD) is an integrated circuit element (a monolithic device) of semiconductor material.
- A silicon wafer (p type or n type) is oxidized to generate a layer of SiO2 on its surface.
- A matrix of metal electrodes is deposited on the oxide layer and is linked to the CCD output leads.
- When light falls onto the CCD element (from an object), a charge packets are generated within the substrate silicon wafer. Now if an external potential is applied to a particular electrode of the CCD, a potential well is formed under the electrode and a charge packet is deposited here. This charge packet can be moved across the CCD to an output circuit.

Image Sensors:

- Imaging device is a sensor, and an image is the sensed data.
- Depending on the imaging device, an image can be of many varieties such as optical, thermal or infrared, x-ray, ultraviolet, acoustic, ultrasound, and so on.

Image Processing and Computer Vision:

- An image may be processed (analyzed) to obtain a more refined image from which useful information such as edges, contours, areas, and other geometrical information can be determined.
- Computer Vision involves higher level operations than image processing and is akin to what humans infer based on what they see.
  1. Filtering: (to remove noise and enhance the image) including directional filtering (to enhance edges, for edge detection)
  2. Thresholding: (to generate a two-level black-and-white image where the gray levels above a set threshold are assigned white and those below the threshold are assigned black)
  3. Segmentation: (to subdivide an enhanced image, identify geometric shapes/objects, and capture properties such as area and dimensions of the identified geometric entities)
  4. Morphological processing: (sequential shrinking, filtering, stretching, etc. to prune out unwanted image components and extract those that are important)
  5. Subtraction: (e.g., subtract the background form the image)
  6. Template matching: (to match a processed image to a template—useful in object detection)
  7. Compression: (to reduce the quantity of data that is needed to represent the useful information of an image)
Hall-Effect Sensor:
- Consider a semiconductor element subject to a dc voltage $v_{ref}$.
- If a magnetic field is applied perpendicular to the direction of this voltage, a voltage $v_o$ will be generated in the third orthogonal direction within the semiconductor element. This is known as the Hall Effect (observed by E.H. Hall in 1879).

Hall-Effect Motion Sensors:
- A Hall-effect sensor may be used for motion sensing in many ways; for example, as an analog proximity sensor, a limit switch (digital), or a shaft encoder.
- Output voltage $v_o$ increases as the distance from the magnetic source to the semiconductor element decreases, the output signal $v_o$ can be used as a measure of proximity.
- Digitally speaking, certain threshold level of the output voltage $v_o$ can be used to generate a binary output, which represents the presence/absence of an object.
- The use of a toothed ferromagnetic wheel (as for a digital tachometer) to alter the magnetic flux will result in a shaft encoder.

Ultrasonic Sensors:
- Ultrasound waves are pressure waves, just like sound waves, but their frequencies are higher (ultra) than the audible frequencies (range of 20 Hz to 20 kHz).
- Ultrasonic sensors are used in many applications, including medical imaging, ranging systems for cameras with autofocusing capability, level sensing, and speed sensing.
- Velocity - Using Doppler Effect.

$$\chi = \frac{ct}{2};$$
- $t$ is the time of flight of the ultrasound pulse (from generator to receiver)
- $x$ is the distance between the ultrasound generator/receiver and the target object
- $c$ is the speed of sound in the medium (typically, air)
Tactile Sensing:

- Tactile sensing is usually interpreted as touch sensing, but tactile sensing is different from a simple clamping where very few discrete force measurements are made.
- In tactile sensing, a force distribution is measured, using a closely spaced array of force sensors and usually exploiting the skin-like properties of the sensor array.
- Tactile sensing is particularly important in two types of operations:
  - Grasping and fine manipulation, and
  - Object identification.
- In **grasping and fine manipulation**, the object has to be held in a stable manner without being allowed to slip and without being damaged.
- **Object identification** includes recognizing or determining the shape, location, and orientation of an object as well as detecting or identifying surface properties (e.g., density, hardness, texture, flexibility), and defects.

Ideally, these tasks would require two types of sensing:

- Continuous spatial sensing of time-variable contact force
- Sensing of surface deformation profiles (time-variable)

- Note that learning also can be an important part of tactile sensing.

- Typical specifications for an industrial tactile sensor are as follows:
  - Spatial resolution of about 1 mm (about 100 sensor elements)
  - Force resolution of about 2 g
  - Dynamic range of 60 dB
  - Force capacity (maximum touch force) of about 1 kg
  - Response time of 5 ms or less (a bandwidth of over 200 Hz)
  - Low hysteresis (low energy dissipation)
  - Durability under harsh working conditions
  - Robustness and insensitivity to change in environmental conditions (temperature, dust, humidity, vibration, etc.)
  - Capability to detect and even predict slip

Dexterity:

Dexterity is an important consideration in sophisticated manipulators and robotic hands that employ tactile sensing:

\[
\text{Motion Dexterity} = \frac{\text{Number of degrees of freedom in the device}}{\text{Motion resolution of the device}}
\]

\[
\text{Force Dexterity} = \frac{\text{Number of degrees of freedom}}{\text{Force resolution}}
\]

Read Example 6.10 and Strain Gauge example 6.11
MEMS Sensors:
- Microelectromechanical systems (MEMS) are microminiature devices consisting of microminiature components such as sensors, actuators, and signal processing integrated and embedded into a single chip while exploiting both electrical/electronic and mechanical features of them.
- The device size can be in the sub-millimeter scale (0.01–1.0 mm) and
- The component size can be as small as a micrometer (micron), in the range 0.001–0.1 mm. Since MEMS exploits the integrated-circuit (IC) technologies in their fabrication, many components can be integrated into a single device (e.g., a few to a million).

