

ASSIGNMENT 2

DEADLINE: March 5, 2017, 23:59:59

P1- A design objective in most control system applications is to achieve small time constants. An exception is the time constant requirement for a piezoelectric sensor. Explain why a large time constant, in the order of 1.0 s, is desirable for a piezoelectric sensor in combination with its signal-conditioning circuit. (**Hint:** You can use the results of the following paragraph to explain this).

An equivalent circuit for a piezoelectric accelerometer, which uses a quartz crystal as the sensing element, is shown in the Fig. P1. The generated charge is denoted by q , and the output voltage at the end of the accelerometer cable is v_o . The piezoelectric sensor capacitance is modeled by C_p , and the overall capacitance experienced at the sensor output, whose primary contribution is due to cable capacitance, is denoted by C_c . The resistance of the electric insulation in the accelerometer is denoted by R . Write a differential equation relating v_o to q . What is the corresponding transfer function? Using this result, show that the accuracy of accelerometer improves when the sensor time constant is large and when the frequency of the measured acceleration is high. For a quartz crystal with $R = 1 \times 10^{11} \Omega$ and $C_p = 300 \text{ pF}$, and a circuit with $C_c = 700 \text{ pF}$, compute the time constant.

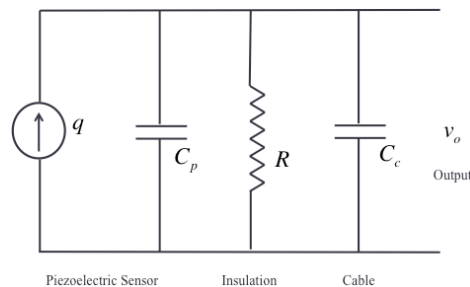


Fig P1.

P2- a. A standard accelerometer that weighs 100 gm is mounted on a test subject that has an equivalent mass of 3 kg . Estimate the accuracy in the first natural frequency of the object measured using this arrangement, considering mechanical loading due to accelerometer mass alone. If a miniature accelerometer that weighs 0.5 gm is used instead, what is the resulting accuracy?

b. A strain-gage accelerometer uses a semiconductor strain gage mounted at the roof of a cantilever element, with the seismic mass mounted at the free end of the cantilever. Suppose that the cantilever element has a square cross-section with dimensions $1.5 \times 1.5 \text{ mm}^2$. The equivalent length of the cantilever element is 25 mm , and the equivalent seismic mass is 0.2 gm . If the cantilever is made of an aluminum alloy with Young's modulus $E = 69 \times 10^9 \text{ N/m}^2$, estimate the useful frequency range of the accelerometer in Hertz.

Hint: When force f is applied to the free end of a cantilever, the deflection y at that location may be approximated by the formula

$$y = \frac{Fl^3}{3EI}$$

where l = cantilever length,

I = second moment area of the cantilever cross-section about the bending axis = $\frac{bh^3}{12}$,

b = cross-section width,

h = cross-section height.

P3- The sensitivity S_s of a strain gage consists of two parts: the contribution from the change in resistivity of the material and the direct contribution due to the change in shape of the strain-gage when deformed. Show that the second part may be approximated by $(1 + 2\nu)$, where ν denotes the Poisson's ratio of the strain gage material.

P4- What is meant by the term bridge sensitivity in strain-gage measurements? Describe methods of increasing bridge sensitivity. Assuming the load resistance to be very high in comparison with the arm resistances in the strain-gage bridge we had in the lecture notes, obtain an expression for the power dissipation p in terms of the bridge resistances and the supply voltage. Discuss how the limitation on power dissipation can affect bridge sensitivity.

P5- Consider a standard strain-gage bridge with R_1 as the only active gage and $R_3 = R_4$. Obtain an expression for R_1 in terms of R_2 , v_o , and v_{ref} . Show that when $R_1 = R_2$, we get $v_o = 0$ -a balanced bridge- as required. Note that the equation for R_1 , assuming that v_o is measured using a high-impedance voltmeter, can be used to detect large resistance changes in R_1 . Now suppose that the active gage R_1 is connected to the bridge using a long, twisted wire pair, with each wire having a resistance of R_c . The bridge circuit has to be modified as in Figure P5 in this case.

Using the expression obtained earlier for R_1 , show that the equation of the modified bridge is given by

$$R_1 = R_2 \left[\frac{v_{ref} + 2v_o}{v_{ref} - 2v_o} \right] + 4R_c \frac{v_o}{[v_{ref} - 2v_o]}$$

Obtain an expression for the fractional error in the R_1 measurement due to cable resistance R_c . Show that this error can be decreased by increasing R_2 and v_{ref} .

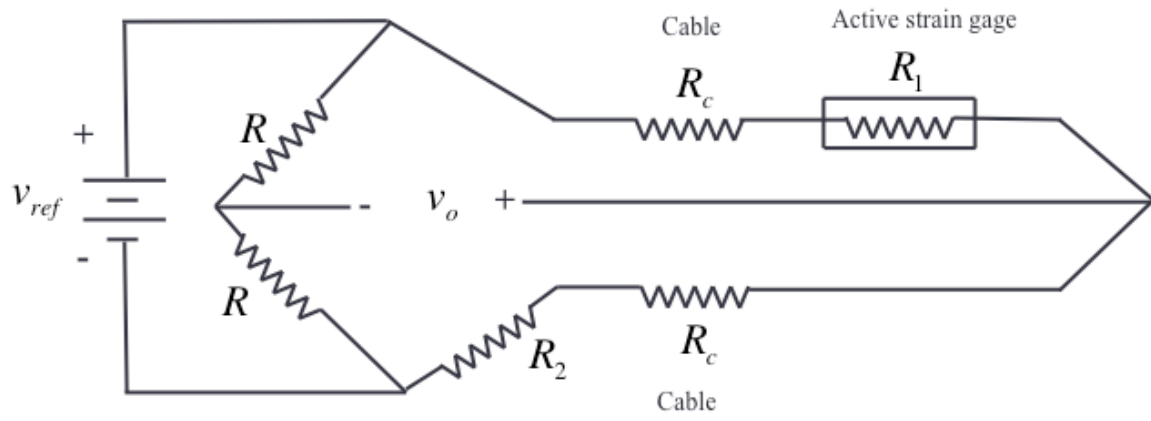


Fig P5.