The advantages of MEMS are primarily the advantages of IC devices which include:
- Microminiature size and weight
- Large surface area to volume ratio (when compared in the same measurement units)
- Large-scale integration (LSI) of components/circuits
- High performance
- High speed (20 ns switching speeds)
- Low power consumption
- Easy mass-production
- Low cost (in mass production)
- In particular, the microminiature size also means negligible mechanical loading, fast response, and negligible power consumption (and related electrical loading).

Energy Conversion Mechanism:
**Piezoelectric:** Mechanical strain in a piezoelectric material causes a charge separation across the material producing a voltage. Strain energy produced by the mechanical work that is needed to deform the material, is converted into electrostatic energy. This is a passive device.

**Electrostatic:** A voltage causes + and – charges to separate into the capacitor plates. The attraction force between the plates is supported by an external mechanical force. If plates move apart, mechanical work is done, capacitance is reduced, and the voltage is increased. Hence, mechanical energy is converted into electrical energy. This is a passive device.

**Electromagnetic:** As a coil moves in a magnetic field, a current is induced in the coil. In this process, mechanical energy is converted into electrical energy. This is a passive device.
Magnetic Circuits:

Figure 15.1 Magnetic fields can be visualized as lines of flux that form closed paths. Using a compass, we can determine the direction of the flux lines at any point. Note that the flux density vector $\mathbf{B}$ is tangent to the lines of flux.

*See - Magnetic Circuits pdf copy on line*
Stepper Motors:
The terms *stepper motor*, *stepping motor*, and *step motor* are synonymous and are often used interchangeably.

There are three basic types of stepper motors:
1. *Variable-reluctance* (VR) stepper motors, which have soft-iron (ferromagnetic) rotors
2. *Permanent-magnet* (PM) stepper motors, which have magnetized rotors
3. *Hybrid* (HB) stepper motors, which have two stacks of rotor teeth forming the two poles of a permanent magnet located along the rotor axis

The VR and PM steppers operate in a somewhat similar manner:
- Specifically, the *stator magnetic field (polarity) is stepped* so as to change the minimum reluctance (or detent) position of the rotor in increments.
- Hence, both types of motors undergo similar changes in reluctance (magnetic resistance) during operation.
- A disadvantage of VR steppers is that as the rotor is not magnetized, the holding torque is practically zero when the stator windings are not energized (i.e., power-off conditions). Hence, it is not capable to hold the mechanical load at a given position under power-off conditions, unless mechanical brakes are employed.
- An HB stepper motor possesses characteristics of both VR steppers and PM steppers.
  - The rotor of an HB stepper motor consists of two rotor segments connected by a shaft.
  - Each rotor segment is a toothed wheel and is called a stack.
  - The two rotor stacks form the two poles of a permanent magnet located along the rotor axis.
  - Hence, an entire stack of rotor teeth is magnetized to be a single pole (which is different from the case of a PM stepper where the rotor has multiple poles).
The rotor polarity of an HB stepper can be provided either by a permanent magnet, or by an electromagnet using a coil activated by a unidirectional dc source and placed on the stator to generate a magnetic field along the rotor axis.

A photograph of the internal components of a two-stack stepping motor is given in figure.

**Permanent-Magnet Stepper Motor**

To explain the operation of a PM stepper motor, consider the simple schematic diagram shown in figure.

- The stator has two sets of windings (i.e., two phases) placed at 90°.
- This arrangement has four *salient poles* in the stator, each pole geometrically separated by a 90° angle from the adjacent one. The rotor is a two-pole permanent magnet. Each phase can take one of the three states 1, 0, and −1, which are defined as follows:
  1. **State 1:** current in a specified direction
  2. **State −1:** current in the opposite direction
  3. **State 0:** no current

**Note that:**

- As −1 is the complement state of 1, in some literature the notation 1′ is used to denote the state −1.
- By switching the currents in the two phases in an appropriate sequence, either a clockwise (CW) rotation or a counterclockwise (CCW) rotation can be generated. The CW rotation sequence is shown.
- \( \phi_i \) denotes the state of the \( i_{th} \) phase.
- The step angle for this motor is 45°. At the end of each step, the rotor assumes the minimum reluctance position that corresponds to the particular magnetic polarity pattern in the stator. This is a stable equilibrium configuration and is known as the detent position for that step.
- Note: *Reluctance measures the magnetic resistance* in a flux path.
When the stator currents (phases) are switched for the next step, the minimum reluctance position changes (rotates by the step angle) and the rotor assumes the corresponding stable equilibrium position and the rotor turns through a single step (45° in this example). Observe that in one complete rotation of the rotor, the state of each phase sweeps through one complete cycle of the switching sequence in figure in the CW direction.

- For CW rotation of the motor, the state of phase 2 ($\phi_2$) lags the state of phase 1 ($\phi_1$) by two steps.
- For CCW rotation, $\phi_2$ leads $\phi_1$ by two steps.
- Hence, instead of eight pairs of numbers, just eight numbers with a delay operation would suffice to generate the phase-switching logic.
- This approach is faster and more effective because the switching logic for a stepper motor.

**Increasing Resolution:**

- A stepping resolution of 45 degrees is too coarse for most applications. We can increase the resolution by adding the rotor pole pairs.
  - e.g.: Stepping Resolution of 30 degrees
  - Common types are 30, 15, and 7.5 degrees.
  - For high precision, this solution is not cost effective.
Variable-Reluctance Stepper Motor
Now consider the VR stepper motor shown.

- The rotor is a non-magnetized soft-iron (ferromagnetic) bar.
- The full-stepping sequence for CW rotation is shown in figure below. The step angle is 60°. Only one phase is energized at a time in order to execute full stepping.
- With VR steppers, the direction of the current (the polarity of a stator pole pair) is not reversed in the full-stepping sequence; only the states 1 and 0 (i.e., on and off) are used for each phase.
- In the case of half stepping, however, two phases have to be energized simultaneously during some steps.
- Furthermore, current reversals are needed in half stepping, thus requiring more elaborate switching circuitry.
- The advantage, however, is that the step angle would be halved to 30°, thereby providing improved motion resolution.
- When two phases are activated simultaneously, the minimum reluctance position is halfway between the corresponding pole pairs (i.e., 30° from the detent position that is obtained when only one of the two phases is energized), which enables half stepping.
- It follows that, depending on the energizing sequence of the phases, either full stepping or half stepping would be possible.
- Micro-stepping provides much smaller step angles achieved by changing the phase currents by small increments (rather than just the states on, off, and reversal) so that the detent (equilibrium) position of the rotor shifts in correspondingly small angular increments.
Polarity Reversal:
The polarity of a stator pole can be reversed in two ways:
- There is only one set of windings for a group of stator poles. This is the case of unifilar windings. Polarity of the poles is reversed by reversing the direction of current in the winding.
- There are two sets of windings for a group of stator poles. This is the case of bifilar windings (i.e., double-file or two-coil windings).
- Only one set of windings is energized at a time, producing one polarity for this group of poles. The other set of windings produces the opposite polarity.
- Note: One winding with a center tap may be used in place of two windings. The other two terminals of the coil are given opposite (i.e., positive and negative) voltages.

Stepper Motor Classification:
- Most classifications of stepper motors are based on the nature of the motor rotor.
- One such classification considers the magnetic character of the rotor. Specifically, as discussed before:
  - VR stepper motor has a soft-iron rotor, whereas
  - PM stepper motor has a magnetized rotor.
- Another practical classification that is based on the number of stacks of teeth (or rotor segments) present on the rotor shaft. In particular, an HB stepper motor has two stacks of teeth.
- Further sub-classifications are possible, depending on the tooth pitch (angle between adjacent teeth) of the stator and the tooth pitch of the rotor.
- In a single-stack stepper motor:
  - The rotor tooth pitch and the stator tooth pitch generally have to be unequal
  - So that not all teeth in the stator are ever aligned with the rotor teeth at any instant.
  - It is the misaligned teeth that exert the magnetic pull, generating the driving torque.
  - In each motion increment, the rotor turns to the minimum reluctance (stable equilibrium) position corresponding to that particular polarity distribution of the stator.
- In multiple-stack stepper motors:
  - Operation is possible even when the rotor tooth pitch is equal to the stator tooth pitch, provided that at least one stack of rotor teeth is rotationally shifted (misaligned) from the other stacks by a fraction of the rotor tooth pitch.
  - In this design, it is this inter-stack misalignment that generates the drive torque for each motion step. It is obvious that unequal-pitch multiple-stack steppers are also a practical possibility. In this design, each rotor stack operates as a separate single-stack stepper motor. The stepper motor classifications described thus far are summarized in Figure 8.8.
Single-Stack Stepper Motors:

Phase Winding \( P = 3 \).

Rotor teeth \( N_r = 8 \); Soft iron.

Stator teeth \( N_s = 12 \).

Poles per phase \( m = 4 \).

Stator pitch \( \theta_s = \frac{360}{N_s} \).

Rotor pitch \( \theta_r = \frac{360}{N_r} \).

Step Angle \( \Delta \theta \): Smallest misalignment between stator pole and adjacent rotor tooth in any stable equilibrium.

\[ \Delta \theta = \theta_r - r\theta_s \quad (\theta_r > \theta_s) \]

\[ \Delta \theta = \theta_s - r\theta_r \quad (\theta_r < \theta_s) \]

\( r \) is the largest positive integer such that \( \Delta \theta > 0 \).

i.e. Largest feasible \( r \) such that a misalignment occurs.

Since \( N_s = 12 \rightarrow \theta_s = \frac{360}{12} = 30^\circ \)

\( N_r = 8 \rightarrow \theta_r = \frac{360}{8} = 45^\circ \)

\[ \Delta \theta = \theta_r - \theta_s = 15^\circ \]; Each switching corresponds to rotation \( \theta \).

If phase-1 is off, phase-2 is on; Rotor will turn 15° CCW.

If phase-3 is on, Rotor will turn 15° CW.

If phase-1 is on & phase-2 is on; 75° is also possible.

Full stepping: 1–2–3–1 CCW; 1–3–2–1 CW

Se have \( p \) phases meaning the whole switching angle is: \( p \Delta \theta \)

From original Equation \( \Delta \theta = \theta_r - \theta_s \)

Substituting \( \theta_r = p \Delta \theta \) above.

\[
\theta_r = r \theta_s + \frac{\theta_r}{p} \quad (\theta_r > \theta_s)
\]

For Equation \( \Delta \theta = \theta_s - r \theta_r \quad (\theta_r < \theta_s) \)

\[
\theta_s = r \theta_r + \frac{\theta_r}{p} \quad (\theta_r < \theta_s)
\]

\( \theta_r \) = rotor tooth pitch angle
\( \theta_s \) = stator tooth pitch angle
\( p \) = # of phases in stator
\( r \) = Largest feasible positive integer

→ pitch angle definition.

\[
360^\circ = \frac{r \times 360}{n_s} + \frac{360}{n_r}
\]

or

\[
\theta_s = r n_r + \frac{n_s}{p} \quad ; (n_s > n_r)
\]

and

\[
\theta_r = r n_s + \frac{n_s}{p} \quad (n_s < n_r)
\]

Finally # of Revolution,

\[
\eta = \frac{360}{\Delta \theta}
\]

Read Examples 8.1, 8.2, 8.3
Advantages of Toothed Construction:

The toothed construction of the stator and the rotor of a stepper motor has many advantages.

1. It improves the motion resolution (step angle), which now depends on the tooth pitch. Very small step angles can be achieved as a result.
2. It enhances the concentration of the magnetic field, which generates the motor torque. This means improved torque characteristics.
3. The torque and motion characteristics become smoother (smaller ripples and less jitter) as a result of the distributed tooth construction.

In the case shown in figure above, the stator teeth are equally spaced but the pitch (angular spacing) is not identical to the pitch of the rotor teeth.

In the toothed-stator construction, \( n_s \) represents the number of teeth rather than the number of poles in the stator. The number of rotor teeth has to be increased in proportion.

Governing Equations:

\[
\begin{align*}
\text{Considering } & \theta_r > \theta_s \text{ (i.e. } n_r < n_s) \\
& \theta_r - \theta_s: \text{ offset between rotor and stator pitch} \\
& \# \text{ of rotor teeth } n_s / m_p \\
& \Delta \theta = \frac{n_s}{m_p} (\theta_r - \theta_s); \theta_r > \theta_s; \ p: \# \text{ of phases} \\
& m: \# \text{ of stator poles per phase} \\
& \Delta \theta = \frac{\theta_r}{p}; \text{ True for toothed construction as well} \\
& \Theta_r = p \Delta \theta \text{ and } n_s = m p \theta_s \\
& \Delta \theta = \frac{n_s}{m_p} (\theta_r - \theta_s) \Rightarrow n_s = n_r + m \quad (n_r < n_s) \\
& \text{General Formula: } n_s = n_r + m
\end{align*}
\]

Read Example 8.4.
Micro-Stepping:

- Microstepping is achieved by properly changing the phase currents in small steps, instead of switching them on and off (as in the case of full stepping and half stepping).
- The principle behind this can be understood by considering two identical stator poles (wound with identical windings), as shown in diagram.

- When the currents through the windings are identical (in magnitude and direction) the resultant magnetic field will lie symmetrically between the two poles.

- If the current in one pole is decreased while the other current is kept unchanged, the resultant magnetic field will move closer to the pole with the larger current.

- As the detent position (equilibrium position) depends on the position of the resultant magnetic field, it follows that very small step angles can be achieved simply by controlling (varying the relative magnitudes and directions of) the phase currents.

- Step angles of 1/125 of a full step or smaller may be obtained through microstepping.

- For example, 10,000 steps/revolution may be achieved.

- Note: The step size in a sequence of microsteps is not identical. This is because stepping is done through microsteps of the phase current, which (and the magnetic field generated by it) has (have) a nonlinear relation with the physical step angle.
Multiple-Stack Stepper Motors

- Both equal pitch and unequal pitch constructions are possible.
- Smaller step angles are possible with unequal construction.
- For the equal pitch case, each stator segment has several poles and all poles of each stator segment are wound to the same phase.
- Misalignment can be achieved by two methods:
  1. The teeth in the *stator segments are aligned* but the *teeth in the rotor segments are misaligned* consecutively by 1/3 pitch angle.
  2. The teeth in the *rotor segments are aligned* but the *teeth in the stator segments are misaligned* consecutively by 1/3 pitch angle.

- Both full stepping and half stepping can be achieved.
- For full stepping the step angle is $\theta_s/3 = \theta_r/3$.
- Stepping sequence 1-2-3-1 would turn the rotor in one direction and 1-3-2-1 would turn in the other.
- For half stepping they are 1-12-2-23-3-31-1 and 1-13-3-32-2-21-1
- The full stepping angle if there are $s$-stacks:
  \[
  \Delta \theta = \frac{\theta_r}{s} = \frac{\theta_s}{s} = \frac{\theta}{s}
  \]
- Very fine angular resolutions can be achieved with unequal multi-stack stepper motors. Switching logic of these stepper motors is more complex.
- Since the step angle of a non-toothed single stack stepper motor is $(\theta_r-\theta_s)$
  \[
  \Delta \theta = \frac{\theta_r - \theta_s}{s}
  \]
- For a tooth-pole multi-stack motor:
  \[
  \Delta \theta = \frac{n_s (\theta_r - \theta_s)}{mps} \quad \Delta \theta = \frac{\theta_r}{ps}
  \]
Equal Pitch and Unequal Pitch:

**Equal-Pitch Multiple-Stack Stepper:**

- For each rotor stack, there is a toothed stator segment around it, whose pitch angle is identical to that of the rotor \( \theta_s = \theta_r \).
- A stator segment may appear to be similar to that of an equal-pitch single-stack stepper, but this is not the case.
- Each stator segment is wound to a single phase, thus the entire segment can be energized (polarized) or de-energized (depolarized) simultaneously. It follows that, in the equal pitch case,
- Meaning \( p = s \); where \( p \) is the \# of phases and \( s \) is the \# of rotor stacks.
- The misalignment that is necessary to produce the motor torque may be introduced in one of two ways:
  1. The teeth in the stator segments are perfectly aligned, but the teeth in the rotor stacks are misaligned consecutively by \( 1/s \times \) pitch angle.
  2. The teeth in the rotor stacks are perfectly aligned, but the teeth in the stator segments are misaligned consecutively by \( 1/s \times \) pitch angle.

- Now consider the three-stack case.
  - Suppose that phase 1 is energized
    - Then the **teeth in the rotor stack 1** will align perfectly with the stator teeth in phase 1 (segment 1).
    - But the teeth in the rotor stack 2 will be shifted from the stator teeth in phase 2 (segment 2) by a one-third-pitch angle in one direction, and
    - The teeth in rotor stack 3 will be shifted from the stator teeth in phase 3 (segment 3) by a two-thirds pitch angle in the same direction (or a one-third-pitch angle in the opposite direction).
  - It follows that **if phase 1** is now de-energized and phase 2 is energized:
    - The rotor will turn through one-third pitch in one direction.
    - If, instead, phase 3 is turned on after phase 1, the **rotor will turn through one-third pitch in the opposite direction**.
    - Clearly, the step angle (for full stepping) is a one-third-pitch angle for the three-stack, three-phase construction.
    - The switching sequence 1-2-3-1 will turn the rotor in one direction,
    - And the switching sequence 1-3-2-1 will turn the rotor in the opposite direction.

In general, for a stepper motor with \( s \) stacks of teeth on the rotor shaft, the full-stepping step angle is given by: \( \Delta \theta = \frac{\theta}{s} = \frac{\theta}{p} \); where \( \theta = \theta_r = \theta_s \) = tooth pitch angle.
• Note that the step angle can be decreased by increasing the number of stacks of rotor teeth.
• Increased number of stacks also means more phase windings with associated increase in the magnetic field and the motor torque.
• However, the length of the motor shaft increases with the number of stacks, and can result in flexural (shaft bending) vibration problems (particularly whirling of the shaft), air gap contact problems, large bearing loads, wear and tear, and increased noise.
• As in the case of a single-stack stepper, half stepping can be accomplished by energizing two phases at a time.
• Hence, in the three-stack stepper, for one direction, the half-stepping sequence is 1-12-2-23-3-31-1;
• In the opposite direction, it is 1-13-3-32-2-21-1.

Unequal-Pitch Multiple-Stack Stepper
• Very fine angular resolutions (step angles) can be achieved by this design without compromising the length of the motor.
• In an unequal-pitch stepper motor, each stator segment has more than one phase (p number of phases).
• Rather than a simple cascading, however, the phases of different stacks are not wound together and can be switched on and off independently. In this manner yet finer step angles are realized, together with an added benefit of increased torque provided by the multistack design.
• For a single-stack non-toothed-pole stepper, we have seen that the step angle is equal to $\theta_r - \theta_s$.
• In a multistack stepper, this misalignment is further subdivided into s equal steps using the interstack misalignment.
• Hence, the overall step angle for an unequal-pitch, multiple-stack stepper motor with nontoothed poles is given by:
  \[ \Delta \theta = \frac{\theta_r - \theta_s}{s} \text{ for } \theta_r > \theta_s \]
• For a toothed-pole multiple-stack stepper motor, we have:
  \[ \Delta \theta = \frac{n_s(\theta_r - \theta_s)}{mps} \text{ for } \theta_r > \theta_s \]
  o m is the number of stator poles per phase.
  o p is the number of phases in each stator segment and
  o s is the number of rotor stacks and s is the number of rotor stacks.
Controller and Driver:

Time Constant and Torque Degradation:

- As the \textit{torque generated by a stepper motor} is \textit{proportional to the phase current}.
- It is desirable for a phase winding to reach its maximum current level as quickly as possible when it is switched on.
- Unfortunately, as a result of \textit{self-induction}, the current in the energized phase does not build up instantaneously when switched on.
- As the stepping rate increases, the time period that is available for each step decreases.
- Consequently, \textit{a phase may be turned off before reaching its desired current level in order to turn on the next phase}, thereby \textit{degrading the generated torque as shown in figure}.
- One way to increase the current level reached by a phase winding would be to simply increase the supply voltage as the stepping rate increases.
- Another approach would be to use a chopper circuit (a switching circuit) to switch on and off at high frequency, a supply voltage that is several times higher than the rated voltage of a phase winding.
- Specifically, \textit{a sensing element (typically, a resistor) in the drive circuit detects the current level and when the desired level is reached, the voltage supply is turned off}.
- When the \textit{current level goes below the rated level, the supply is turned on} again. The required switching rate (chopping rate) is governed by the electrical time constant of the motor. The electrical Time constant is given by: \( \tau_e = \frac{L}{R} \)

\begin{itemize}
  \item \text{L} - Inductance of the energized phase winding
  \item \text{R} - Resistance of the energized circuit, including winding resistance
\end{itemize}

\textit{Self inductance} is defined as the \textit{induction} of a voltage in a current-carrying wire when the current in the wire itself is changing. In the case of \textit{self-inductance}, the magnetic field created by a changing current in the circuit itself \textit{induces} a voltage in the same circuit. Therefore, the voltage is \textit{self-induced}.
The current increase (build-up) equation is given by:  
\[ i = \frac{v}{R} \exp \left( 1 - \frac{t}{\tau_e} \right); \quad \tau_e = \frac{L}{R} \]

- The larger the electrical time constant the slower the current buildup.
- The driving torque of the motor decreases due to the lower phase current.
- Also, because of self-induction, the current does not die out instantaneously when the phase is switched off.

- The torque characteristics of a stepper motor can be improved (particularly at high stepping rates) and the harmful effects of induced voltages can be reduced by decreasing the electrical time constant.

- A convenient way to accomplish this is by increasing the resistance R.

- Note that we want this increase in R to be effective only during the transient periods (at the instants of switch-on and switch-off).

- During the steady period, we like to have a smaller R, which will give a larger current (and magnetic field), producing a higher torque, and furthermore lower power dissipation (and associated mechanical and thermal problems) and reduction of efficiency.

- This can be accomplished by using a diode and a resistor ΔR, connected in parallel with the phase winding, as shown in figure above.

- In this case, the current will loop through R and ΔR, as shown, during the switch-on and switch-off periods, thereby decreasing the electrical time constant to:  
  \[ \tau_e = \frac{L}{R + \Delta R} \]
Chapter-9: DC Motors:

- A dc motor converts dc electrical energy into rotational mechanical energy.
- A major part of the torque generated in the rotor (armature) of the motor is available to drive an external load.
- DC motors are still widely used in numerous engineering applications including robotic manipulators, vehicles, transport mechanisms, disk drives, positioning tables, machine tools, biomedical devices, and servo-valve actuators.
- In view of effective control techniques that have been developed for ac motors, they are rapidly becoming popular in applications where dc motors had dominated. Still, dc motor is the basis of the performance of an ac motor which is judged in such applications.

Principle of Operation:
The principle of operation of a dc motor is illustrated in figure shown.

- Consider an electric conductor placed in a steady magnetic field at right angles to the direction of the field.
- Flux density B is assumed constant.
- If a dc current is passed through the conductor, the magnetic flux is formed due to the current loops around the conductor, as shown in the figure.
- Consider a plane through the conductor, parallel to the direction of flux of the magnet.
- On one side of this plane, the current flux and the field flux are additive; on the opposite side, the two magnetic fluxes oppose each other. As a result, an imbalance magnetic force F is generated on the conductor, normal to the plane.
- This force (Lorentz’s force) is given by the Lorentz’s law: \( F = B \times i \times l \)
  - B is the flux density of the original field,
  - i is the current through the conductor and
  - l is the length of the conductor
- The active components of i, B, and F are mutually perpendicular and form a right-hand triad, as shown in figure. OR In other words, in the vector representation of these three quantities, the vector F can be interpreted as the cross product of the vectors i and B. Specifically, \( F = i \times B \).
If the conductor is free to move, the generated force moves it at some velocity $v$ in the direction of the force.

As a result of this motion in the magnetic field $B$, a voltage is induced in the conductor. This is known as the back electromotive force or back e.m.f., and is given by: $v_b = Blv$

According to Lenz’s law, the flux due to the back e.m.f. $v_b$ opposes the flux due to the original current through the conductor, thereby trying to stop the motion. This is the cause of electrical damping in motors.

**Static Torque Characteristics:**
For static torque we assume that the motor speed is low so that the dynamic effects need not be explicitly included in the discussion.

- Consider a two-pole permanent magnet stator and a planar coil that is free to rotate about the motor axis, as shown in figure-a.
- The coil (rotor, armature) is energized by current $i_a$ as shown.
- The flux density vector of the stator magnetic field is $B$ and the unit vector normal to the plane of the coil is $n$.
- The angle between $B$ and $n$ is $\delta$, which is known as the torque angle.
- It should be clear from figure-b that the torque $T$ generated in the rotor is given by $T = F \times 2r \sin \delta$.
- Which becomes $T = Bi_a l \times 2r \sin \delta$, or $T = A i_a B \sin \delta$;
  - $l$ is the axial length of the rotor
  - $r$ is the radius of the rotor
  - $A$ is the face area of the planar rotor
Continuing from last page:

- Suppose that the rotor rotation starts by coinciding with the commutation plane, where $\delta = 0$ or $\pi$, and the rotor rotates through an angle of $2\pi$.
- The corresponding torque profile is shown in figure on top.

- Next suppose that the rotor has three planar coil segments placed at 60° apart, and denoted by 1, 2, and 3, as in figure #2.
- Note that current switching occurs at every 60° rotation, and in a given instant two coil segments are energized.

- Figure shows the torque profile of each coil segment and the overall torque profile due to the three-segment rotor in Figure#3.
- Note that the torque profile has improved (i.e., larger torque magnitude and smaller variation) as a result of the multiple coil segments, with shorter commutation angles.
- The torque profile can be further improved by incorporating still more coil segments, with correspondingly shorter commutation angles, but the design of the split-ring and brush arrangement becomes more challenging then.

- Hence, there is a design limitation to achieving uniform torque profiles in a dc motor.
- It should be clear from Figure#2 that if the stator field can be made radial, then $B$ is always perpendicular to $n$ and hence $\sin \delta$ becomes equal to 1. In that case, the torque profile is uniform, under ideal conditions.
Brushless DC Motors:
Before we get into Brushless DC Motors, we should look at shortcomings of the slip-ring and brush mechanisms:
- Rapid wear out, mechanical loading,
- Heat generation due to sliding friction,
- Contact bounce,
- Excessive noise, and
- Electrical sparks with the associated dangers in hazardous (e.g., chemical) environments,
- Problems of oxidation,
- Problems in applications that require wash down (e.g., in food processing), and voltage ripples at current switching instants.
- Conventional remedies to these problems—such as the use of improved brush designs and modified brush positions to reduce sparking—are inadequate in more demanding and sophisticated applications.
- Cooling of the coils is typically needed in long-period operation of heavy-duty motors which may be achieved through forced convection of air or water.
- In addition, the required maintenance (to replace brushes and resurface the split-ring commutator) can be rather costly and time consuming.
- Electronic communication, as used in brushless dc motors, is able to overcome these problems.

Permanent-Magnet Motors:
- Brushless dc motors have permanent-magnet rotors.
- Since in them the polarities of the rotor cannot be switched as the rotor crosses a commutation plane, commutation is accomplished by electronically switching the current in the stator winding segments.
- Note that this is the reverse of what is done in brushed commutation,
  o where the stator polarities are fixed and
  o The rotor polarities are switched when crossing a commutation plane.
- The stator windings of a brushless dc motor can be considered the armature windings, whereas for a brushed dc motor, rotor is the armature.
- The torque–speed characteristics of dc motors are different from those of stepper motors or ac motors.
- Brushless DC motors are commonly used for smooth torque transitions and speed control whereas, stepper motors are commonly used for stepping precision motion control.
DC Motor Equations:

- Consider a dc motor with separate windings in the stator and the rotor.
- Each coil has a resistance (R) and an inductance (L).
- When a voltage (v) is applied to the coil, a current (i) flows through the circuit, thereby generating a magnetic field.
- Forces are produced in the rotor windings, and an associated torque ($T_m$), which turns the rotor.
- The rotor speed ($\omega_m$) causes the magnetic flux linkage with the rotor coil from the stator field to change at a corresponding rate, thereby generating a voltage (back e.m.f.) in the rotor coil.
- Equivalent circuits for the stator and the rotor of a conventional dc motor are shown.
- Since the field flux is proportional to the field current $i_f$, we can express the magnetic torque of the motor as: $T_m = k_f i_a = k_m i_a$

Using Equation $F = Bl$ and $V_b = Blv$

Back emf generated in Armature is \[ V_b = k_f \omega_m = k'm \omega_m \]

\[ k_f = k_m \]

Field Current $i_f$

Armature Current $i_a$

Angular Speed of Motor $\omega_m$

$k, k'$: Motor Constants: dependent on dimensions, turns, $L$, Resistance etc.

Note that: Ideal Electrical to Mechanical Conditions => $I_m \times \omega_m = V_b \times i_a$

Under ideal conditions $k = k'$

or $k_m = k_m'$
Obtaining equations for Field Circuit, Armature Circuit and Mechanical Dynamic:

Field Circuit Equations:
- Assuming stator magnetic field is not affected by the rotor magnetic field.
  \[ V_f = R_f i_f + L_f \frac{di_f}{dt} \]
- Stator L' not affected by rotor and no eddy currents in stator.

Armature Circuit
- Equation for Armature (Rotor) circuit is
  \[ V_a = R_a i_a + L_a \frac{di_a}{dt} + b \]

Mechanical Dynamics:
- Using Newton's 2nd Law to rotor.
- Motor drives some load, requiring torque \( T_L \) to operate.
- Frictional resistance in armature can be modelled as
  \[ J_m \frac{d\omega_m}{dt} = T_m - T_L - b_m \omega_m \]
**Assumptions:**
In obtaining equations for this dynamic model for the system, we have made several assumptions and approximations. In particular, we have either approximated or neglected the following factors:

1. Coulomb friction and associated *dead-band effects*.
2. Magnetic hysteresis (particularly in the stator core, but in the armature as well if not a brushless motor)
3. Magnetic saturation (in both stator and the armature)
4. Eddy current effects (laminated core reduces this effect)
5. Nonlinear constitutive *relations for magnetic induction (in which case inductance L is not constant)*
6. In split-ring and brush commutation, brush contact electrical resistance and friction, finite width contact of brushes, and other types of noise and nonlinearities
7. The effect of the rotor magnetic flux (armature flux) on the stator magnetic flux (field flux)
Steady-State Characteristics:
- In selecting a motor for a given application, its steady-state characteristics are a major determining factor.
- Steady-state torque-speed curves are employed for this purpose.
- The rationale is that, if the motor is able to meet the steady-state operating requirements, with some design conservatism, it should be able to tolerate some deviations under transient conditions of short duration.
- In the separately excited case shown in top figure, where the armature circuit and field circuit are excited by separate and independent voltage sources, it can be shown that the steady-state torque-speed curve is a straight line.

\[ V_f = R_f i_f + L_f \frac{di_f}{dt} \]  \( \text{(1)} \)

\[ V_a = R_a i_a + L_a \frac{di_a}{dt} + V_b \]  \( \text{(2)} \)

Take derivative and set \( \frac{di}{dt} = 0 \) ⇒ steady state conditions

\( \hat{e}_f \) is constant for fixed supply \( V_f \)  \( \text{(3)} \)

Also previously \( T_m = k_i f l_a = k_m l_a \) and \( V_b = k_i f \omega_m = k_m \omega_m \)

Substituting (4) and (5) into (2) We get \( V_a = \frac{R_a}{k_i f} T_m + k_i f \omega_m \)  \( \text{(4)} \)

Under steady state in field circuit from (1). \( V_f = i_f R_f \)

Torque speed may be expressed as:

\[ \frac{W_m + R_a R_f^2}{k^2 \eta^2 f} \frac{T_m}{k^2 f} = \frac{V_a}{k_m} \]

\[ \frac{W_m + T_m}{T_s} = 1 \]  \( \text{defining} \ T_s \ (\text{stalling/starting) Torque,} \ W_0 \ \text{(no-load speed & no-damping)} \)

Read Example 9.1 Pa. 667 Text
Output Power:

Output Power of a motor is given by:

\[ P = T_m \omega_m \]

Using:

\[ \frac{\omega_m}{\omega_0} + \frac{T_m}{T_s} = 1 \]

Magnetic Torque (stalling Torque)

\[ \text{no load speed} \]

Substitute \( \frac{\omega_m}{\omega_0} \) in 1:

\[ P = T_s \left(1 - \frac{\omega_m}{\omega_0}\right) \omega_m \]

Plot of Max Power: differentiate 3 w.r.t speed \( \omega_m \):

\[ \frac{dP}{d\omega_m} = T_s \left(1 - \frac{\omega_m}{\omega_0}\right) - \frac{T_s \omega_m}{\omega_0} = T_s \left(1 - 2 \frac{\omega_m}{\omega_0}\right) = 0 \]

\[ \frac{dP}{d\omega_m} = T_s \left(1 - 2 \frac{\omega_m}{\omega_0}\right) = 0 \]

\[ \omega_m(\text{max}) = \frac{\omega_0}{2} \]

Max Power is given by \( \frac{1}{2} \) no load speed.

\[ P_{\text{max}} = \frac{1}{4} T_s \omega_0 \]

Shown in graphic form as quadratic shape
Combined Excitation of Motor Windings:
The shape of the steady-state speed–torque curve will change if a common voltage supply is used to excite both the field windings and the armature windings. Here, the two windings have to be connected together.

There are three common ways the windings of the rotor and the stator are connected.
1. Shunt-wound motor
2. Series-wound motor
3. Compound-wound motor

- **In a shunt-wound motor**, the armature windings and the field windings are connected in parallel.
- **In the series-wound motor**, they are connected in series.
- **In the compound-wound motor**, part of the field windings is connected with the armature windings in series and the other part is connected in parallel.
- Note that in a shunt-wound motor at steady state, the back e.m.f. $v_b$ depends directly on the supply voltage.
- Since the back e.m.f. $v_b$ is proportional to the speed, it follows that speed controllability is good with the shunt-wound configuration.

In a series-wound motor,
- The relation between $v_b$ and the supply voltage is coupled through both the armature windings and the field windings.
- Hence its speed controllability is relatively poor.
- But in this case, a relatively large current flows through both windings at low speeds of the motor (when the back e.m.f. is small), giving a higher starting torque.
- Also, the operation is approximately at constant power in this case. These properties are summarized in Table.
- Since both speed controllability and higher starting torque are desirable characteristics, compound-wound motors are used to obtain a performance in between the two extremes.

<table>
<thead>
<tr>
<th>TABLE 9.1</th>
<th>Influence of the Winding Configuration on the Steady-State Characteristics of a DC Motor</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC Motor Type</td>
<td>Field Coil Resistance</td>
</tr>
<tr>
<td>Shunt-wound</td>
<td>High</td>
</tr>
<tr>
<td>Series-wound</td>
<td>Low</td>
</tr>
<tr>
<td>Compound-wound</td>
<td>Parallel high, series low</td>
</tr>
</tbody>
</table>